Weighing Neutrinos with Galaxy Surveys

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We show that galaxy redshift surveys sensitively probe the neutrino mass, with eV mass neutrinos suppressing power by a factor of 2. The Sloan Digital Sky Survey can potentially detect N nearly degenerate massive neutrino species with mass $m_\nu \gtrsim 0.65(\Omega_m h^2/0.1N)^{0.8}$ eV at better than 2σ once microwave background experiments measure two other cosmological parameters. Significant overlap exists between this region and that implied by the Liquid Scintillator Neutrino Detector experiment, and even $m_\nu \sim 0.01-0.1$ eV, as implied by the atmospheric anomaly, can affect cosmological measurements. [S0031-9007(98)06410-2]

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Current neutrino experiments reveal anomalies resolvable by nonzero neutrino masses and flavor oscillations. The Liquid Scintillator Neutrino Detector direct detection experiment suggests ν_μ to ν_e oscillations with $\delta m^2_{\mu e} \gtrsim 0.2$ eV^2 [1]. The deficit of μ neutrinos in atmospheric showers indicates mixing between ν_μ and another species with $\delta m^2_{\mu i} \sim 10^{-3}-10^{-2}$ eV^2 [2]. Finally, the solar neutrino deficit requires $\delta m^2_{ei} \sim 10^{-5}$ eV^2 (see, e.g., [3] for recent assessments). These results are consistent with one to three weakly interacting neutrinos in the eV mass range [4].

Cosmological measurements provide an independent, albeit indirect [5], means of determining neutrino masses in the above-mentioned range. Massive neutrinos would produce a strong suppression in the clustering of galaxies, with even a 10% neutrino contribution making a 100% difference in the power [6]. Detecting this suppression would measure the absolute mass of the neutrinos, in contrast to the mass splittings measured by the oscillation effects described above.

While the general effect is well known, most work to date has focused on a combined neutrino mass around 5 eV, as this is the minimum needed to affect cosmology at the current observational sensitivities [7,8]. There are three reasons why this situation is likely to change soon. First, evidence continues to mount that we live in a low-density universe (e.g., [9]). Since the cosmological effects depend on the density *fraction* supplied by neutrinos, our sensitivity to the neutrino mass increases roughly in inverse proportion to the density parameter. Second, the cosmic microwave background (CMB) experiments currently under development should establish a cosmological framework (e.g., [10]) that is as secure as the standard model of particle physics. The parameters left unspecified by the model may then be measured with confidence. Finally, upcoming high precision galaxy surveys such as the Sloan Digital Sky Survey (SDSS) [11] should be able to measure the total power on the relevant scales to $\sim$1% accuracy. The combination of these developments implies that galaxy surveys will soon provide either an interesting constraint on or a detection of the mass of the neutrinos.

Although the effects of massive neutrinos are very large, variations in other cosmological parameters may mimic the signal. Therefore, to qualify as a true detection, all other aspects of the cosmology that similarly affect the power spectrum must be previously or simultaneously determined.

In this Letter, we evaluate the ability of galaxy surveys to distinguish between these possibilities and thereby measure the mass of the neutrinos. We establish the physical basis of the measurement, evaluate the uncertainties caused by our ignorance of other aspects of cosmology, and present the 2σ detection threshold in mass for SDSS. These results depend on two assumptions: that CMB observations will confirm that structure forms through the gravitational instability of cold dark matter, and that the galaxy bias is linear, i.e., the galaxy power spectrum is proportional to the underlying mass power spectrum. The second assumption is relaxed in the concluding remarks.

Neutrino signature.—In a universe with the standard thermal history [12], the temperature of the background neutrinos is $(4/11)^{1/3}$ that of the CMB. This implies $\Omega_\nu h^2 \approx N m_\nu/94$ eV for N massive neutrino species of nearly identical mass m_ν. Here and below, Ω_i is the fraction of the critical density contributed by the ith matter species (ν = neutrinos, b = baryons, m = all matter species) and $H_0 = 100h$ km s^{-1} Mpc^{-1}. We assume this thermal history and a power-law spectrum of initial adiabatic density fluctuations with $P(k) \propto k^n$ throughout.

