Self-accelerating Massive Gravity: Exact solutions for any isotropic matter distribution

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We present an exact solution to the equations of massive gravity that display cosmological constant-like behavior for any spherically symmetric distribution of matter, including arbitrary time dependence. On this solution, the new degrees of freedom from the massive graviton generate a cosmological constant-like contribution to stress-energy that does not interact directly with other matter sources. When the effective cosmological constant contribution dominates over other sources of stress energy the cosmological expansion self-accelerates, even when no other dark-energy-like ingredients are present. The new degrees of freedom introduced by giving the graviton the mass do not respond to arbitrarily large radial or homogeneous perturbations from other matter fields on this solution. We comment on possible implications of this result.

I. INTRODUCTION

More than seventy years have elapsed since Pauli and Fierz made the first attempt at writing a theory of gravity with a massive graviton [1]. In the intervening years, daunting challenges to realizing such a theory have been found, including the scylla of incompatibility with Solar System tests [2, 3] and the charybdis of ghost-like degrees of freedom [4]. Recently, de Rham, Gabadadze, and Tolley have constructed a theory of massive gravity [5–7] that evades these dangers [8, 9]. This theory also contains a vacuum solution that recovers exactly a Schwarzschild-de Sitter solution [10, 11]. Moreover, in the flat matter dominated limit, the theory has a solution that responds to the presence of matter by producing an effective cosmological constant contribution to the stress tensor at the cost of introducing inhomogeneous solutions for the Stückelberg fields that describe the new degrees of freedom that come from massive gravity [12]. For an open universe, a related solution has been explicitly shown to evolve into self-acceleration [13].

In this paper, we generalize considerations in [12, 13] to an arbitrary spatially isotropic metric. We find cosmological constant type solutions in the presence of any isotropic distribution of matter. Such solutions connect the perturbative flat matter dominated solution [12] to the de Sitter solution [10, 11] allowing a cosmological expansion history identical to the ΛCDM model even in the presence of spherically symmetric matter perturbations.

II. MASSIVE GRAVITY

The covariant Lagrangian density for a theory of massive gravity will have, in addition to the usual Einstein-Hilbert term, a mass term, which we write as the potential $\mathcal{U}$,

$$\mathcal{L}_G = \frac{M_{\rm pl}^2}{2}\sqrt{-g}\left[R - \frac{m^2}{4}\mathcal{U}(g_{\mu\nu}, \mathcal{K}_{\mu\nu})\right]. \tag{1}$$

$M_{\rm pl}^2 = 1/8\pi G$ and $\hbar = c = 1$ throughout. $\mathcal{K}_{\mu\nu}$ is a tensor that characterizes metric fluctuations away from a fiducial (flat) space time. At the linearized level, the potential must take on the Fierz-Pauli structure to be ghost free; but any purely linear theory will exhibit the vDVZ discontinuity [2, 3], where an additional helicity mode couples to matter even in the $m \to 0$ limit. Nonlinear extensions to the Fierz-Pauli potential can evade this problem via a strong coupling phenomenon known as the Vainshtein mechanism [14], where the extra coupling is suppressed near matter sources. However, these extensions typically contain an unhealthy ghost-like degree of freedom [4].

For a theory of massive gravity to be free from this ghost, the potential term must take a special form built out of expressions that have the form of total derivatives in absence of dynamics [7]. These can be written as contractions of the tensor

$$\mathcal{K}^\mu{}_\nu = \delta^\mu{}_\nu - \sqrt{\mathbf{\Sigma}}^\mu{}_\nu. \tag{2}$$

The matrix $\sqrt{\mathbf{\Sigma}}$ is understood to denote $\sqrt{\mathbf{\Sigma}}^\mu{}_\alpha\sqrt{\mathbf{\Sigma}}^\alpha{}_\nu \equiv \Sigma^\mu{}_\nu$. The potential-generating matrix is defined as

$$\Sigma^\mu{}_\nu \equiv g^{\mu\alpha}\partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab} \equiv g^{\mu\alpha}\Sigma_{\alpha\nu}, \tag{3}$$

where ϕ^a are the 4 Stückelberg fields introduced to restore diffeomorphism invariance. The ϕ^a fields transform as scalars, while Σ, $\sqrt{\mathbf{\Sigma}}$ and $\mathcal{K}$ transform as tensors under general coordinate transforms.

