The Astrophysical Journal, 504:L57–L60, 1998 September 10

COSMIC COMPLEMENTARITY: H_0 AND Ω_m FROM COMBINING COSMIC MICROWAVE BACKGROUND EXPERIMENTS AND REDSHIFT SURVEYS

Daniel J. Eisenstein, Wayne Hu, and Max Tegmark[1]
Institute for Advanced Study, Princeton, NJ 08540
Received 1998 May 19; accepted 1998 July 9; published 1998 July 31

ABSTRACT

We show that the detection of acoustic oscillations in both upcoming cosmic microwave background (CMB) satellite experiments and large-redshift surveys can yield 5% determinations of H_0 and Ω_m, an order-of-magnitude improvement over CMB data alone. CMB anisotropies provide the sound horizon at recombination as a standard ruler. For reasonable baryon fractions, this scale is imprinted on the galaxy power spectrum as a series of spectral features. Measuring these features in redshift space determines the Hubble constant, which in turn yields Ω_m once combined with CMB data. Since the oscillations in both power spectra are frozen-in at recombination, this test is insensitive to low-redshift cosmology.

Subject headings: cosmic microwave background — cosmology: theory — dark matter — large-scale structure of universe

1. INTRODUCTION

In the usual cosmological paradigm, the cosmic microwave background (CMB) contains a vast amount of information about cosmological parameters (Hu, Sugiyama, & Silk 1997). With upcoming experiments, most notably the two satellite missions *Microwave Anisotropy Probe* (*MAP*)[2] and *Planck*,[3] detailed measurements of the angular power spectra of its anisotropy and polarization may accurately determine many cosmological parameters (Jungman et al. 1996; Bond, Efstathiou, & Tegmark 1997; Zaldarriaga, Spergel, & Seljak 1997). However, certain changes in the cosmological parameters can conspire to leave the CMB power spectra unchanged, resulting in degenerate directions in the parameter space (Bond et al. 1994, 1997; Zaldarriaga et al. 1997; Huey et al. 1998). For example, since the Hubble constant H_0 and the matter density Ω_m can be varied while keeping the angular diameter distance and the matter-radiation ratio fixed, their values remain uncertain but highly correlated. Such degeneracies must be broken with cosmological information from other sources.

Upcoming redshift surveys for the study of large-scale structure hold the potential for resolving this issue. In particular, the 2dF survey[4] and the Sloan Digital Sky Survey (SDSS)[5] should measure the galaxy power spectrum on large enough scales to allow detailed comparisons with the mass power spectra predicted by cosmological theories. In this Letter and a companion paper (Eisenstein, Hu, & Tegmark 1998b, hereafter EHT), we explore the potential of combining redshift surveys and CMB anisotropy data for the purpose of parameter estimation. Here we focus on the dramatic improvement possible in the measurement of H_0 and Ω_m. Neither data set yields tight limits by itself, yet together they could yield errors better than 5% on H_0 and 10% on Ω_m.

The key to this improvement is the presence of features in the matter power spectrum on scales exceeding 60 h^{-1} Mpc. With a nonnegligible baryon fraction, the acoustic oscillations that exist before recombination are imprinted not only on CMB anisotropies but also on the linear power spectrum (Holtzman 1989; Hu & Sugiyama 1996; Eisenstein & Hu 1998a). CMB anisotropies accurately calibrate their characteristic length scale; measurement of this standard ruler in the redshift survey power spectrum yields H_0. With this added information, the CMB returns a significantly more precise measure of Ω_m.