This initial power spectrum is processed by the gravitational instability of the fluctuations. The large momentum of cosmological eV mass neutrinos prevents them from clustering with the cold components on scales smaller than the neutrinos can move in a Hubble time. The growth of the fluctuations is therefore suppressed on all scales below the horizon when the neutrinos become nonrelativistic [6]

$$k_{\rm nr} \approx 0.026\left(\frac{m_\nu}{1\ \rm eV}\right)^{1/2} \Omega_m^{1/2} h\,{\rm Mpc}^{-1}. \quad (1)$$

The small-scale suppression is given by

$$\left(\frac{\Delta P}{P}\right) \approx -8\,\frac{\Omega_\nu}{\Omega_m} \approx -0.8\left(\frac{m_\nu}{1\ \text{eV}}\right)\left(\frac{0.1N}{\Omega_m h^2}\right). \quad (2)$$

Galaxy surveys such as the SDSS bright red galaxy (BRG) survey (assumed to be volume limited to $1h^{-1}$ Gpc [11]) should measure the power between $(0.1-0.2)h\ \text{Mpc}^{-1}$ to $\sim 1\%$. A more detailed analysis shows that only masses below

$$m_{\min} \approx 0.02(\Omega_m h^2/0.1N)\ \text{eV} \quad (3)$$

make less than a 2σ change in the power spectrum measured by the BRG survey.

As an example, we plot in Fig. 1 the power spectrum with and without a single 1 eV massive neutrino species for an $\Omega_m = 0.2$, $h = 0.65$ model (lower curves) and an $\Omega_m = 1.0$, $h = 0.5$ model (upper curves). The expected 1σ error boxes from the BRG survey shows that the two models are clearly distinguishable. For comparison, the difference between these models in the CMB power spectrum at degree angular scales is roughly 3% (1%) and never exceeds 5% (4%) for multipoles $\ell < 2000$ for the open variant of the low (high) $\Omega_0 h^2$ cases (cf. [13]).

Parameter degeneracies.—Although the suppression of power caused by massive neutrinos is large, we must consider whether other cosmological effects can mimic this signal. The suppression begins at $k_{\rm nr}$ [Eq. (1)] and approaches the constant factor of Eq. (2) at smaller scales. Many cosmological effects can produce the gross effect of a change in the ratio of large to small scale power; we must rely on the detailed differences between these mechanisms in order to distinguish one from another.

We consider variants of the adiabatic cold dark matter model. The power spectrum is then a function of the normalization A, tilt n, h, $\Omega_m h^2$, $\Omega_b h^2$, and $\Omega_\nu h^2$. The spatial curvature, cosmological constant, and the linear bias parameter are implicitly included through the normalization [6]. We estimate the accuracy with which these parameters can be jointly measured from the SDSS BRG survey using the technique described in [14]. Here the 6×6 covariance matrix of the 6 parameter estimates is approximated by the inverse of the so-called Fisher information matrix $\mathbf{F}$. Its elements $\mathbf{F}_{ij}$ are obtained by integrating $(\partial_i \ln P)(\partial_j \ln P)$ over the range $k_{\min} \le k \le k_{\max}$ discussed below, weighted by a function that incorporates the relevant aspects of the survey geometry and sampling density. Here the derivatives are with respect to the ith and jth parameters, evaluated at a fiducial model.

If a small variation in one parameter can be mimicked by joint variations in other parameters, then one of these functions $\partial_i \ln P$ can be approximated by a linear combination of the others. This situation is referred to as a *parameter degeneracy,* since it makes $\mathbf{F}$ nearly singular and leads to extremely poor determinations (large variance $\mathbf{F}_{ii}^{-1}$) for the parameters involved.

Clearly, the ability to estimate parameters comes only from scales on which one has both precise measurements and reliable theoretical predictions. On large scales, linear perturbation theory is accurate, but the survey volume (about $1h^{-3}$ Gpc^3 for the BRG survey) is limited; hence $k_{\min} \approx 0.005h\ \text{Mpc}^{-1}$. On small scales, linear theory fails near $k \approx 0.2h\ \text{Mpc}^{-1}$. While we expect that detailed data analysis will push $k_{\max}$ to slightly smaller scales by including mild corrections to linear theory, we simply use the linear power spectrum for this work and adopt $k_{\max} \approx 0.2h\ \text{Mpc}^{-1}$. We shall see that if $k_{\min} \lesssim k_{\rm nr} \lesssim k_{\max}$, then the unique signature of massive neutrinos can be identified and m_ν measured.

