Astro 242

The Physics of Galaxies and the Universe: Lecture Notes

Wayne Hu

Syllabus

- Text: An Introduction to Modern Astrophysics 2nd Ed., Carroll and Ostlie
- First class Wed Jan 9. Reading period Mar 13-14
- Jan 7: Milky Way Galaxy
- Jan 14: Nature of Galaxies
- Jan 21: Galactic Evolution
- Jan 28: Active Galaxies
  Midterm
- Feb 4: Structure of the Universe
- Feb 11-18: Cosmology
- Feb 25-Mar 3: Early Universe

Set 1:

Milky Way Galaxy

Astrophysical units

- Length scales
- 1AU = 1.496 $\times 10^{13}$cm – Earth-sun distance – used for solar system scales
- 1pc = 3.09 $\times 10^{18}$cm = 2.06 $\times 10^{5}$AU – 1AU subtends 1arcminute on the sky at 1pc – distances between nearby stars

  Defined by measuring parallax of nearby stars to infer distance - change in angular position during Earth's orbit: par(allax arc)sec(ond)

$$\frac{1\mathrm{AU}}{1\mathrm{pc}} = \frac{1}{2.06 \times 10^5} = 4.85 \times 10^{-6} = \frac{\pi}{60 * 60 * 180} = 1''$$

- 1kpc = 10^3 pc – distances in the Galaxy
- 1Mpc = 10^6 pc - distances between galaxies
- 1Gpc = 10^9 pc - scale of the observable universe

Astrophysical units

- Fundamental observables are the brightness of the sky or flux and angular position of objects (energy per unit time per unit area) in a given frequency band

$$F = \frac{L}{4\pi d^2}$$

- Frequency band defined by filters - in the limit of bands are infinitesimal the whole frequency spectrum measured
- Relative flux easy to measure - absolute flux requires calibration of filter: (apparent) magnitudes (originally defined by the eye as a filter)

$$m_1 - m_2 = -2.5 \log(F_1/F_2)$$

Astrophysical units

- Absolute magnitude: apparent magnitude of an object at a distance of 10pc

$$m - M = -2.5 \log(d/10\text{pc})^2 \rightarrow \frac{d(m-M)}{10\text{pc}} = 10^{(m-M)/5}$$

- If frequency spectrum has lines, Doppler shift of lines gives relative or radial velocity of object V_r

$$1 + z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{rest}}} = 1 + \frac{V_r}{c}$$

(where $V_r > 0$ denotes recession and redshift) used to measure velocity for dynamics of systems, including universe as whole

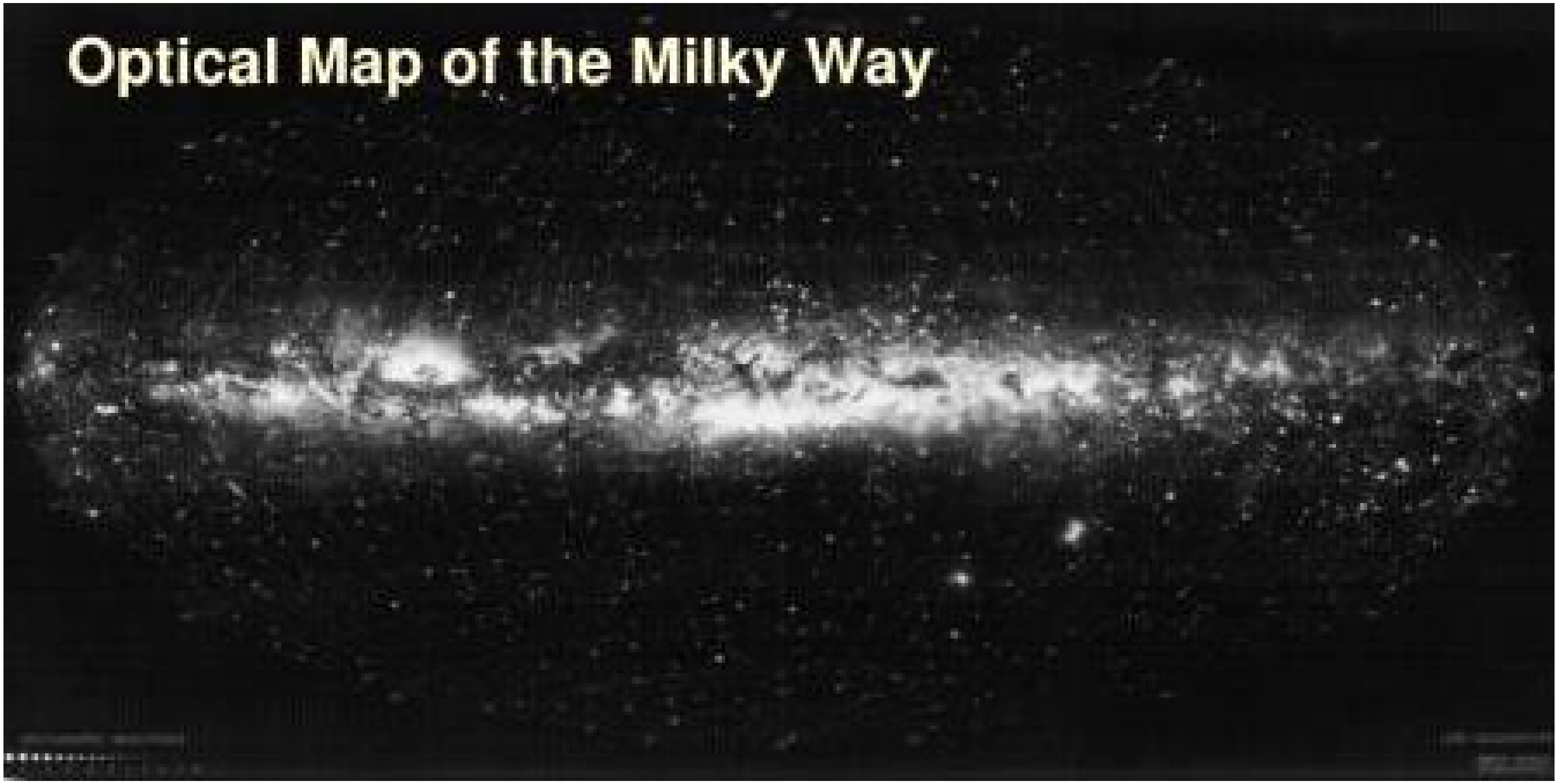
Astrophysical units

- Masses in units of solar mass $M_\odot = 1.989 \times 10^{33}$g
- Mass measurement always boils down to inferring the gravitational force necessary to keep test object of mass m with a velocity v bound

