Astro 242
The Physics of Galaxies and the Universe: Lecture Notes
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Syllabus

- Text: An Introduction to Modern Astrophysics 2nd Ed., Carroll and Ostlie
- First class Wed Jan 6. Reading period Mar 11-12
- Jan 6: Milky Way Galaxy
- Jan 13: Nature of Galaxies
- Jan 20: Galactic Evolution
- Jan 27: Active Galaxies
  Midterm Feb 10 on material above
- Feb 3: Structure of the Universe
- Feb 10: In class Midterm
- Feb 17, 24: Cosmology
- Mar 3, 10: Early Universe
Common Themes

- **Mapping** out the Universe marching out in **distance from Earth**
  
  Start with closest system: **Galaxy**
  
  End with furthest system: whole **Universe**

- **Limitations** imposed by the ability to measure only a **handful** of quantities, all from our vantage point in the Galaxy
  
  Common tools: **distance measures, number counts**

- Inferences on the **dynamical nature** of the systems by using **physical laws** to interpret observations
  
  Common tools: **mass inferences** from Newtonian dynamics, General Relativity
Set 1:
Milky Way Galaxy
Astrophysical units

- **Length scales**
  - **1AU** = \(1.496 \times 10^{13}\) cm – *Earth-sun* distance – used for solar system scales
  - **1pc** = \(3.09 \times 10^{18}\) cm = \(2.06 \times 10^5\) AU – 1AU subtends 1 arcsecond on the sky at 1pc – distances between nearby stars

Defined by measuring parallax of nearby stars to infer distance - change in angular position during Earth’s orbit: par(allax arc)sec(ond)

\[
\frac{1\text{AU}}{1\text{pc}} = \frac{1}{2.06 \times 10^5} = 4.85 \times 10^{-6} = \frac{\pi}{60 \times 60 \times 180} = 1''
\]

- **1kpc** = \(10^3\) pc – distances in the *Galaxy*
- **1Mpc** = \(10^6\) pc - distances between *galaxies*
- **1Gpc** = \(10^9\) pc - scale of the *observable universe*
Astrophysical units

- Fundamental observables are the flux $F$ (energy per unit time per unit area) or brightness (+ per unit solid angle) and angular position of objects in a given frequency band.

- Related to the physical quantities, e.g. the luminosity of the object $L$ if the distance to the object is known:

\[ F = \frac{L}{4\pi d^2} \]

- Solar luminosity:

\[ L_\odot = 3.839 \times 10^{26} \text{W} = 3.839 \times 10^{33} \text{erg/s} \]

- Frequency band defined by filters - in limit of infinitesimal bands, the whole frequency spectrum measured – “spectroscopy”
Astrophysical units

- **Relative flux** easy to measure - absolute flux requires calibration of filter: (apparent) magnitudes (originally defined by eye as filter)

\[ m_1 - m_2 = -2.5 \log\left(\frac{F_1}{F_2}\right) \]

- **Absolute magnitude**: apparent magnitude of object at \( d = 10\text{pc} \)

\[ m - M = -2.5 \log\left(\frac{d}{10\text{pc}}\right)^2 \rightarrow \frac{d(m - M)}{10\text{pc}} = 10^{(m-M)/5} \]

- If frequency spectrum has lines, **Doppler shift** gives relative or radial velocity of object \( V_r \) aka redshift \( z \)

\[ 1 + z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{rest}}} = 1 + \frac{V_r}{c} \]

(where \( V_r > 0 \) denotes recession and redshift) used to measure velocity for dynamics of systems, including universe as whole
Astrophysical units

- Masses in units of solar mass \( M_{\odot} = 1.989 \times 10^{33} \text{g} \)

- Mass measurement always boils down to inferring gravitational force necessary to keep test object of mass \( m \) with a velocity \( v \) bound

- For circular motion - centripetal force

\[
\frac{mv^2}{r} \approx \frac{GmM}{r^2} \rightarrow M \approx \frac{v^2r}{G}
\]

- Requires a measurement of velocity and a measurement or estimate of size

- Various systems will have order unity correction to this circular-motion based relation
Starlight: Optical Image

- Color overlay: microwave background
Galactic Census

- From such data, infer the structure of the galaxy
- Sun is embedded in a stellar disk
  $\sim 8$ kpc from the galactic center
- Extent of disk
  $\sim 25$ kpc radius, spiral structure
- Thickness of neutral gas disk
  $< 0.1$ kpc
- Thickness of thin disk of young stars $\sim 0.35$ kpc
- Thickness of thick disk $\sim 1$ kpc
Galactic Census