2. METHODOLOGY

We seek to quantify the potential sensitivity of these data sets to various cosmological parameters. For this, we use the Fisher matrix formalism (see Tegmark, Taylor, & Heavens 1997 for a review), which yields a lower limit on the statistical errors on cosmological parameters achievable by a set of experiments. This formalism operates within the context of a parameterized cosmological model. For this, we use a 12-variable parameterization of the adiabatic cold dark matter (CDM) model, described in detail in EHT. It includes CDM, baryons, massive neutrinos, a cosmological constant Ω_Λ, curvature Ω_K ($\equiv 1 - \Omega_\Lambda - \Omega_m$), the Hubble constant $H_0 \equiv 100\ h$ km s^{-1} Mpc^{-1}, a reionization optical depth, and a primordial helium fraction. It assumes an initial scalar power spectrum $P_i(k) \propto k^{n_S+\alpha \log(k/k_p)}$ ($k_p \equiv 0.025$ Mpc^{-1}) with tilt n_S, a logarithmic running of the tilt α, and an unknown amplitude as well as scale-invariant tensor contributions with an unconstrained amplitude. Finally, it allows an unknown linear bias to adjust the galaxy power spectrum relative to the mass ($P_{\rm gal} = b^2 P_{\rm mass}$). All of the above parameters are determined simultaneously from the data.

For CMB anisotropies, we use the experimental specifications of the *MAP* and *Planck* satellites for temperature and polarization (EHT). We assume that foregrounds and systematics can be eliminated with negligible loss of cosmological information. For large-scale structure, we use the projected specifications of the bright red galaxy (BRG) sample of the SDSS to determine its sensitivity to the linear power spectrum (Tegmark 1997). On small scales, the observed power spectrum reflects nonlinear effects and galaxy formation issues. We therefore employ only wavenumbers less than $k_{\rm max} = 0.1\ h$ Mpc^{-1}, under the assumption that the linear power spectrum on these scales can be reconstructed (up to the unknown linear bias)

[1] Hubble Fellow.

[2] See http://map.gsfc.nasa.gov, maintained at NASA by D. N. Spergel and G. Hinshaw.

[3] See http://astro.estec.esa.nl/SA-general/Projects/Planck, maintained at ESA by J. Tauber.

[4] See http://meteor.anu.edu.au/~colless/2dF, maintained at MSSSO by M. Colless.

[5] See http://www.astro.princeton.edu/BBOOK/, maintained at Princeton University by R. Lupton.

TABLE 1
ERRORS ON H_0 AND Ω_m FOR ΛCDM

EXPERIMENT	ΔH_0 General	ΔH_0 Flat	$\Delta\Omega_m$ General	$\Delta\Omega_m$ Flat
MAP (no polarization)	135	15	1.4	0.23
With SDSS	3.0	2.5	0.042	0.037
MAP (with polarization)	23	6.7	0.25	0.10
With SDSS	2.9	2.4	0.037	0.036
Planck (no polarization)	113	5.3	1.2	0.079
With SDSS	2.5	2.3	0.035	0.035
Planck (with polarization)	13	1.6	0.14	0.024
With SDSS	2.2	1.4	0.027	0.020

NOTE.—The fiducial model has $\Omega_m = 0.35$, $H_0 = 65$ km s^{-1} Mpc^{-1}, and $\Omega_B = 0.05$. We use $k_{\rm max} = 0.1$ h Mpc^{-1} for SDSS. Errors are 1 σ; ΔH_0 errors are in units of km s^{-1} Mpc^{-1}. General: Ω_K estimated from data. Flat: $\Omega_K = 0$ by fiat.

despite the effects of extinction, evolution, and redshift-space distortions (Hamilton 1998; Tegmark et al. 1998b). We vary $k_{\rm max}$ in § 4.2.

3. CMB RESULTS

Parameter degeneracies occur when changes in the model parameters leave the power spectra essentially unchanged relative to the size of the experimental uncertainties. In particular, since cosmic variance is substantial at large angular scales, changes that affect large angles while leaving the acoustic peaks unchanged will be difficult to detect.

The angular diameter distance d_A to the last scattering surface contains the most important degeneracy for the present discussion. The CMB acoustic peaks are a high-redshift pattern viewed at distance d_A. The pattern may be held fixed by keeping $\Omega_m h^2$ and the baryon density $\Omega_B h^2$ constant. However, d_A depends on the low-redshift effects of a cosmological constant or curvature. Changing Ω_Λ and Ω_K so as to keep the angular diameter distance constant leaves the acoustic peaks unchanged. Only large-angle ($\ell \lesssim 50$) gravitational redshift effects or small-angle ($\ell \gtrsim 1000$) gravitational lensing effects can resolve this ambiguity.