$$\frac{mv^2}{r} \approx \frac{GmM}{r^2} \rightarrow M \approx \frac{v^2 r}{G}$$

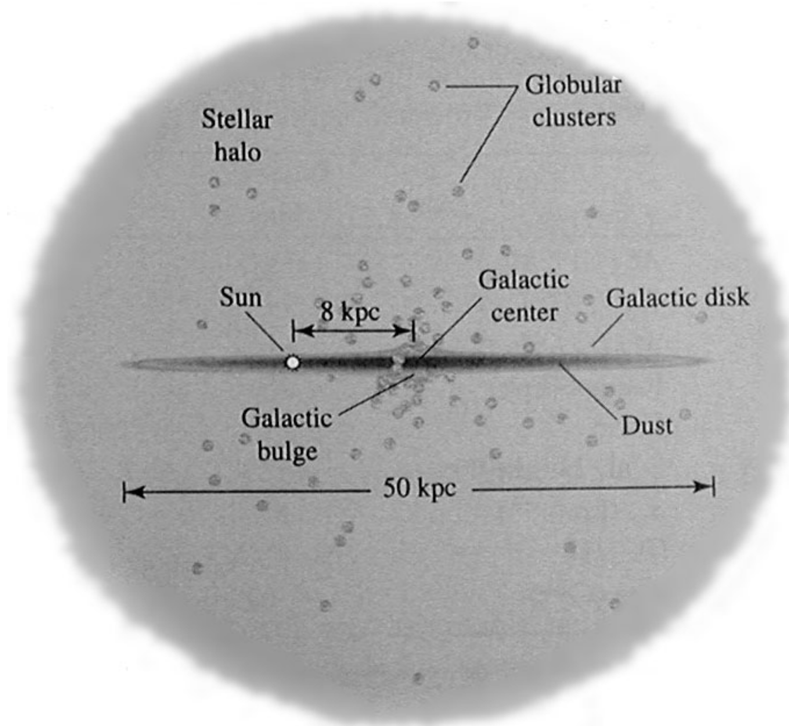
- Requires a measurement of velocity and a measurement or estimate of size
- Various systems will have an order unity correction to this approximate relation

Starlight: Optical Image



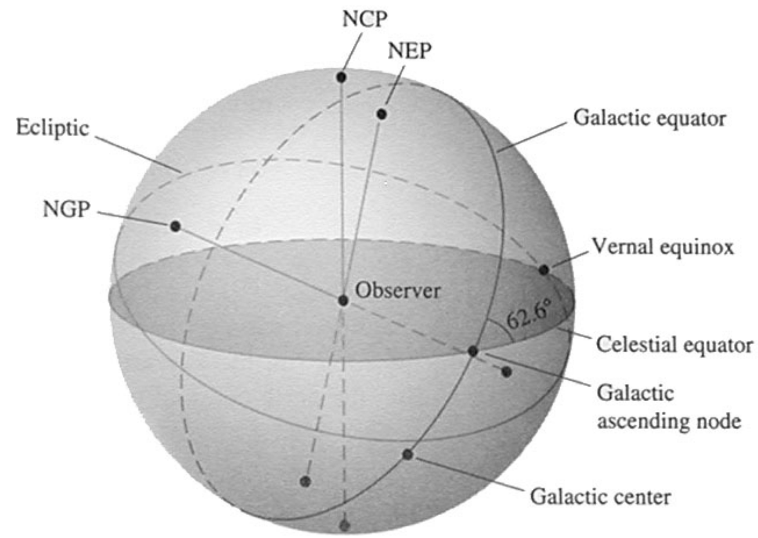
Galactic Components and Scale

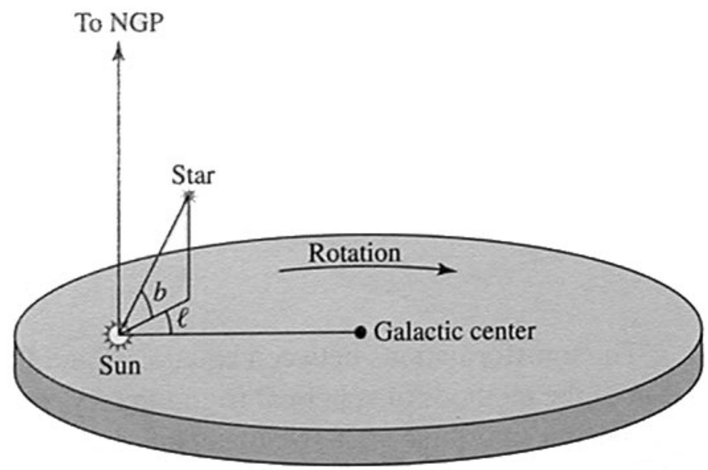
- From such data,
  infer the structure of the galaxy
- Sun is embedded in a stellar disk
  ~ 8 kpc from the galactic center
- Extent of disk
  ~ 25 kpc radius, spiral structure
- Thickness of neutral gas disk
  < 0.1 kpc
- Thickness of thin disk of young stars ~ 0.35 kpc
- Thickness of thick disk ~ 1 kpc



Galactic Components and Scale

- Central stellar bulge
  radius ~ 4 kpc, with central bar
- Supermassive black hole, inferred
  from mass < 120AU of center
- Extended spherical stellar
  halo with globular
  clusters, radius > 100 kpc
- Extended dark matter halo,
  radius > 200 kpc





Mass and Luminosity

- Neutral gas disk: $M \sim 0.5 \times 10^{10} M_\odot$
- Thin disk: $M \sim 6 \times 10^{10} M_\odot$, $L_B \sim 1.8 \times 10^{10} L_\odot$
- Thick disk: $M \sim 0.2 - 0.4 \times 10^{10} M_\odot$, $L_B \sim 0.02 \times 10^{10} L_\odot$
- Bulge: $M \sim 1 \times 10^{10} M_\odot$, $L_B \sim 0.3 \times 10^{10} L_\odot$
- Supermassive black hole mass $3.7 \pm 0.2 \times 10^6 M_\odot$
- Stellar halo: $M \sim 0.3 \times 10^{10} M_\odot$, $L_B \sim 0.1 \times 10^{10} L_\odot$
- Dark matter halo: $M \sim 2 \times 10^{12} M_\odot$
- Total $M \sim 2 \times 10^{12} M_\odot$, $L_B \sim 3.6 \times 10^{10} L_\odot$

