Set 3:
Galaxy Evolution
Galaxies are clustered, found in groups like the local group up to large clusters of galaxies like the Coma cluster.

Small satellite galaxies like the LMC and SMC are merging into the Milky way. Recent discovery of other satellites like the Sagitarius dwarf and tidal streams.
Environment

- cD galaxies in centers of rich galaxy clusters are the products of frequent mergers in the cluster environment.
- HST images of galaxies in the process of merging
- Theoretically, structure in the universe is thought to form bottom up from the merger of small objects into large objects
- Over the lifetime of the universe, galaxy evolution is a violent process
Antennae Galaxies
N-body and Hydro Simulations

• To understand the physical processes behind the observations, $N$-body and hydrodynamic simulations are used.

• In an interaction between galaxies, stars and dark matter essentially never physically collide - act as collisionless point particles or “Nbodies” that interact gravitationally.

• Gas is more complicated and can shock, etc - use hydrodynamic techniques + cooling and star formation.
Interactions and Mergers

- $N$-body simulations reproduce the main features of mergers in terms of stars.
- As galaxies approach, tidal forces pull stars out into tidal streams much like tides on the Earth - features like the Antennae galaxies or the Magellenic stream.
- Much like mass loss in a binary stellar system but with equipotential surfaces constantly changing as the galaxy centers move with respect to one another.
In a minor merger, a satellite galaxy can warp the disk of a larger galaxy in a major merger. Two spirals may have their disks disrupted and become an elliptical.

Eventually the merger completes. Though collisionless, the stars interact gravitationally, and their motion dissipates through dynamical friction.
Dynamical Friction

- Consider a single encounter of an object of mass $M$ with a (smaller) mass $m$

- Two-body encounter can be re-expressed as a single particle of the reduced mass in the potential of the combined mass: Newton’s third law

$$M\ddot{x}_M = F_{Mm} = -F_{mM} = -m\ddot{x}_m$$

- Center of mass $x_{cm} = (Mx_M + mx_m)/(M + m)$ has zero acceleration (uniform velocity)

$$\ddot{x}_{cm} = 0 = \frac{M}{M + m}\ddot{x}_M + \frac{m}{M + m}\ddot{x}_m$$

- Separation $\mathbf{R} = x_m - x_M$ obeys

$$\ddot{\mathbf{R}} = \ddot{x}_m - \ddot{x}_M$$
Dynamical Friction

- Eliminate $x_M$

$$\ddot{x}_M = -\frac{m}{M}\ddot{x}_m$$

$$\ddot{R} = \left(1 - \frac{m}{M}\right)\ddot{x}_m$$

- Gravitational acceleration

$$\ddot{x}_m = -\frac{GM}{R^2}\hat{r}$$

$$\ddot{R} = -\frac{G(M + m)}{R^2}\hat{r}$$

- Test particle moving in gravitational potential of combined mass (equivalently particle with reduced mass $\mu = mM/(m + M)$ with the force $GMm/r^2$) if $M \gg m$ then $m$ is essentially the test mass and center of mass frame is rest frame of $M$
Dynamical Friction

- Want to find the change in velocity of $M$ due to interactions with $m$ given kinematics of the reduced mass $V = \dot{R}$

\[ \Delta v_m - \Delta v_M = \Delta V \]

\[ m\Delta v_m + M\Delta v_M = 0 \]

\[ \Delta v_M = -\left(\frac{m}{m + M}\right)\Delta V \]

- Now determine $\Delta V$ from single particle kinematics. Consider an initial relative velocity $V$ and an impact parameter $b$, the initial separation transverse to $V$

- If the impact parameter is sufficiently large then the encounter is weak and the trajectory of the test particle is only slightly deflected
Dynamical Friction