In short, the CMB data sets will yield precision information on the physical properties at high redshift, notably $\Omega_m h^2$, $\Omega_B h^2$, and $d_A(\Omega_m h^2, \Omega_\Lambda, \Omega_K)$, but not on H_0 and Ω_m individually. A similar situation occurs in quintessence models with trade-offs between Ω_Q and the equation of state of the Q-field (Huey et al. 1998).

In Table 1, we present the error bars on H_0 and Ω_m attainable by upcoming CMB satellite experiments within our 12-dimensional parameter space. One sees that when varying both Ω_Λ and Ω_K, the constraints on H_0 and Ω_m are poor, although the polarization information does provide considerable help. Even if one assumes a flat cosmological model ($\Omega_K = 0$), *MAP*, with its partial coverage of the acoustic peaks, will not yield tiny errors on H_0 and Ω_m.

There are several caveats. The Fisher matrix expansion of the likelihood function is not accurate for large steps in parameter space, which means that large error bars accurately detect a degenerate direction but may inaccurately reflect its magnitude (Zaldarriaga et al. 1997). One artifact of this is that the ellipses in Figure 1 follow straight lines rather than curves (e.g., constant $\Omega_m h^2$ in the CMB case). Moreover, the errors are slightly overestimated in the case of *Planck* because we have not included gravitational lensing (Seljak 1996; Metcalf & Silk 1997), by which the differences in growth factor between otherwise degenerate models will alter the small-angle power spectra. Nevertheless, the point remains that the CMB alone will not constrain H_0 and Ω_m to a level capable of strong consistency checks against other cosmological tests.

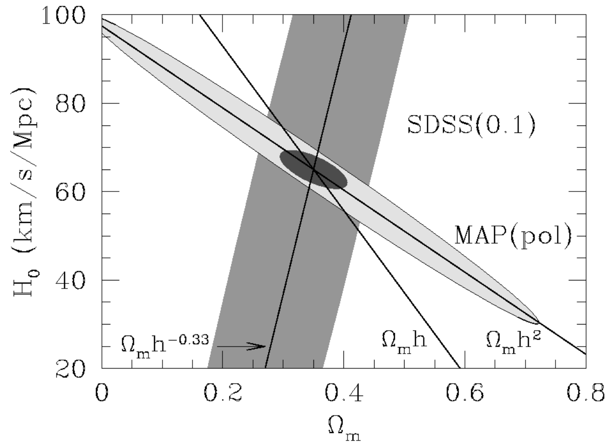

FIG. 1.—The 68% allowed regions for *MAP* (with polarization) alone, SDSS ($k_{\rm max} = 0.1$ h Mpc^{-1}) alone, and the two combined. The lines in the directions of constant $\Omega_m h^2$, $\Omega_m h$, and $\Omega_m h^{-0.33}$ are shown.

4. RESULTS WITH REDSHIFT SURVEYS

4.1. *Linear Analysis*

When we include the Fisher information matrix from SDSS, the error bars on H_0 and Ω_m drop by an order of magnitude. In Table 1, we see that for a fiducial ΛCDM model, the errors on H_0 are below 3 km s^{-1} Mpc^{-1} while those on Ω_m are around 0.035. EHT discuss improvements on other parameters of the model; however, none are nearly as dramatic. Figure 1 displays the situation. Note that the results are roughly independent of whether polarization information is available or not.

The reason for this dramatic improvement lies with the baryons. Table 2 shows the error bars on H_0 for a sequence of fiducial models with increasing baryon fraction; those on Ω_m behave similarly. Increasing the baryon fraction from ~1% to ~15% results in a dramatic increase in the information provided by SDSS. A baryon fraction exceeding 10% is strongly indicated by cluster gas fractions (White et al. 1993; David, Jones, & Forman 1995; White & Fabian 1995; Evrard 1997).