Methods: Star Counts

- One of the oldest methods for inferring the structure of the galaxy from 2D skymaps is from star counts
- History: Kapteyn (1922), building on early work by Herschel, used star counts to map out the structure of the galaxy
- Fundamental assumptions
  - Stars have a known (distribution in) absolute magnitude
  - No obscuration
- Consider a star with known absolute magnitude M (magnitude at 10pc). It's distance can be inferred from the inverse square law from its observed m as

$$\frac{d(m-M)}{10\mathrm{pc}} = 10^{(m-M)/5}$$

Methods: Star Counts

- Combined with the angular position on the sky, the 3d position of the star can be measured - mapping the galaxy
- In more detail, one would like to use the star counts to determine the number density of stars in each patch of sky.
- A fall off in the number density in radius would determine the edge of the galaxy
- Suppose there is an indicator of absolute magnitude like spectral type that allows stars to be selected to within dM of M
- Describe the underlying quantity to be extracted as the spatial number density within dM of M: $n_M(M, \mathbf{r})dM$ with total number density

Methods: Star Counts

- The observable is say the total number of stars brighter than a limiting apparent magnitude m in a solid angle $d\Omega$ implicitly integrated over radial distance
- Stars at a given M can only be observed out to a distance $d(m-M)$ before their apparent magnitude falls below the limit
- Total number observed out to in solid angle $d\Omega$ within dM of M

$$N_M dM = \left[\int_0^{d(m-M)} n_M(M,r) r^2 dr \right] d\Omega dM$$

- Differentiating with respect to $d(m-M)$ provides a measurement of $n_M(M,r)$.

Methods: Star Counts

- More generally, the selection criteria is a perfect indicator of M and so dM is not infinitesimal and some stars in the range will be missed - $S(M)$ and M is integrated over

$$n(\mathbf{r}, S) = \int_{-\infty}^{\infty} n_M(M, \mathbf{r}) S(M) dM$$

- Alternately use all stars but assume a functional form for n_M, e.g. derived from local estimates and assumed to be the same at larger r. In this case, measurements determine the normalization of a distribution with fixed shape and determine

$$n(\mathbf{r}) = \int_{-\infty}^{\infty} n_M(M, \mathbf{r}) dM$$

- Similar method applies to mapping out the universe with galaxies

Methods: Star Counts

- Kapteyn used all of the stars (assumed to have the same n_M shape in r)
- He inferred a flattened spheroidal system of $<$ 10kpc extent in plane and $<$ 2kpc out of plane: too small
- Missing: interstellar extinction dims stars dropping them out of the sample at a given limiting magnitude

Methods: Variable Stars

- With a good indicator of absolute magnitude or “standard candle” one can use individual objects to map out the structure of the galaxy (and cosmology)
- History: Shapley (1910-1920) used RR Lyrae and W Virginis variable stars - period-luminosity relation [calibrated by moving cluster and other methods] (radial oscillations with a density dependent sound speed - luminosity and density related on the instability strip)
- Measure the period of oscillation, infer a luminosity and hence an absolute magnitude, infer a distance from the observed apparent magnitude
- Inferred a 100kpc scale for the galaxy - overestimate due to differences in types of variable stars and interstellar extinction in calibrating the period-luminosity relation

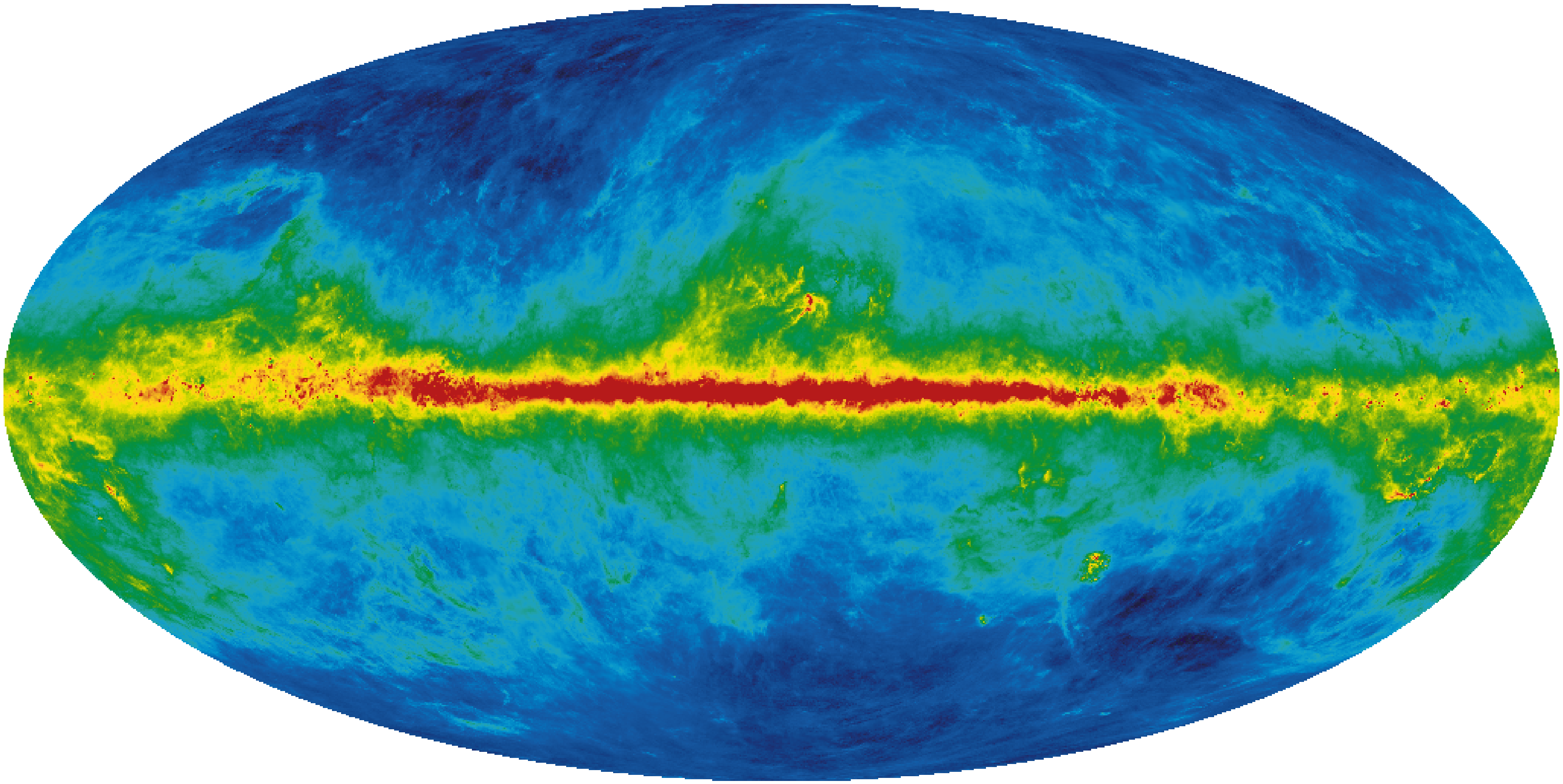
Interstellar Extinction

- Apparent magnitude is dimmed by extinction leading to a distance inference that is too large
- Dust (silicates, graphite, hydrocarbons) in ISM (Chap 12) dims stars at visible wavelengths making true distance less than apparent
- Distance formula modified to be