The solid lines in Fig. 2 show the standard deviation of a measurement of m_ν (or, equivalently, $\Omega_\nu h^2$) as a function of $k_{\max}$ if all relevant cosmological parameters are determined simultaneously from the SDSS BRG data set. Consider first the low $\Omega_m h^2$ case (Fig. 2, bottom panel). The standard deviation drops rapidly near $k_{\max} = 0.05h\ \text{Mpc}^{-1}$, well below the scale at which the neutrinos begin to affect the power spectrum (see Fig. 1). If we use only information from $k \lesssim 0.05h\ \text{Mpc}^{-1}$, we find that the neutrino signal can be accurately duplicated by variations in other parameters. For example, a change in normalization and tilt would be indistinguishable within the BRG survey error bars. When considering smaller scales, more subtle combinations still exist; these near degeneracies reduce the parameter sensitivity more than 100-fold. A similar situation occurs in the high $\Omega_m h^2$ case (Fig. 2, top panel) but at a smaller scale [Eq. (1)].

If we possess external information on the other cosmological parameters, the situation improves dramatically because parameters may no longer be shifted arbitrarily so as to mimic the neutrino signal. Indeed, upcoming

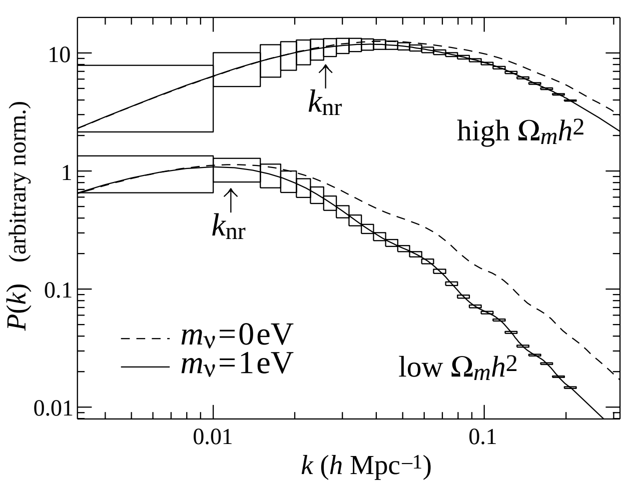

FIG. 1. Effect of a 1 eV neutrino on the BRG power spectrum compared with expected precision of the SDSS BRG survey (1σ error boxes, assuming $\sigma_8 = 2$ for the BRGs). Upper curves: an $\Omega_m = 1.0$, $h = 0.5$, $\Omega_b h^2 = 0.0125$, $n = 1$ model with and without a 1 eV neutrino mass. Lower curves: The same but for an $\Omega_m = 0.2$, $h = 0.65$ model.

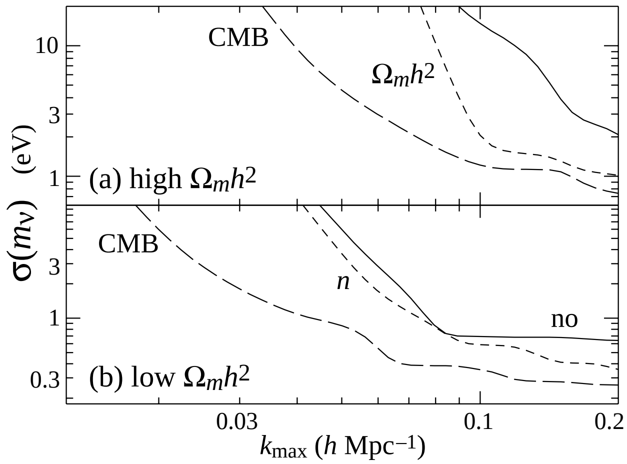

FIG. 2. Standard deviation $\sigma(m_\nu)$ as a function of the upper cutoff k_{max} for several different choices of prior cosmological constraints. Models are the same as in Fig. 1 and have $m_\nu = 1$ eV. (a) High $\Omega_m h^2$: no priors, solid line; single prior of $\sigma(\Omega_m h^2) = 0.04$, dashed line; full CMB prior (see text), long-dashed line. (b) Low $\Omega_m h^2$: as in (a), save that the single prior is $\sigma(n) = 0.06$, dashed line.