As the baryon fraction increases, significant acoustic oscillations develop in the matter power spectrum (see Fig. 2, Hu & Sugiyama 1996, and Eisenstein & Hu 1998a). There is a characteristic scale of these oscillations that is known as the sound horizon; morphologically, the power spectrum acquires a sharp break near the sound horizon and a series of oscillations marking the harmonics of this scale. The size of the sound horizon can be calculated given knowledge of the physical conditions at high redshift, particularly $\Omega_m h^2$ and $\Omega_B h^2$. Since

TABLE 2
ERRORS ON H_0 AS FUNCTION OF Ω_B

Ω_B	Ω_B/Ω_m (%)	*MAP* (with polarization)	With SDSS
0.005	1.4	36	12
0.02	5.7	29	9.2
0.05	14	23	2.9
0.10	29	24	1.3

NOTE.—Same parameters as in Table 1 save for the baryon fraction.

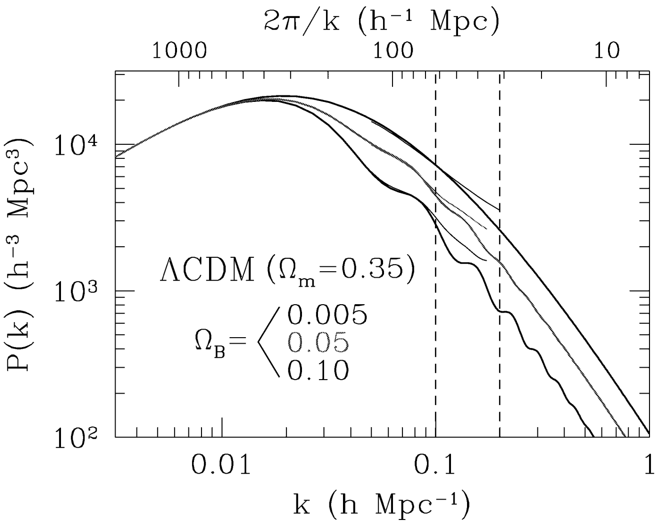

FIG. 2.—Power spectra for three ΛCDM models, showing a progression of Ω_B (0.005, 0.05, and 0.1). The spectra are normalized on large scales. The thin curves show the power spectrum extended to second order (Jain & Bertschinger 1994), assuming $\sigma_8 = 1$ for the linear power spectrum. The perturbation formalism uses $\Omega_m = 1$, but this is a good approximation (Bernardeau 1994).

these are exactly the quantities that are well constrained by the relative heights of the CMB acoustic peaks, we can accurately infer the scale of these features in real space. Its measurement in the redshift-space power spectrum then yields an accurate measure of H_0.

At low baryon fractions, the SDSS power spectrum still reduces the error bars on H_0 and Ω_m. This results from the one scale left in the matter power spectrum, that of the horizon at matter-radiation equality. In redshift space, this yields a measure of $\Gamma \equiv \Omega_m h$, with considerable inaccuracies due to confusion with scalar tilt. However, the fact that the SDSS ellipse in Figure 1 lies along a line very different from constant $\Omega_m h$ indicates that at moderate baryon fractions, this feature is not providing the primary leverage on H_0. Note that the break and oscillation morphology of the baryonic features cannot be mimicked by the effects of tilt or massive neutrinos.

In short, baryonic features yield a standard ruler, whose length can be accurately inferred by the CMB and can be measured in redshift space using large-redshift surveys. The comparison of lengths yields H_0; combining this with $\Omega_m h^2$ gives Ω_m. This inference is independent of the angular diameter distance to last scattering (provided that the peaks are visible at all!), so this method will function regardless of cosmological constant, spatial curvature, or more exotic smooth components (see, e.g., Turner & White 1997). With Ω_m known, the location of the CMB peaks and the information from Type Ia supernovae (Perlmutter et al. 1998; Riess et al. 1998) may be focused on distinguishing these low-redshift effects (Tegmark et al. 1998a; Hu et al. 1998).