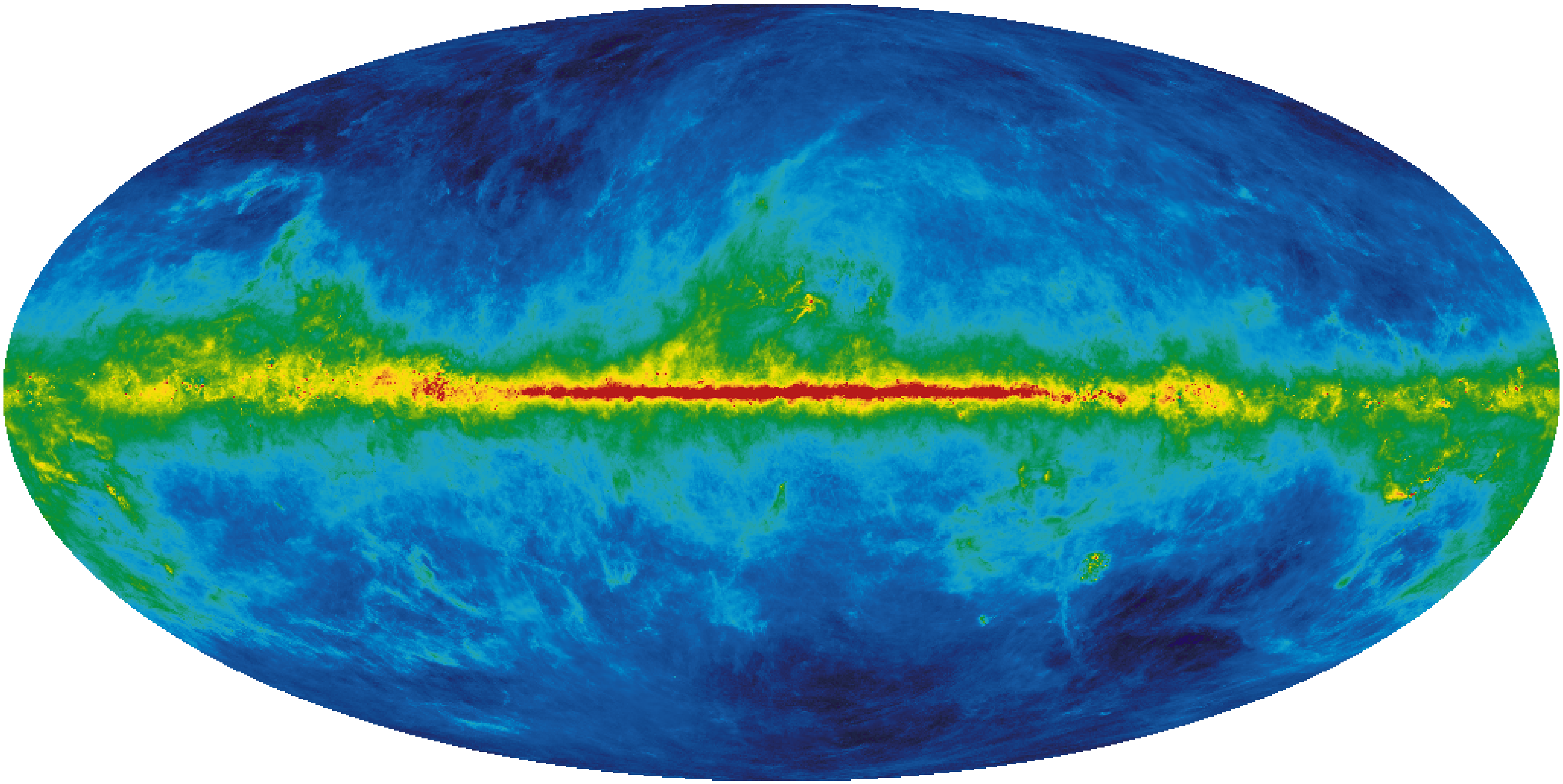
$$\frac{d}{10\mathrm{pc}} = 10^{(m_\lambda - M_\lambda - A_\lambda)/5}$$

  where the extinction coefficient $A_\lambda \geq 0$ depends on wavelength λ
- Extinction also depends on direction, e.g. through the disk, through a giant molecular cloud, etc. Typical value at visible wavelengths and in the disk is 1 mag/kpc
- Dust emits or reradiates starlight in the infrared - maps from these frequencies [IRAS, DIRBE] can be used to calibrate extinction

Dust Emission



Extinction Correction



Kinematic Distances to Stars

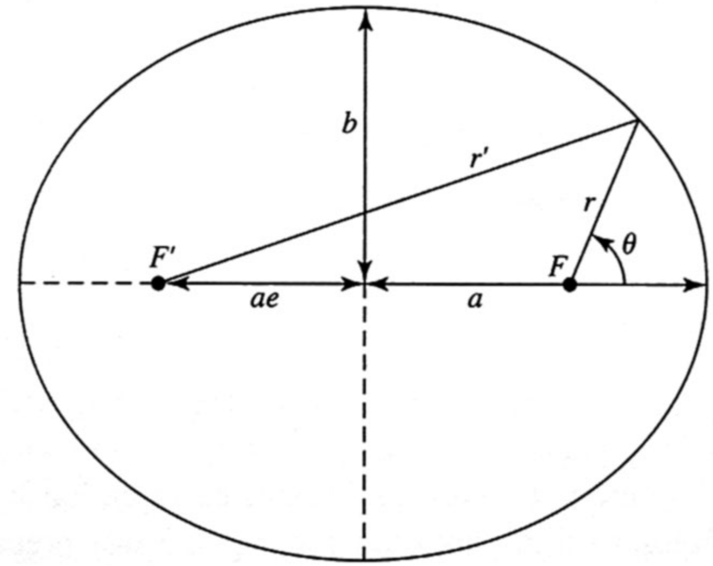
- Only nearby stars have their distance measured by parallax - further than a parsec the change in angle is < 1 arcsec: $p(\mathrm{arcsec}) = 1\mathrm{pc}/d$
- If proper motion across the sky can be measured from the change in angular position μ in rad/s

$$v_t = \mu d$$

- Often v_t can be inferred from the radial velocity and a comparison with μ gives distance d given assumption of the dynamics
- Example: Keplerian orbits of stars around galactic center $R_0 = 7.6 \pm 0.3$kpc
- Example: Stars in a moving cluster share a single total velocity whose direction can be inferred from apparent convergent motion (see Fig 24.30)