CMB anisotropy experiments should yield precise measurements of cosmological parameters critical to this situation. We therefore show the effect in Fig. 2 (long-dashed curve) of placing CMB constraints on the cosmological parameters: $\sigma(\ln A) = 0.40$, $\sigma(n) = 0.06$, $\sigma(\Omega_m h^2) = 0.04$, $\sigma(\Omega_b h^2) = 0.1\Omega_b h^2$, and $\sigma(h) = 0.1$, where $\sigma(i)$ denotes the standard deviation of i. We view these constraints as quite conservative, since they are weaker than those predicted for the MAP satellite [15,16] and ignore the tight correlation between the marginalized error bars [17].

Which one prior is most important depends upon the fiducial model. For low $\Omega_m h^2$ models with small neutrino fractions, one cannot accurately probe the scales on which the neutrino suppression is small since $k_{min} \gtrsim k_{nr}$. This enables the tilt n to produce much of the desired effect. We show the error bars resulting from including only the tilt prior in Fig. 2 (short-dashed line). While this is the most important prior at $k_{max} = 0.2h$ Mpc^{-1}, the others combined have a non-negligible effect.

For the high $\Omega_m h^2$ case, one has precise measurements around k_{nr}, so that the onset of neutrino effects can be distinguished from tilt (cf. Fig. 1). However, altering $\Omega_m h^2$ or h causes $P(k)$ to slide horizontally (leaving the largest scales unchanged); as one can see in Fig. 1, this is roughly degenerate with the neutrino effect. The $\Omega_m h^2$ prior is most important in this case; we show this situation in Fig. 2 (upper panel, short-dashed line).

We also test how $\sigma(m_\nu)$ increases as we double each prior in turn. The results change by more than a few percent only for $\Omega_m h^2$ (20%) and n (10%) in the high $\Omega_m h^2$ model and for n (40%) in the low $\Omega_m h^2$ model.

Results.—Given the confusion with variations in other cosmological parameters, what is the minimum detectable neutrino mass m_{det}? In Fig. 3, we show the 2σ detection threshold [i.e., $m_\nu = 2\sigma(m_\nu)$] assuming the CMB priors given above, $k_{max} = 0.2h$ Mpc^{-1}, and a family of fiducial models with $\Omega_b h^2 = 0.0125$, $h = 0.5$, and $n = 1$. The choice of a fiducial model does not amount to fixing cosmological parameters; all parameters are determined by the galaxy data or by the prior constraints.

With these choices, SDSS can detect the neutrinos if

$$m_\nu \geq m_{det} \approx 0.65\left(\frac{\Omega_m h^2}{0.1N}\right)^{0.8} \text{ eV}. \tag{4}$$

If the exponent here were unity, it would correspond to a fixed fractional suppression of power [Eq. (2)]. In practice, one does slightly better at larger $\Omega_m h^2/N$ because k_{nr} [Eq. (1)] is larger and thus better resolved.

This result is fairly insensitive to changes in the fiducial model or survey parameters. Choosing $h = 0.8$ increases m_{det} by 15% at low $\Omega_m h^2$; doubling the baryon density does the opposite. Neither matters at high $\Omega_m h^2$. Altering n or A affects the answer very little. Reverting from the deeper BRG survey to the main SDSS North survey [11] increases m_{det} by less than 25%.

As for the assumptions implicit in Fig. 3, only the prior constraints on the tilt n in the low $\Omega_0 h^2$ regime and $\Omega_0 h^2$ itself in the high regime are essential. We have taken conservative priors from the CMB here and save a full joint analysis for future work [17]. Decreasing k_{max} to $0.13h$ Mpc^{-1} increases m_{det} by $\sim$40% at large $\Omega_m h^2$ but makes little difference at small $\Omega_m h^2$ (cf. Fig. 2).