TABLE 3

ERRORS ON H_0 FOR DIFFERING SDSS ASSUMPTIONS

	ΔH_0		$\Delta\Omega_m$	
k_{max}	$P(k)$	$P_S(k)$	$P(k)$	$P_S(k)$
MAP alone	23	23	0.25	0.25
0.025	16	15	0.17	0.16
0.05	9.7	10.7	0.098	0.11
0.1	2.9	10.0	0.037	0.11
0.2	1.2	9.0	0.016	0.10
0.4	0.9	8.6	0.014	0.10

NOTE.—*MAP* with polarization has been taken in each case. Limits with the actual linear power spectrum $P(k \le k_{max})$ are compared with those from a smoother analytic form $P_S(k \le k_{max})$ (see text). Same model and notation as in Table 1.

4.2. *Nonlinearities*

Baryonic oscillations are a feature of the *linear* power spectrum. Nonlinear evolution erases these signatures. Hence, we should test the extent to which the above results depend upon our use of linear theory.

Heretofore, we have assumed that we could use the linear power spectrum on scales longward of $k_{max} = 0.1\ h$ Mpc^{-1}. This is close to the point at which nonlinearities will smooth the oscillations. We therefore display in Table 3 the effect of altering k_{max}, again simply ignoring information on all smaller scales. For several fiducial models, we find that moving k_{max} from 0.1 to 0.2 h Mpc^{-1} decreases the errors on H_0 and Ω_m by about a factor of 2.5. However, these gains saturate as one extends k_{max} to 0.4 h Mpc^{-1}; the acoustic oscillations there are of such small amplitude that little information is gained.

We also consider an alternative formulation in which we model the matter power spectrum by a fitting formula (Eisenstein & Hu 1998b) that includes the break at the sound horizon but not the oscillations. The resulting errors are shown in Table 3. For $k_{max} \gtrsim 0.08\ h$ Mpc^{-1}, the performance is significantly worse than that achieved with the actual linear power spectrum. This is close to the location of the first bump in this fiducial model. Note that including the featureless power spectrum on scales from 0.1 to 0.4 h Mpc^{-1} adds very little additional information on H_0 or Ω_m.

We therefore conclude that detection of at least the first of the acoustic oscillations ($k \approx 0.07\ h$ Mpc^{-1} in this model) is critical to enabling a precision measure of H_0 and Ω_m. Detecting additional peaks improves the possible error bars, but with diminishing returns, because the oscillations damp down in amplitude. In Figure 2, we present the results of second-order perturbation theory for the power spectrum (Jain & Bertschinger 1994) assuming a linear power spectrum normalization of $\sigma_8 = 1.0$. For this value, linear theory indeed holds to $k_{max} \approx 0.1\ h$ Mpc^{-1}, so that the first baryonic peak survives while higher peaks are smeared out. The second-order correction scales as σ_8^2; note that a linear $\sigma_8 = 1.0$ would yield an observed (nonlinear) value well in excess of unity. Cosmological simulations normalized to the cluster abundance reach similar conclusions (Meiksin, Peacock, & White 1998).

5. DISCUSSION

Detection of acoustic oscillations in the matter power spectrum would be a triumph for cosmology, since it would confirm the standard thermal history and the gravitational instability paradigm. Moreover, because the matter power spectrum displays these oscillations in a different manner than does the CMB, we would gain new leverage on cosmological parameters. In particular, we have shown in this Letter that the combination of power spectrum measurements from a galaxy redshift survey with anisotropy measurements from CMB satellite experiments could yield a precision measurement of H_0 and Ω_m.