Methods: Stellar Kinematics

- Can infer more than just distance: SMBH
- Galactic center: follow orbits of stars close to galactic center
- One star: orbital period 15.2yrs, eccentricity $e = 0.87$, perigalacticon distance (closest point on orbit to F 120 AU=1.8×10^{13}m



- Estimate mass: $a = ae - r_p$ so semimajor axis

$$a = \frac{r_p}{1-e} = 1.4 \times 10^{14}\text{m}$$

Methods: Stellar Kinematics

- Kepler's 3rd law

$$M = \frac{4\pi^2 a^3}{GP^2} = 7 \times 10^{36}\mathrm{kg} = 3.5 \times 10^6 M_\odot$$

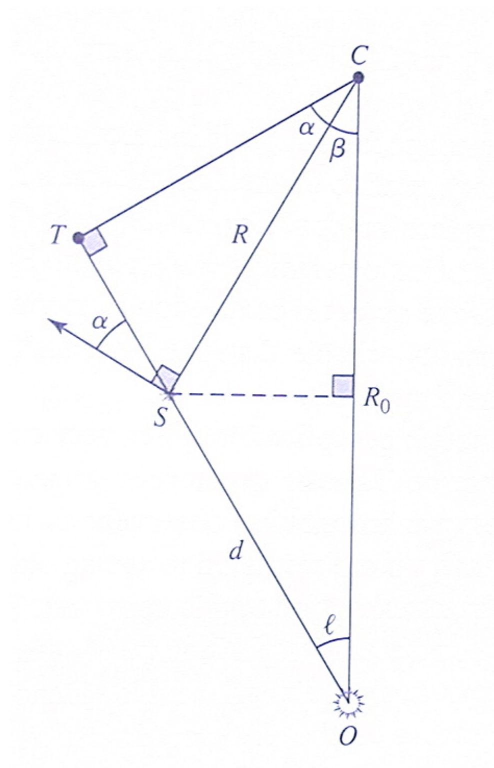
- That much mass in that small a radius can plausibly only be a (supermassive) black hole
- Note that this is an example of the general statement that masses are estimated by taking

$$M \approx \frac{v^2 r}{G} = \frac{(2\pi a)^2 a}{GP^2} = \frac{4\pi^2 a^3}{GP^2}$$

Methods: Stellar Kinematics

- Stars around sun - higher velocity, lower metalicity stars: thick disk, lower velocity higher metalicity stars orbiting with sun in thin disk
- Halo stars vs sun or LSR suggests orbital speed of Θ_0 =220 km/s
- Differential rotation $\Theta(R) = R\Omega(R)$ where $\Omega(R)$ is the angular velocity curve– observables are radial and tangential motion with respect to LSR

$$v_r = R\Omega\cos\alpha - R_0\Omega_0\sin\ell$$
$$v_t = R\Omega\sin\alpha - R_0\Omega_0\cos\ell$$



Methods: Stellar Kinematics

- d (parallax) and R_0 are known observables, R is not - eliminate with trig relations

$$R\cos\alpha = R_0 \sin\ell \qquad R\sin\alpha = R_0\cos\ell - d$$

- Eliminate R

$$\begin{aligned} v_r &= (\Omega - \Omega_0) R_0 \sin\ell \\ v_t &= (\Omega - \Omega_0) R_0 \cos\ell - \Omega d \end{aligned}$$

solve for $\Omega(R)$ locally where

$$\begin{aligned} \Omega - \Omega_0 &\approx \frac{d\Omega}{dR}(R - R_0) \\ &\approx \frac{1}{R_0}\left(\frac{d\Theta}{dR} - \frac{\Theta_0}{R_0}\right)(R - R_0) \quad [\Omega = \Theta/R] \end{aligned}$$

and $d \ll R_0$, $\cos\beta \approx 1$

Methods: Stellar Kinematics

- Reduce with trig identities

$$R_0 = d\cos\ell + R\cos\beta \approx d\cos\ell + R$$

$$R - R_0 \approx -d\cos\ell$$

$$\cos\ell\sin\ell = \frac{1}{2}\sin 2\ell$$

$$\cos^2\ell = \frac{1}{2}\left(\cos 2\ell + 1\right)$$

to obtain

$$v_r \approx Ad\sin 2\ell$$

$$v_t \approx Ad\cos 2\ell + Bd$$

Methods: Stellar Kinematics

- Oort constants

$$A = -\frac{1}{2}\left[\frac{d\Theta}{dR} - \frac{\Theta_0}{R_0}\right] = -\frac{R_0}{2}\frac{d\Omega}{dR}$$

$$B = -\frac{1}{2}\left[\frac{d\Theta}{dR} + \frac{\Theta_0}{R_0}\right]$$

- Observables v_r, v_t, ℓ, d: solve for Oort's constants. From Hipparcos

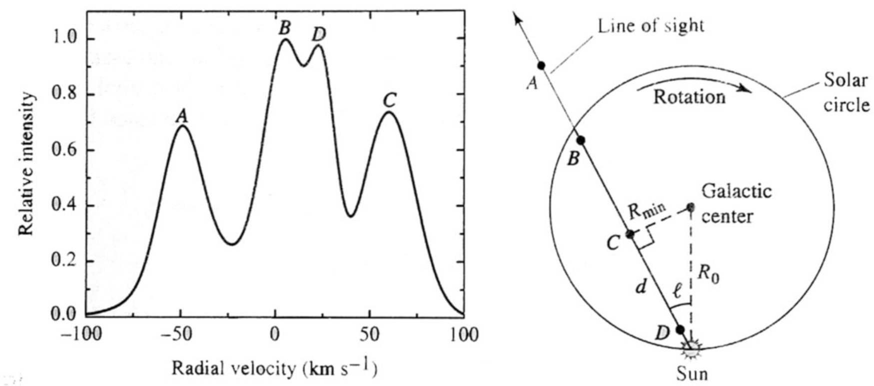
$$A = 14.8 \pm 0.8\mathrm{km/s/kpc}$$

$$B = -12.4 \pm 0.6\mathrm{km/s/kpc}$$

- Angular velocity $\Omega = v/r$ decreases with radius: differential rotation. Physical velocity $\Theta(R)$: $d\Theta/dR|_{R_0} = -(A + B) = -2.4$ km/s/kpc decreases slowly compared with 220km/s - flat rotation curve