- Central stellar bulge radius $\sim 4 \text{kpc}$, with central bar
- Supermassive black hole, inferred from large mass within $120 \text{AU}$ (solar system scale) of center
- Extended spherical stellar halo with globular clusters, radius $> 100 \text{kpc}$
- Extended dark matter halo, radius $> 200 \text{kpc}$
Mass and Luminosity

- Neutral gas disk: $M \sim 0.5 \times 10^{10} M_\odot$
- Thin disk: $M \sim 6 \times 10^{10} M_\odot$, $L_B \sim 1.8 \times 10^{10} L_\odot$
- Thick disk: $M \sim 0.2 - 0.4 \times 10^{10} M_\odot$, $L_B \sim 0.02 \times 10^{10} L_\odot$
- Bulge: $M \sim 1 \times 10^{10} M_\odot$, $L_B \sim 0.3 \times 10^{10} L_\odot$
- Supermassive black hole mass 3.7 $\pm$ 0.2 $\times$ 10$^6 M_\odot$
- Stellar halo: $M \sim 0.3 \times 10^{10} M_\odot$, $L_B \sim 0.1 \times 10^{10} L_\odot$
- Dark matter halo: $M \sim 2 \times 10^{12} M_\odot$
- Total: $M \sim 2 \times 10^{12} M_\odot$, $L_B \sim 3.6 \times 10^{10} L_\odot$
Methods: Star Counts

- One of the oldest methods for inferring the structure of the galaxy from 2D sky maps is from star counts.

- History: Kapteyn (1922), building on early work by Herschel, used star counts to map out the structure of the galaxy.

- Fundamental assumptions
  - Stars have a known (distribution in) absolute magnitude.
  - No obscuration.

- Consider a star with known absolute magnitude $M$ (magnitude at 10pc). Its distance can be inferred from the inverse square law from its observed $m$ as

\[
\frac{d(m - M)}{10\text{pc}} = 10^{(m - M)/5}
\]
Methods: Star Counts

- Combined with the **angular position** on the sky, the **3d position** of the star can be measured - mapping the galaxy
- Use the star counts to determine **statistical properties**: number density of stars in each patch of sky
- A **fall off** in the number density in radial distance would determine the **edge** of the galaxy
- Suppose there is an **indicator** of absolute magnitude like spectral type that allows stars to be **selected** to within $dM$ of $M$
- Describe the underlying quantity to be extracted as the **spatial number density** within $dM$ of $M$: $n_M(M, r)dM$
Methods: Star Counts

• The observable is say the total number of stars brighter than a limiting apparent magnitude \( m \) in a solid angle \( d\Omega \).

• Stars at a given \( M \) can only be observed out to a distance \( d(m - M) \) before their apparent magnitude falls below the limit.

• So there is radial distance limit to the volume observed.

• Total number observed out to in solid angle \( d\Omega \) within \( dM \) of \( M \) is integral to that limit.

\[
N_M dM = \left[ \int_0^{d(m - M)} n_M(M, r) r^2 dr \right] d\Omega dM
\]

• Differentiating with respect to \( d(m - M) \) provides a measurement of \( n_M(M, r) \).
Methods: Star Counts

- So dependence of counts on the limiting magnitude \( m \) determines the number density and e.g. the edge of the system.

- In fact, if there were no edge to system the total flux would diverge as \( m \to 0 \) - volume grows as \( d^3 \) flux decreases as \( d^{-2} \): Olber’s paradox.

- Generalizations of the basic method:

- Selection criteria is not a perfect indicator of \( M \) and so \( dM \) is not infinitesimal and some stars in the range will be missed - \( S(M) \) and \( M \) is integrated over - total number

\[
N = \int_{-\infty}^{\infty} dM S(M) N_M
\]
Methods: Star Counts

- Alternately use all stars [weak or no $S(M)$] but assume a functional form for $n_M$ e.g. derived from local estimates and assumed to be the same at larger $r$

In this case, measurements determine the normalization of a distribution with fixed shape and determine

$$n(r) = \int_{-\infty}^{\infty} n_M(M, r) dM$$

- Similar method applies to mapping out the Universe with galaxies
Methods: Star Counts

- **Kapteyn** used all of the stars (assumed to have the same $n_M$ shape in $r$)

- He inferred a **flattened spheroidal system** of $< 10$ kpc extent in plane and $< 2$ kpc out of plane: too small

- Missing: **interstellar extinction** dims stars dropping them out of the sample at a given limiting magnitude
Methods: Variable Stars

- With a good *indicator* of absolute magnitude or “standard candle” one can use *individual objects* to map out the structure of the Galaxy (and Universe)

- History: *Shapley* (1910-1920) used RR Lyrae and W Virginis variable stars - with a *period-luminosity* relation

  Radial oscillations with a *density* dependent sound speed - luminosity and density related on the *instability strip*