The potential measurement of H_0 and Ω_m depends critically on the ability of the redshift survey to detect the baryonic features in the linear power spectrum. The best possible error bars are a strong function of the baryon fraction but are surprisingly good even if the fraction is ∼10%, roughly the minimum implied by cluster observations. For such cases, the fractional limits achievable with the SDSS are 5% for H_0 and 10% for Ω_m, if only the first acoustic peak in $P(k)$ is detected. Detecting the smaller scale peaks could allow an additional factor of 3 refinement; the exact limits would depend on the scale at which nonlinear effects smooth out the power spectrum. The results depend only mildly on the details of the CMB experiment: we find only slight gains as our presumed CMB data set improves from *MAP* without polarization to *Planck* with polarization. While we have quoted numbers for SDSS, it is possible that the 2dF survey will be able to make significant progress on the detection of features in the power spectrum on very large scales. Unfortunately, the hints of excess power on 100 h^{-1} Mpc scales are not likely to be due to baryons (Eisenstein et al. 1998a).

We have treated the galaxy power spectrum by assuming linear bias on large scales. There is some theoretical motivation for this (Scherrer & Weinberg 1997); moreover, if bias tends toward unity as structure grows (Fry 1996; Tegmark & Peebles 1998), then scale dependences in the bias at the time of formation will be suppressed. Most importantly, this method of measuring H_0 and Ω_m depends on extracting an oscillatory feature from the power spectrum. While one cannot prove that scale-dependent bias should be monotonic on the largest scales, this seems more likely than an oscillation! Finally, the assumption of linearity can be tested by constructing the power spectrum with different types of galaxies (see, e.g., Peacock 1997); future redshift surveys will allow this to be done on very large scales with good statistics.

The method proposed here yields H_0 independent of local distance measurements and Ω_m without the complications inherent in dynamical methods; i.e., this method is free of many confusing astrophysical problems. On the other hand, it does depend on restricting oneself to a class of models with observable acoustic oscillations in both CMB anisotropies and the galaxy power spectrum. This assumption will be definitively tested from the data itself.

If the method described in this Letter can yield tight constraints on H_0 and Ω_m, it will then be very important to compare these with other measurements of these quantities. In the coming decade, there will be a number of paths toward a precision measure of H_0, such as the local distance ladder (see, e.g., Freedman et al. 1998), gravitational lensing (see, e.g., Blandford & Kundic 1997), and the Sunyaev-Zeldovich effect (see, e.g., Cooray et al. 1998). Similarly, good estimates of Ω_m may be possible from velocity fields (see, e.g., Dekel 1997), cluster evolution (Carlberg et al. 1997a; Bahcall, Fan, & Cen 1997), and M/L measurements (see, e.g., Carlberg, Yee, & Ellingson 1997b). If the results from these diverse sets of measurements are found to agree, we will have a secure foundation on which to base our cosmology.

Numerical power spectra were generated with CMBFAST (Seljak & Zaldarriaga 1996). We thank Martin White for useful discussions. D. J. E. is supported by a Frank and Peggy Taplin Membership; D. J. E. and W. H. by NSF-9513835; W. H. by the Keck Foundation and a Sloan Fellowship; M. T. by NASA through grant NAG5-6034 and a Hubble Fellowship HF-01084.01-96A from STScI, operated by AURA, Inc., under NASA contract NAS4-26555.