Methods: 21 cm

- Spin interaction of the electron and proton leads to a spin flip transition in neutral hydrogen with wavelength 21cm

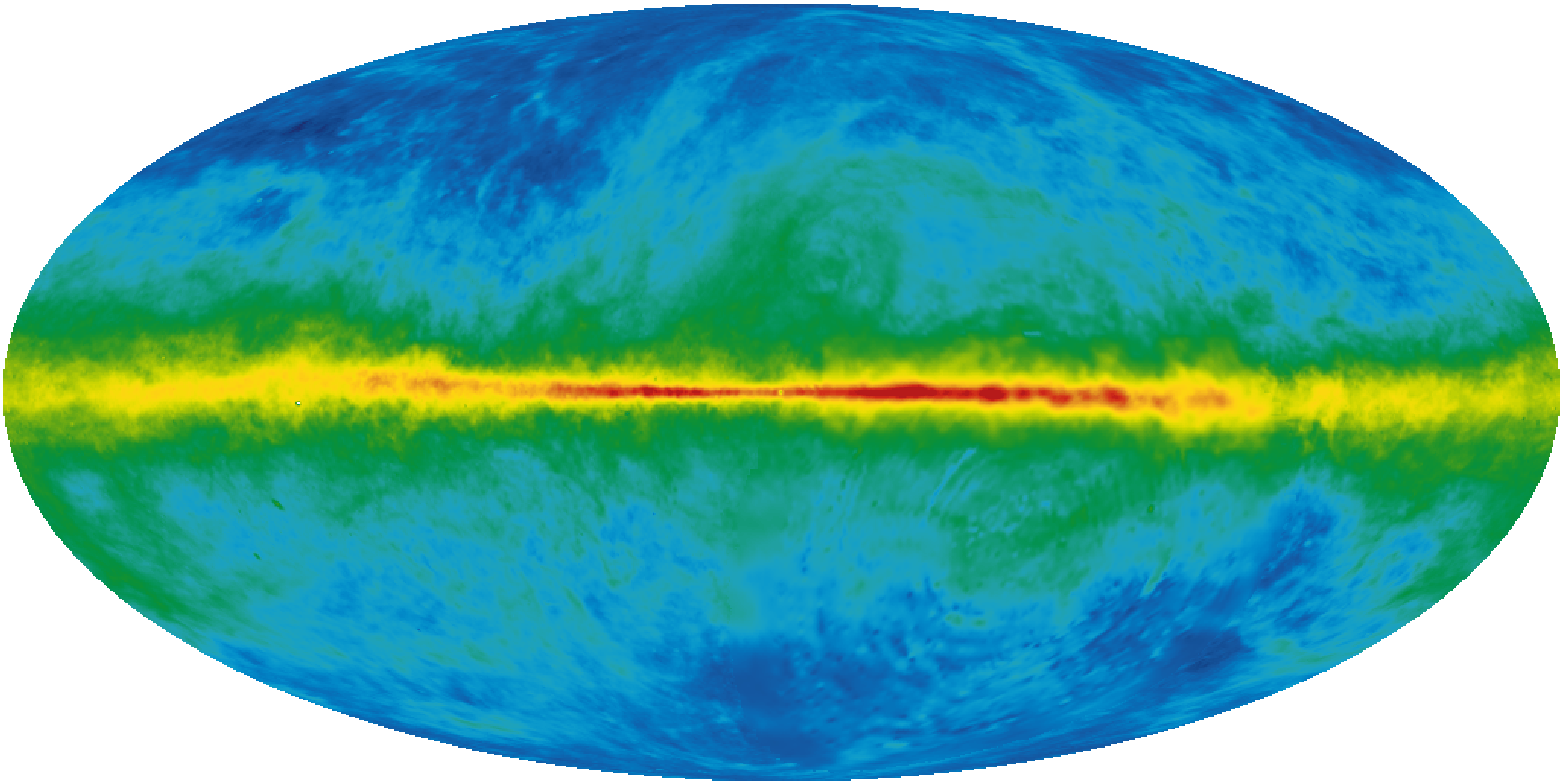


- Line does not suffer substantial extinction and can be used to probe the neutral gas and its radial velocity from the Doppler shift throughout the galaxy
- No intrinsic distance measure
- Neutral gas is distributed inhomogeneously in clouds leading to distinct peaks in emission along each sight line