- The test particle then experiences the potential on the unperturbed trajectory: “Born approximation”. The force perpendicular to the velocity

\[ \dot{V}_\perp = -\frac{G(M + m)}{b^2 + x^2(t)} \frac{b}{\sqrt{b^2 + x^2}} \]

where \( x(t) = V_0 t \) if \( t = 0 \) and \( x = 0 \) is set to be at the closest approach

\[ |\Delta V_\perp| = \int_{-\infty}^{\infty} dt \frac{G(M + m)b}{(b^2 + V_0^2t^2)^{3/2}} = \frac{2G(M + m)}{bV_0} \]

- Change in \( V_\perp \) has no net effect since there is an equal probability of an impact with \(-b\).
Dynamical Friction

- There is a coherent effect on $V_\parallel$. Energy conservation says that the speed $V_0$ is conserved so that $V_\parallel$ is reduced

$$\theta_{\text{def}} \approx \sin \theta_{\text{defl}} = \frac{|\Delta V_\perp|}{V_0} = \frac{2G(M + m)}{bV_0^2}$$

$$|\Delta V_\parallel| = V_0(1 - \cos \theta_{\text{defl}}) \approx \frac{1}{2}V_0\theta_{\text{defl}}^2 \approx \frac{2G^2(m + M)^2}{b^2V_0^3}$$

with a direction opposite to $V_0$

- Back to the change in the velocity of the real mass $M$

$$|\Delta v_M| = \frac{2G^2m(m + M)}{b^2V_0^3}$$

with the same direction as $V_0$ - i.e. $M$ will get a kick in the direction of oncoming $m$ particles
Dynamical Friction

- Now consider the mass $M$ to be moving through a sea of particles $m$ with number density $n$ and mass density $\rho = mn$.

- Rate of encounters at an impact parameter $db$ around $b$ will be $nV_0\sigma$ where $\sigma$ is the cross sectional area:

  \[ nV_0 \times 2\pi bdb \]

- Total rate of change of velocity is the integral over all allowed impact parameters:

  \[
  \left| \frac{dv_M}{dt} \right| = \int_{b_{\text{min}}}^{b_{\text{max}}} V_0 n |\Delta v_M| 2\pi bdb
  \]

  \[
  \left| \frac{dv_M}{dt} \right| = \frac{4\pi G^2 mn(m + M)}{V_0^2} \ln \frac{b_{\text{max}}}{b_{\text{min}}} = \frac{4\pi G^2 \rho(m + M)}{V_0^2} \ln \frac{b_{\text{max}}}{b_{\text{min}}}
  \]
Dynamical Friction

- Rate depends weakly (logarithmically) on the limits for the impact parameter. $b_{\text{max}}$ is size of $m$ system. $b_{\text{min}}$ is set by the validity of the “Born approximation”

$$\Delta V_\perp = \frac{2G(M + m)}{b_{\text{min}}V_0} \approx V_0$$

$$b_{\text{min}} \approx \frac{2G(M + m)}{V_0^2}$$

- For $b_{\text{max}} < b_{\text{min}}$ this term must go to zero and a better calculation from Chandrashekar (see Binney & Tremaine) replaces the log “Gaunt” factor with

$$\ln \frac{b_{\text{max}}}{b_{\text{min}}} \rightarrow \ln \left[ 1 + \left( \frac{b_{\text{max}}V_0^2}{G(M + m)} \right)^2 \right]^{1/2} \equiv \ln \Lambda$$
Dynamical Friction

- Considering $M$ to be falling into a body of density $\rho$ whose particles $m \ll M$ have no net velocity $V_0 = -v_M$ there is a frictional force that will stop the body

$$M \frac{dv_M}{dt} \approx - \left[ \frac{4\pi G^2 \rho M^2}{v_M^2} \ln \Lambda \right] \hat{v}_M$$
Galaxy Formation

• The same process of merging but with smaller proto-Galactic objects of $10^6 - 10^8 M_\odot$ can eventually assemble the galaxies of $10^{12} M_\odot$ we see today. Theoretical tools include cosmological $N$-body and hydro simulations.