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In matrix notation, the potential can be written [5–7]

$$
\begin{aligned}
-\frac{\mathcal{U}}{4} &= [\mathcal{K}]^2 - [\mathcal{K}^2] \\
&\quad + \alpha_3\left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right) \\
&\quad + \alpha_4\big([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 8[\mathcal{K}][\mathcal{K}^3] \\
&\qquad + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]\big),
\end{aligned} \tag{4}
$$

where brackets denote traces, $[\mathbf{A}] \equiv A^\mu{}_\mu$, and α_3, α_4 are free parameters. Using Eq. (2), we can reexpress the potential in terms of traces of products of $\sqrt{\mathbf{\Sigma}}$

$$
\begin{aligned}
\frac{\mathcal{U}}{4} &= -12 + 6[\sqrt{\mathbf{\Sigma}}] + [\mathbf{\Sigma}] - [\sqrt{\mathbf{\Sigma}}]^2 \\
&\quad + \alpha_3\Big(-24 + 18[\sqrt{\mathbf{\Sigma}}] - 6[\sqrt{\mathbf{\Sigma}}]^2 + [\sqrt{\mathbf{\Sigma}}]^3 \\
&\qquad - 3[\mathbf{\Sigma}]([\sqrt{\mathbf{\Sigma}}] - 2) + 2[\mathbf{\Sigma}^{3/2}]\Big) \\
&\quad + \alpha_4\Big(-24 + 24[\sqrt{\mathbf{\Sigma}}] - 12[\sqrt{\mathbf{\Sigma}}]^2 - 12[\sqrt{\mathbf{\Sigma}}][\mathbf{\Sigma}] \\
&\qquad + 6[\sqrt{\mathbf{\Sigma}}]^2[\mathbf{\Sigma}] + 4[\sqrt{\mathbf{\Sigma}}]^3 + 12[\mathbf{\Sigma}] - 3[\mathbf{\Sigma}]^2 \\
&\qquad - 8[\mathbf{\Sigma}^{3/2}]([\sqrt{\mathbf{\Sigma}}] - 1) + 6[\mathbf{\Sigma}^2] - [\sqrt{\mathbf{\Sigma}}]^4\Big).
\end{aligned} \tag{5}
$$

Variation of the action with respect to the metric yields the modified Einstein equations

$$
G_{\mu\nu} = m^2 T^{(\mathcal{K})}_{\mu\nu} + \frac{1}{M_{\rm pl}^2} T^{(m)}_{\mu\nu}, \tag{6}
$$

where $G_{\mu\nu}$ is the usual Einstein tensor and $T^{(m)}_{\mu\nu}$ is the matter stress energy tensor. Here

$$
\begin{aligned}
T^{(\mathcal{K})}_{\mu\nu} &= \frac{1}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}\sqrt{-g}\,\frac{\mathcal{U}}{4} \\
&= -\frac{1}{2}\Big\{\frac{\mathcal{U}}{4} g_{\mu\nu} - 2\Sigma_{\mu\nu} - 2(3 - [\sqrt{\mathbf{\Sigma}}])\sqrt{\mathbf{\Sigma}}_{\mu\nu} \\
&\quad + \alpha_3\Big[-3\left(6 - 4[\sqrt{\mathbf{\Sigma}}] + [\sqrt{\mathbf{\Sigma}}]^2 - [\mathbf{\Sigma}]\right)\sqrt{\mathbf{\Sigma}}_{\mu\nu} \\
&\qquad + 6\left([\sqrt{\mathbf{\Sigma}}] - 2\right)\Sigma_{\mu\nu} - 6\mathbf{\Sigma}^{3/2}_{\mu\nu}\Big] \\
&\quad + \alpha_4\Big[-24\left(\mathbf{\Sigma}^2_{\mu\nu} - ([\sqrt{\mathbf{\Sigma}}] - 1)\mathbf{\Sigma}^{3/2}_{\mu\nu}\right) \\
&\qquad - 12\left(2 - 2[\sqrt{\mathbf{\Sigma}}] - [\mathbf{\Sigma}] + [\sqrt{\mathbf{\Sigma}}]^2\right)\Sigma_{\mu\nu} \\
&\qquad - \Big(24 - 24[\sqrt{\mathbf{\Sigma}}] + 12[\sqrt{\mathbf{\Sigma}}]^2 - 4[\sqrt{\mathbf{\Sigma}}]^3 \\
&\qquad -12[\mathbf{\Sigma}] + 12[\mathbf{\Sigma}][\sqrt{\mathbf{\Sigma}}] - 8[\mathbf{\Sigma}^{3/2}]\Big)\sqrt{\mathbf{\Sigma}}_{\mu\nu}\Big]\Big\}
\end{aligned} \tag{7}
$$

is the dimensionless effective stress energy tensor provided by the mass term. Note that this effective stress energy depends explicitly on the metric itself. To solve the modified Einstein equation, we first parameterize the metric and then solve for the joint effect of the matter and mass term.