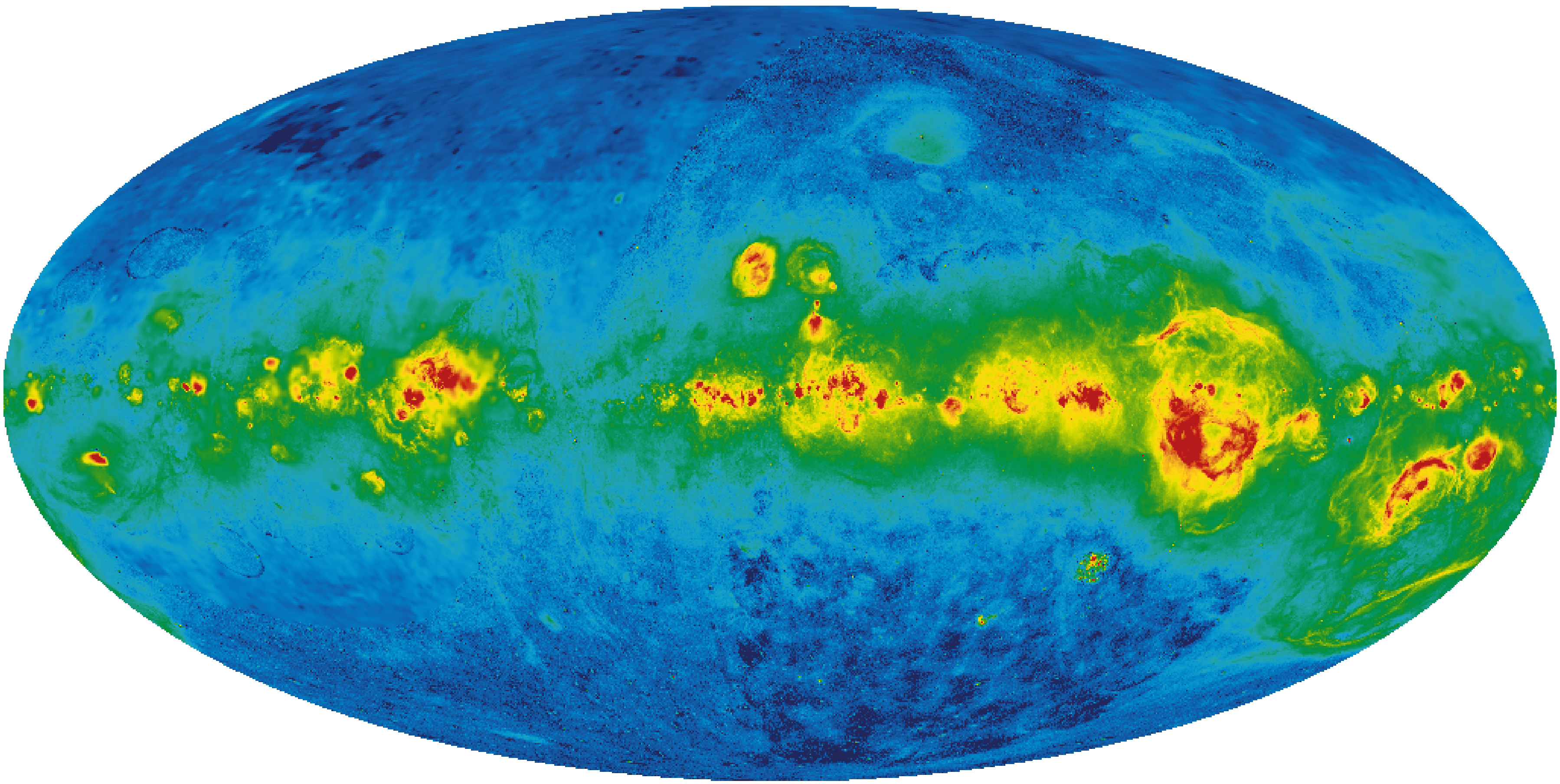
Methods: 21 cm

- Due to projection of velocities along the line of sight and differential rotation, the highest velocity occurs at the closest approach to the galactic center or tangent point
- Build up a rotation curve interior to the solar circle $R < R_0$
- Rotation curve steeply rises in the interior $R < 1$kpc, consistent with near rigid body rotation and then remains flat out through the solar circle