Quasilinear evolution near k_{max} presents a complication to the analysis, but so long as the power spectrum can be calculated as a function of cosmological parameters through simulations or analytic approximations [18], this need not necessarily degrade the parameter estimation. However, to the extent that evolution washes out features

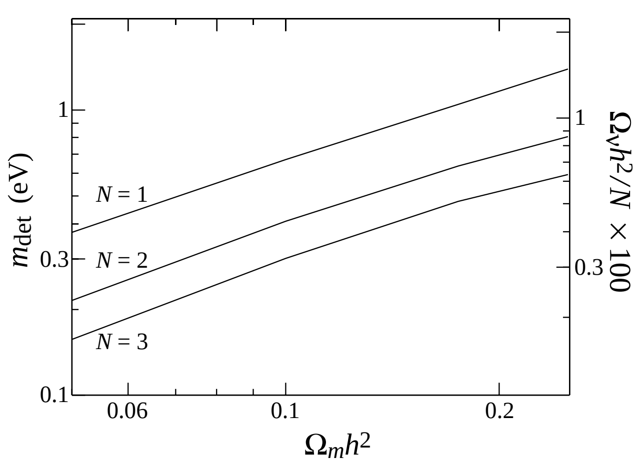

FIG. 3. The 2σ detection threshold for m_ν from the SDSS BRG survey as a function of the matter density $\Omega_m h^2$ for the number of degenerate mass neutrinos $N = 1-3$. We have used $h = 0.5$, $\Omega_b h^2 = 0.0125$, $n = 1$, and $k_{max} = 0.2h$ Mpc^{-1}; variations on these produce only mild shifts.

in the power spectrum, degeneracies may appear that are not present in linear calculation. Hence, this issue merits further investigation, although we view our choice of $k_{\max}$ as conservative. Fortunately, as shown in Fig. 2, prior information from the CMB assists in making the results robust against changes in $k_{\max}$.

Our fundamental assumption is that the power spectrum of the galaxies is proportional to that of the underlying mass, i.e., that the galaxy bias is linear. This assumption is well motivated in the linear regime [19]. Bias that develops a scale dependence as fluctuations become nonlinear introduces only moderate uncertainties because it does not accurately mimic the neutrino signature. Adopting the prescription of [20] [their Eq. (20)] and marginalizing over the scale-dependence parameter b_2 degrades the result in the models of Fig. 2 by a factor of 2.5 implying a similar effect on the detection threshold of Eq. (4). The degradation stems not from the presence of nonlinear bias but from our ignorance of its amplitude. The latter can be constrained by the scale dependence of redshift space distortions and the *relative* bias of different galaxy populations. Peacock [21] determines $\sigma(b_2) = 0.01$ for the relative bias of IRAS and optical galaxies; bounding scale-dependent bias at this level would restore the limits of Eq. (4).

In summary, although galaxy surveys can measure power to $\sim 1\%$, isolating the mass of the neutrino at 2σ requires $\sim 50\%$ power variations. In principle, this means that a 50-fold improvement of $m_{\rm det}$ to $m_{\rm min}$ [Eq. (3)] would be available if other cosmological parameters were known perfectly. While this is unrealistic, a measurement with a precision of $\sigma(\ln A) \sim 0.1$ and $\sigma(n) \sim 0.03$ would yield a factor of 2 improvement in $m_{\rm det}$ and is potentially within reach of planned experiments.

For m_ν between $m_{\rm min}$ and $m_{\rm det}$, the effects on the power spectrum are significant yet cannot be robustly attributed to massive neutrinos. As this brackets the mass range implied by atmospheric neutrinos, the possibility of a light massive neutrino species must be considered when measuring other cosmological parameters.

Massive neutrinos present an example where the galaxy power spectrum provides cosmological information on fundamental physics not available in CMB measurements, but where CMB measurements are nonetheless needed for an unambiguous detection. This illustrates the complementary nature of galaxy surveys and CMB anisotropies.

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