III. EXACT SOLUTION

Generalizing [12, 13], we consider an arbitrary spatially isotropic metric,

$$
ds^2 = -b^2(r,t)dt^2 + a^2(r,t)(dr^2 + r^2 d\Omega^2). \tag{8}
$$

We correspondingly take a spherically symmetric ansatz for the Stückelberg fields:

$$
\begin{aligned}
\phi^0 &= f(t,r), \\
\phi^i &= g(t,r)\frac{x^i}{r},
\end{aligned} \tag{9}
$$

and look for solutions to the functions $g(t,r)$ and $f(t,r)$. The potential matrix (3) then takes the form

$$
\mathbf{\Sigma} = \begin{pmatrix}
\dfrac{\dot f^2 - \dot g^2}{b^2} & \dfrac{\dot f f' - \dot g g'}{b^2} & 0 & 0 \\
\dfrac{\dot g g' - \dot f f'}{a^2} & \dfrac{-f'^2 + g'^2}{a^2} & 0 & 0 \\
0 & 0 & \dfrac{g^2}{a^2 r^2} & 0 \\
0 & 0 & 0 & \dfrac{g^2}{a^2 r^2}
\end{pmatrix}, \tag{10}
$$

where primes denote derivatives with respect to r and overdots with respect to t.

The resulting, rather involved, calculation is made easier by isolating the upper-left-hand 2×2 submatrix of $\mathbf{\Sigma}$ and using the Cayley-Hamilton theorem, which states that a matrix solves its own characteristic polynomial. For a 2×2 matrix $\mathbf{A}$, this means

$$
[\mathbf{A}]\mathbf{A} = \mathbf{A}^2 + (\det\mathbf{A})\,\mathbf{I}_2,
$$

where $\mathbf{I}_2$ is the 2×2 identity matrix. We can then use that $\det\mathbf{A}^n = (\det\mathbf{A})^n$ to find the square root of $\mathbf{\Sigma}_2$, the upper-left-hand 2×2 submatrix of $\mathbf{\Sigma}$:

$$
\sqrt{\mathbf{\Sigma}_2} = \frac{1}{\sqrt{X}}\left[\mathbf{\Sigma}_2 + W\mathbf{I}_2\right], \tag{11}
$$

where

$$
\begin{aligned}
X &\equiv \left(\frac{\dot f}{b} + \mu\frac{g'}{a}\right)^2 - \left(\frac{\dot g}{b} + \mu\frac{f'}{a}\right)^2, \\
W &\equiv \frac{\mu}{ab}\left(\dot f g' - \dot g f'\right),
\end{aligned} \tag{12}
$$

and $\mu = \mathrm{sgn}(\dot f g' - \dot g f')$.

With Eq. (11), traces of $\mathbf{\Sigma}^n$ become

$$
\begin{aligned}
[\sqrt{\mathbf{\Sigma}}] &= \sqrt{X} + \frac{2g}{ar}, \\
[\mathbf{\Sigma}] &= X - 2W + \frac{2g^2}{a^2r^2}, \\
[\mathbf{\Sigma}^{3/2}] &= X^{3/2} - 3W\sqrt{X} + \frac{2g^3}{a^3r^3}, \\
[\mathbf{\Sigma}^2] &= X^2 - 2W(2X - W) + \frac{2g^4}{a^4r^4},
\end{aligned} \tag{13}
$$

and the potential is given by

$$\frac{\mathcal{U}}{4} = P_0\left(\frac{g}{ar}\right) + \sqrt{X} P_1\left(\frac{g}{ar}\right) + W P_2\left(\frac{g}{ar}\right), \quad (14)$$

where the P_n polynomials are

$$\begin{aligned} P_0(x) &= -12 - 2x(x-6) - 12(x-1)(x-2)\alpha_3 \\ &\quad - 24(x-1)^2\alpha_4, \\ P_1(x) &= 2(3-2x) + 6(x-1)(x-3)\alpha_3 + 24(x-1)^2\alpha_4, \\ P_2(x) &= -2 + 12(x-1)\alpha_3 - 24(x-1)^2\alpha_4. \end{aligned} \quad (15)$$