REFERENCES

Bahcall, N. A., Fan, X., & Cen, R. 1997, ApJ, 485, L53
Bernardeau, F. 1994, ApJ, 433, 1
Blandford, R. D., & Kundic, T. 1997, in The Extragalactic Distance Scale, ed. M. Livio, M. Donahue, & N. Panagia (Cambridge: Cambridge Univ. Press), in press (astro-ph/9611229)
Bond, J. R., Crittenden, R., Davis, R. L., Efstathiou, G., & Steinhardt, P. J. 1994, Phys. Rev. Lett., 72, 13
Bond, J. R., Efstathiou, G., & Tegmark, M. 1997, MNRAS, 291, L33
Carlberg, R. G., Morris, S. L., Yee, H. K. C., & Ellingson, E. 1997a, ApJ, 479, L19
Carlberg, R. G., Yee, H. K. C., & Ellingson, E. 1997b, ApJ, 478, 462
Cooray, A. R., Carlstrom, J. E., Joy, M., Grego, L., Holzapfel, W. L., & Patel, S. K. 1998, in Proc. Dark Matter Workshop, ed. D. Cline, in press (astro-ph/9804149)
David, L. P., Jones, C., & Forman, W. 1995, ApJ, 445, 578
Dekel, A. 1997, in Third ESO-VLT Workshop, Galaxy Scaling Relations: Origins, Evolution, and Applications, ed. L. da Costa & A. Renzini (Springer: Berlin), 245
Eisenstein, D. J., & Hu, W. 1998a, ApJ, 496, 605
———. 1998b, ApJ, submitted
Eisenstein, D. J., Hu, W., Silk, J., & Szalay, A. 1998a, ApJ, 494, L1
Eisenstein, D. J., Hu, W., & Tegmark, M. 1998b, ApJ, submitted (astro-ph/9807130) (EHT)
Evrard, A. E. 1997, MNRAS, 292, 289
Freedman, W. L., Mould, J. R., Kennicut, R. C., & Madore, B. F. 1998, IAU Symp. 183, Cosmological Parameters and the Evolution of the Universe, ed. K. Sato (Dordrecht: Kluwer), in press (astro-ph/9801080)
Fry, J. N. 1996, ApJ, 461, 65
Hamilton, A. J. S. 1998, in Ringberg Workshop on Large-Scale Structure, ed. D. Hamilton (Dordrecht: Kluwer), in press (astro-ph/9708102)
Holtzman, J. A. 1989, ApJS, 71, 1
Hu, W., Eisenstein, D. J., Tegmark, M., & White, M. 1998, Phys. Rev. D, submitted (astro-ph/9806362)
Hu, W., & Sugiyama, N. 1996, ApJ, 471, 542
Hu, W., Sugiyama, N., & Silk, J. 1997, Nature, 386, 37
Huey, G., Wang, L., Dave, R., Caldwell, R. R., Steinhardt, P. J. 1998, preprint (astro-ph/9804285)
Jain, B., & Bertschinger, E. 1994, ApJ, 431, 495
Jungman, G., Kamionkowski, M., Kosowsky, A. & Spergel, D. N. 1996, Phys. Rev. D, 54, 1332
Meiksin, A., Peacock, J. A., & White, M. 1998, in preparation
Metcalf, R. B., & Silk, J. 1997, ApJ, 489, 1
Peacock, J. A. 1997, MNRAS, 284, 885
Perlmutter, S., et al. 1998, Nature, 391, 51
Riess, A. G., et al. 1998, AJ, in press (astro-ph/9805201)
Scherrer, R. J., & Weinberg, D. H. 1997, preprint (astro-ph/9712192)
Seljak, U. 1996, ApJ, 463, 1
Seljak, U., & Zaldarriaga, M. 1996, ApJ, 469, 437
Tegmark, M. 1997, Phys. Rev. Lett., 79, 3806
Tegmark, M., Eisenstein, D., Hu, W., & Kron, R. 1998a, ApJL, submitted (astro-ph/9805117)
Tegmark, M., Hamilton, A., Strauss, M., Vogeley, M., & Szalay, A. 1998b, ApJ, 499, 555
Tegmark, M., & Peebles, P. J. E. 1998, ApJ, 500, L79
Tegmark, M., Taylor, A. N., & Heavens, A. F. 1997, ApJ, 480, 22
Turner, M. S., & White, M. 1997, Phys. Rev. D, 56, 4439
White, D. A., & Fabian, A. C. 1995, MNRAS, 273, 72
White, S. D. M., Navarro, J. F., Evrard, A. E., & Frenk, C. S. 1993, Nature, 366, 429
Zaldarriaga, M., Spergel, D. N., & Seljak, U. 1997, ApJ, 488, 1