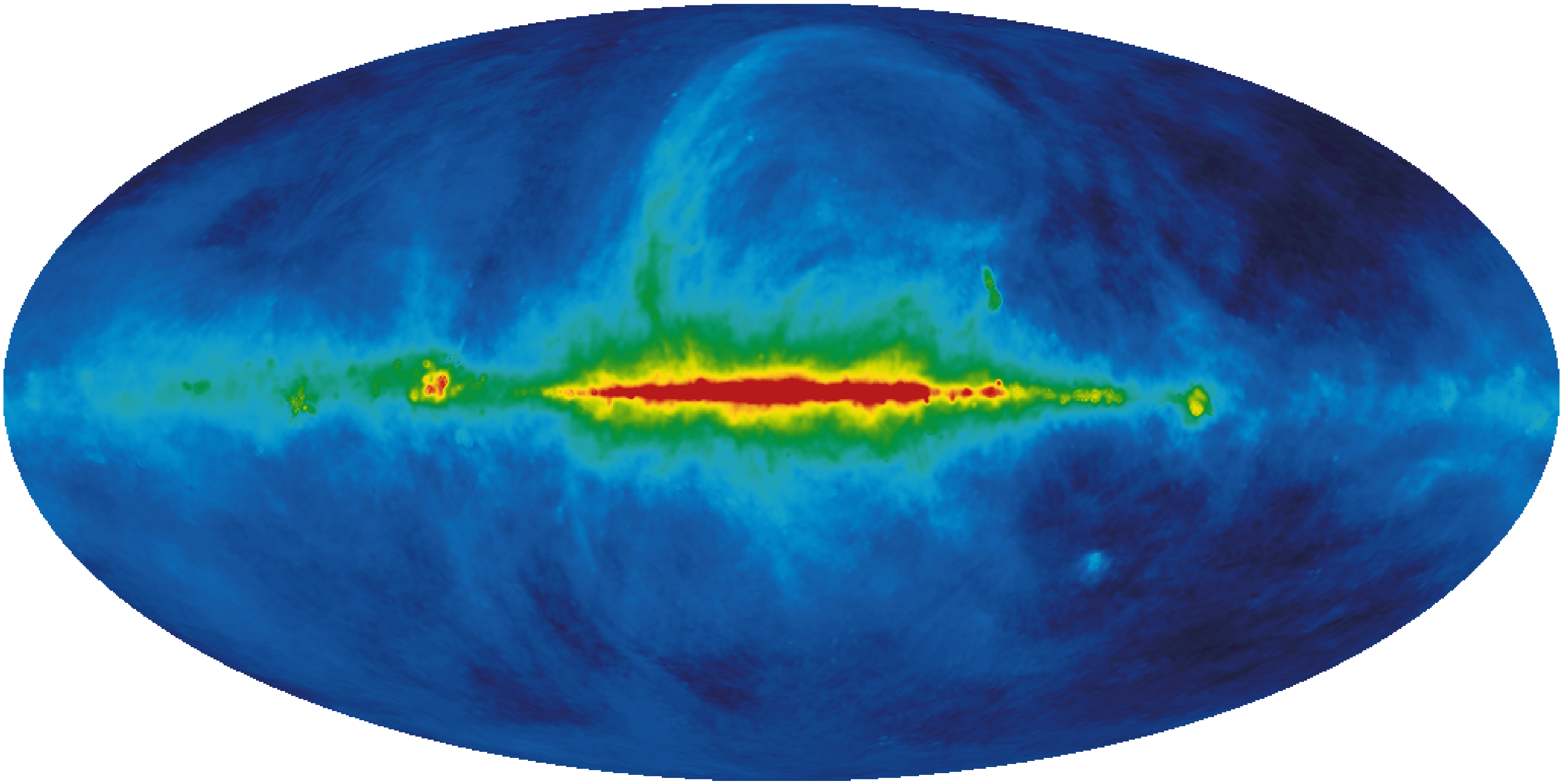
Neutral Gas: 21cm Emission



Ionized Gas: $H\alpha$ Line Emission

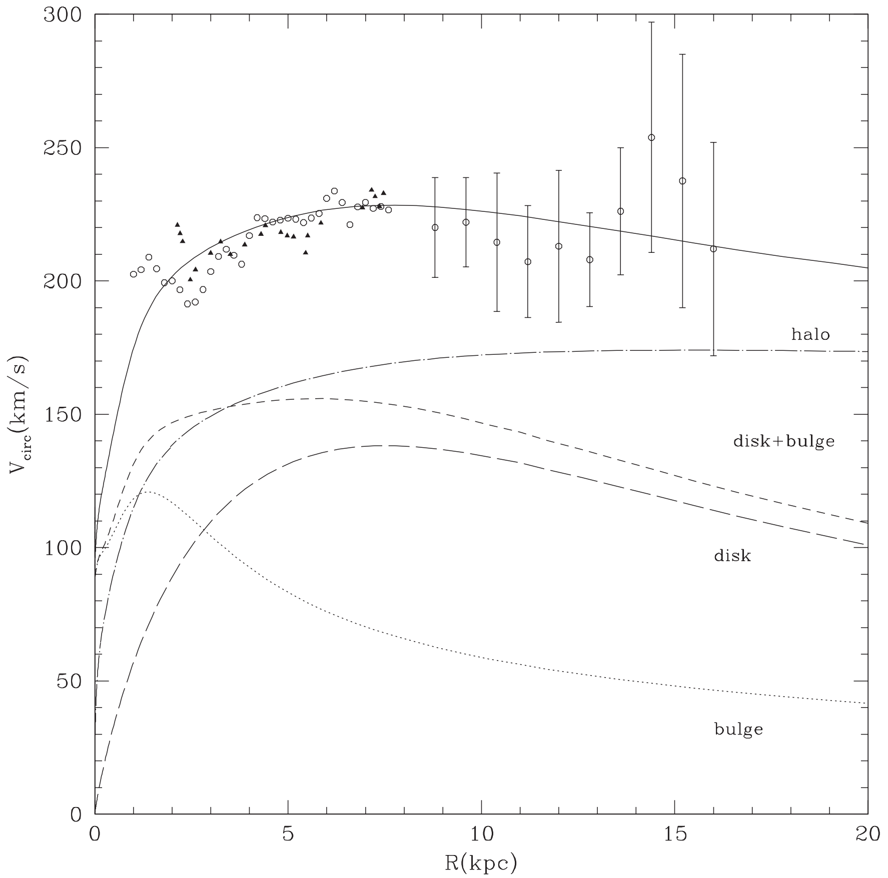


Cosmic Rays in B Field: Synchrotron



Methods: Rotation Curves

- Extending the rotation curve beyond the solar circle with objects like Cepheids whose distances are known reveals a flat curve out to $\sim$ 20kpc
- Mass required to keep rotation curves flat much larger than implied by stars and gas. Consider a test mass m orbiting at a radius r around an enclosed mass $M(r)$



Methods: Rotation Curves

- Setting the centripetal force to the gravitational force

$$\frac{mv^2(r)}{r} = \frac{GM(r)m}{r^2}$$

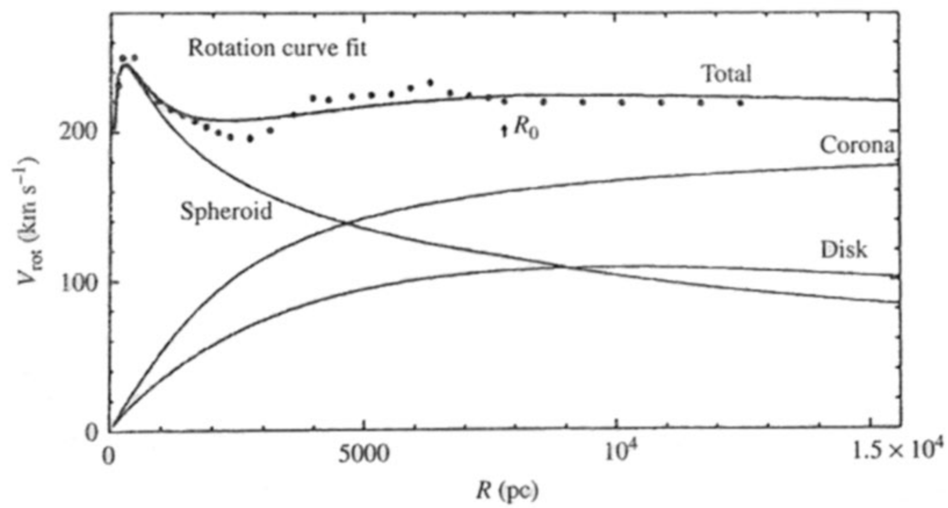
$$v(r) = \left(\frac{GM(r)}{r}\right)^{1/2}$$

Side note: this is the fundamental way masses are measured - balance internal motions of luminous matter with gravitational force - other examples: virial theorem with velocity dispersion, hydrostatic equilibrium with thermal motions

- Measuring the rotation curve $v(r)$ is equivalent to measuring the mass profile $M(r)$ or density profile $\rho(r) \propto M(r)/r^3$

Methods: Rotation Curves

- Flat rotation curve $v(r)$ =const implies $M \propto r$ - a mass linearly increasing with radius
- Rigid rotation implies $\Omega = v/r$=const. $v \propto r$ or $M \propto r^3$ or ρ =const



- Rotation curves in other galaxies show the same behavior: evidence that “dark matter” is ubiquitous in galaxies

Methods: Rotation Curves

- Consistent with dark matter density given by

$$\rho(r) = \frac{\rho_0}{1 + (r/a)^2}$$

- Also consistent with the NFW profile predicted by cold dark matter (e.g. weakly interacting massive particles or WIMPs)

$$\rho(r) = \frac{\rho_0}{(r/a)(1 + r/a)^2}$$

Methods: Gravitational Lensing

- Rotation curves leave open the question of what dark matter is
- Alternate hypothesis: dead stars or black holes - massive astrophysical compact halo object "MACHO"
- MACHOs have their mass concentrated into objects with mass comparable to the sun or large planet
- A MACHO at an angular distance $u = \theta/\theta_E$ from the line of sight to the star will gravitationally lens or magnify the star by a factor of

$$A(u) = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}}$$

where θ_E is the Einstein ring radius in projection

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{d_S - d_L}{d_S d_L}}$$

Methods: Gravitational Lensing

- A MACHO would move at a velocity typical of the disk and halo $v \sim 200$km/s and so the star behind it would brighten as it crossed the line of sight to a background star. With $u_{\rm min}$ as the distance of closest approach at $t = 0$

$$u^2(t) = u^2_{\rm min} + \left(\frac{vt}{d_L \theta_E}\right)^2$$

- Monitor a large number of stars for this characteristic brightening. Rate of events says how much of the dark matter could be in MACHOs.

Methods: Gravitational Lensing

- In the 1990's
  large searches measured
  the rate of microlensing
  in the halo and
  bulge and determined that
  only a small fraction of its
  mass could be in MACHOs