Varying the action with respect to f and g yields the Stückelberg field equations

$$\begin{aligned} &\partial_t \left[\frac{a^3 r^2}{\sqrt{X}} \Big(\frac{\dot{f}}{b} + \mu \frac{g'}{a} \Big) P_1 + \mu a^2 r^2 g' P_2 \right] \\ &- \partial_r \left[\frac{a^2 b r^2}{\sqrt{X}} \Big(\mu \frac{\dot{g}}{b} + \frac{f'}{a} \Big) P_1 + \mu a^2 r^2 \dot{g} P_2 \right] = 0, \end{aligned} \quad (16)$$

and

$$\begin{aligned} &-\partial_t \left[\frac{a^3 r^2}{\sqrt{X}} \Big(\frac{\dot{g}}{b} + \mu \frac{f'}{a} \Big) P_1 + \mu a^2 r^2 f' P_2 \right] \\ &+ \partial_r \left[\frac{a^2 b r^2}{\sqrt{X}} \Big(\mu \frac{\dot{f}}{b} + \frac{g'}{a} \Big) P_1 + \mu a^2 r^2 \dot{f} P_2 \right] \\ &= a^2 b r \left[P_0' + \sqrt{X} P_1' + W P_2' \right], \end{aligned} \quad (17)$$

where $P_n'(x) \equiv dP_n/dx = ar\partial P/\partial g$. By inspection, we find that a solution to the f equation of motion, Eq. (16), is given by $P_1(x_0) = 0$, or

$$x_0 = \frac{1 + 6\alpha_3 + 12\alpha_4 \pm \sqrt{1 + 3\alpha_3 + 9\alpha_3^2 - 12\alpha_4}}{3(\alpha_3 + 4\alpha_4)}, \quad (18)$$

and hence $g = x_0 ar$. Note that if $\alpha_3 = \alpha_4 = 0$, $P_1(x)$ becomes linear and $g = 3ar/2$ is the solution.

The equation of motion for g evaluated on the solution provides a constraint on f

$$\sqrt{X} P_1' = \left(\frac{2P_2}{x_0} - P_2' \right) W - P_0', \quad (19)$$

where the P_n functions are evaluated at x_0 and we have used the fact that

$$W = \frac{\mu}{b} \Big(\dot{f} + \frac{a'}{a} r \dot{f} - \frac{\dot{a}}{a} r f' \Big) x_0. \quad (20)$$

An explicit solution for f is not required for the computation of the stress energy tensor. After straightforward but tedious algebra, we find that its nonzero components are:

$$\begin{aligned} T^{\mathcal{K}}_{00} &= \frac{1}{2} P_0(x_0) b^2, \\ T^{\mathcal{K}}_{rr} &= -\frac{1}{2} P_0(x_0) a^2, \\ T^{\mathcal{K}}_{\theta\theta} = \frac{T^{\mathcal{K}}_{\phi\phi}}{\sin^2\theta} &= -\frac{1}{2} P_0(x_0) a^2 r^2. \end{aligned} \quad (21)$$

The $T^{\mathcal{K}}_{00} = -T^{\mathcal{K}}_{rr}$ pieces can be easily checked from Eq. (14) by direct variation with respect to g^{tt} and g^{rr}, noting that the polynomial pieces come from the angular metric. The angular pieces can be similarly analyzed by separately tracking the equal θ and ϕ contributions to $2(g/ar)^n$ terms in the traces of Eq. (13). Their separate variations can then be reduced with Eq. (19).

Hence, the effective energy density and pressure are

$$(m^2 M_{\rm pl}^2) T^{\mu(\mathcal{K})}_{\ \nu} = \begin{pmatrix} -\rho_{\mathcal{K}} & 0 & 0 & 0 \\ 0 & p_{\mathcal{K}} & 0 & 0 \\ 0 & 0 & p_{\mathcal{K}} & 0 \\ 0 & 0 & 0 & p_{\mathcal{K}} \end{pmatrix}, \quad (22)$$

where

$$\rho_{\mathcal{K}} = -p_{\mathcal{K}} = \frac{1}{2} m^2 M_{\rm pl}^2 P_0(x_0). \quad (23)$$

This shows that a cosmological constant type solution exists for general isotropic metrics. Conversely, the modified Einstein equation for arbitrary spherically symmetric distributions of matter becomes the ordinary Einstein equation plus a cosmological constant on this solution.