• Proto-galactic objects can form if cooling is sufficiently rapid that the heating of the gas during collapse (which would prevent collapse due to pressure, internal motions) can be overcome.

• Recall virial theorem supplies estimate of thermal kinetic energy

\[-2\langle K \rangle = \langle U \rangle\]

\[-2N \frac{1}{2} \mu m_H \sigma^2 = -\frac{3}{5} \frac{GM^2}{R}\]

where $\mu m_H$ is the average mass of particles in the gas, $M$ is the total mass including dark matter and $\sigma$ is the rms velocity.
Galaxy Formation

- Solve for velocity dispersion

\[ \sigma = \left( \frac{3 \ GM}{5 \ R} \right)^{1/2} \]

- Associate the average kinetic energy with a temperature, called the virial temperature

\[ \frac{1}{2} \mu m_H \sigma^2 = \frac{3}{2} k T_{\text{virial}} \]

where \( \mu \) is the mean molecular weight. Solve for virial temperature

\[ T_{\text{virial}} = \frac{\mu m_H \sigma^2}{3k} = \frac{\mu m_H}{5k} \frac{GM}{R} \approx \frac{\mu m_H}{5k} GM^{2/3} \left( \frac{4\pi \rho}{3} \right)^{1/3} \]

- Cooling is a function of the gas temperature through the cooling function.
Galaxy Formation

- Cooling rate (luminosity) per volume

\[ r_{\text{cool}} = n^2 \Lambda(T) \]

\( n^2 \) (number density squared) comes from the fact that cooling is usually a 2 body process - for \( T > 10^6 \text{K} \) thermal bremsstrahlung and Compton scattering, for \( T \sim 10^4 - 10^5 \text{K} \) from the collisional excitation of atomic lines of hydrogen and helium

- Galaxy formation only starts when dark matter mass makes the virial temperature exceed \( T \sim 10^4 \text{K} \) when cooling becomes efficient \( M \sim 10^8 M_\odot \) - first objects and current dwarf ellipticals
Galaxy Formation

- Cooling time is the time required to radiate away all of the thermal energy of the gas

\[ r_{\text{cool}} V t_{\text{cool}} = \frac{3}{2} N kT_{\text{virial}} \]

\[ t_{\text{cool}} = \frac{3}{2} \frac{kT_{\text{virial}}}{n\Lambda} \]

- Compared with the free fall time - dimensionally \( t_{\text{ff}} \propto (G\rho)^{-1/2} \) with the proportionality given for the time of collapse for a homogenous sphere of initial density \( \rho \)

\[ t_{\text{ff}} = \left( \frac{3\pi}{32 \frac{1}{G\rho}} \right)^{1/2} \]
Galaxy Formation

- If $t_{\text{cool}} < t_{\text{ff}}$ then the object will collapse essentially in free fall - fragment and form stars. If opposite, then gravitational potential energy heats the gas making it stabilized by pressure establishing virial equilibrium

$$\left( \frac{t_{\text{ff}}}{t_{\text{cool}}} \right) > \left( \frac{3\pi}{32} \frac{1}{G\rho} \right)^{1/2} \frac{2}{3} \frac{n\Lambda}{kT_{\text{virial}}}$$

- Taking typical numbers $T \sim 10^6 \text{K}$ and $n \sim 5 \times 10^4 \text{m}^{-3}$ and with the density of the collapsing medium being associated with the gas $\rho = \mu m_H n$ gives an upper limit on the gas mass that can cool of $10^{12} M_\odot$ comparable to a large galaxy.
Disk Formation

- Proto-galactic gas fragment and collide retaining initial angular momentum provided from torques from other proto-galactic systems

- Rotationally supported gas disk, cooling in dense regions until HI clouds form from which star formation occurs - thick disk

- Cool molecular gas settles to midplane of thick disk efficiently forming stars - thinness is self-regulating - if disk continued to get thinner then density and star formation goes up heating the material and re-puffing out the disk