For example, the spatially flat FRW space-time is a subset where $a(r,t) = a(t)$ is the scale factor, $b(r,t) = 1$ and the modified Einstein equation (6) just becomes the usual Friedmann equation

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{1}{3M_{\rm pl}^2} (\rho_{\mathcal{K}} + \rho_m). \quad (24)$$

The open or closed FRW space-time is also included with a suitable conformal factor $a(r,t) \neq a(t)$. Note that for the FRW metric, this solution applies for radiation and matter domination as well as for a self-accelerated epoch where the massive graviton itself provides the cosmological constant-like dark energy. It also allows for arbitrary isotropic perturbations around the FRW metric with

$$\begin{aligned} a^2(r,t) &= a^2(t)[1 + 2\Phi(r,t)], \\ b^2(r,t) &= [1 + 2\Psi(r,t)]. \end{aligned} \quad (25)$$

Thus the solution remains of the cosmological constant type for arbitrary spherically symmetric matter distributions. Furthermore, the matter only sees the effects of the mass term as a cosmological constant with no direct coupling to the Stückelberg fields on the exact solution.

It is straightforward to verify that in vacuum ($T^{(m)}_{\mu\nu}$=0) our solution recovers exactly the static Schwarzschild-de Sitter solution from [10, 11] and is in agreement with the approximate self-accelerating solution found in the decoupling limit [15]. It is also similar to other Schwarzschild-de Sitter [16, 17] and open universe [13] solutions.

IV. DISCUSSION

The solution we have found is a perfect analog for a cosmological constant. Because the solution exists for any isotropic distribution of matter, it recovers static solutions like Schwarzschild-de Sitter in vacuum and generalizes them to dynamical cases such as the FRW cosmology. In each of these cases, the presence of other isotropic sources of stress-energy does not alter the cosmological constant-like behavior of massive gravity. Hence, we can have a truly self-accelerating gravitational background that coexists peacefully with a standard cosmological history; the self acceleration begins in precisely the same manner as cosmological-constant-driven acceleration would begin, only here the size of the apparent cosmological constant is set by the graviton mass and the other free parameters of the theory (α_3 and α_4).

In this paper, we have restricted ourselves to isotropic situations. Note that although we have assumed our Stückelberg fields to be in a radially symmetric configuration, their effective center in space disappears in their effective stress energy, which is homogeneous and isotropic. Indeed, we can recover fully homogeneous background solutions supported by the Stückelberg fields. This suggests that perhaps even more general inhomogeneity in the matter fields does not drive inhomogeneity in the observable effective stress energy of the Stückelberg fields.

On general solutions of massive gravity, we expect the new degrees of freedom present in the Stückelberg fields to "mix" with the usual gravitational degrees of freedom. That is, we would expect to find cross-talk between the helicity-2 parts of the graviton and the helicity-0 part of the graviton. Since we are working in Jordan frame in this paper, this mixing would result in a direct coupling between the new degrees of freedom and matter sources in Einstein frame. However, on the background solution we have found, this mixing vanishes, despite the presence of arbitrarily large radial perturbations in the metric. This result appears to be a generalization of a similar finding for perturbations around the vacuum self-accelerating solutions in the decoupling limit [15]. The physical upshot of this finding is that the new gravitational degrees of freedom captured by the Stückelberg fields do not interact directly with radial perturbations in matter sources. In the decoupling limit, where these perturbations are easier to study, this lack of interaction extends to all first order perturbations. In practice, this suggest that there may be no easily measurable deviations from GR around these self-accelerating solutions – the gravitational-strength fifth forces that usually appear in modifications to gravity are absent here.

Moreover, on our solution the Stückelberg fields do not appear to have any kinetic terms in their action. Indeed, we have found our solution (Eq. 18) precisely by demanding that the prefactor of the kinetic energy terms in the action vanish. If we look to the decoupling limit results for guidance, we might expect that this lack of a kinetic term will not persist when we study arbitrary perturbations around our solution. However, general scalar perturbations around the related self-accelerating solutions found in [13] are also found to have no kinetic terms in [18]; see also [19]. In light of these considerations, a careful study of general anisotropic perturbations to this solution will be an important area for future work.

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