  Calibrated locally by moving cluster and other methods

- Measure the *period* of oscillation, infer a *luminosity* and hence an *absolute magnitude*, infer a *distance* from the observed *apparent magnitude*
Methods: Variable Stars

- Inferred a 100kpc scale for the Galaxy - overestimate due to differences in types of variable stars and interstellar extinction

- Apparent magnitude is dimmed by extinction leading to the variable stars being less distant than they appear

- Both Kapteyn and Shapley off because of dust extinction: discrepancy between two independent methods indicates systematic error

- Caveat emptor: in astronomy always want to see a cross check with two or more independent methods before believing result you read in the NYT!
Interstellar Extinction

- Dust (silicates, graphite, hydrocarbons) in ISM (Chap 12) dims stars at visible wavelengths making true distance less than apparent

- Distance formula modified to be

\[ \frac{d}{10\text{pc}} = 10^{(m_\lambda - M_\lambda - A_\lambda)/5} \]

where the extinction coefficient \( A_\lambda \geq 0 \) depends on wavelength \( \lambda \)

- Extinction also depends on direction, e.g. through the disk, through a giant molecular cloud, etc. Typical value at visible wavelengths and in the disk is 1 mag/kpc

- Dust emits or reradiates starlight in the infrared - maps from these frequencies [IRAS, DIRBE] can be used to calibrate extinction
Dust Emission
Kinematic Distances to Stars

- Only nearby stars have their distance measured by parallax - further than a parsec the change in angle is $< 1$ arcsec:
  \[ p(\text{arcsec}) = \frac{1 \text{pc}}{d} \]

- If proper motion across the sky can be measured from the change in angular position $\mu$ in rad/s
  \[ v_t = \mu d \]

- Often $v_t$ can be inferred from the radial velocity and a comparison with $\mu$ gives distance $d$ given assumption of the dynamics

- Example: Keplerian orbits of stars around galactic center
  \[ R_0 = 7.6 \pm 0.3 \text{kpc} \]

- Example: Stars in a moving cluster share a single total velocity whose direction can be inferred from apparent convergent motion (see Fig 24.30)
Methods: Stellar Kinematics

- Can infer more than just distance: SMBH
- Galactic center: follow orbits of stars close to galactic center
- One star: orbital period 15.2 yrs, eccentricity $e = 0.87$, perigalacticon distance (closest point on orbit to $F$) 120 AU = $1.8 \times 10^{13}$ m
- Estimate mass: $a = ae - r_p$ so semimajor axis

$$a = \frac{r_p}{1 - e} = 1.4 \times 10^{14} \text{ m}$$
Methods: Stellar Kinematics

- Kepler’s 3rd law

\[
M = \frac{4\pi^2a^3}{GP^2} = 7 \times 10^{36}\text{kg} = 3.5 \times 10^6 M_\odot
\]

- That much mass in that small a radius can plausibly only be a (supermassive) black hole

- Note that this is an example of the general statement that masses are estimated by taking

\[
M \approx \frac{v^2r}{G} = \frac{(2\pi a)^2a}{GP^2} = \frac{4\pi^2a^3}{GP^2}
\]
Methods: Stellar Kinematics

- Stars around sun - higher velocity, lower metalicity stars: thick disk, lower velocity higher metalicity stars orbiting with sun in thin disk

- Halo stars vs sun or LSR suggests orbital speed of $\Theta_0 = 220$ km/s

- Differential rotation $\Theta(R) = R\Omega(R)$ where $\Omega(R)$ is the angular velocity curve– observables are radial and tangential motion with respect to LSR

\[
\begin{align*}
  v_r &= R\Omega \cos \alpha - R_0\Omega_0 \sin \ell \\
  v_t &= R\Omega \sin \alpha - R_0\Omega_0 \cos \ell
\end{align*}
\]
Methods: Stellar Kinematics

- \( d \) (parallax) and \( R_0 \) are known observables, \( R \) is not - eliminate with trig relations

\[
R \cos \alpha = R_0 \sin \ell \quad R \sin \alpha = R_0 \cos \ell - d
\]

- Eliminate \( R \)

\[
v_r = (\Omega - \Omega_0) R_0 \sin \ell
\]

\[
v_t = (\Omega - \Omega_0) R_0 \cos \ell - \Omega d
\]

solve for \( \Omega(R) \) locally where

\[
\Omega - \Omega_0 \approx \frac{d \Omega}{d R} (R - R_0)
\]

\[
\approx \frac{1}{R_0} \left( \frac{d \Theta}{d R} - \frac{\Theta_0}{R_0} \right) (R - R_0) \quad [\Omega = \Theta/R]
\]

and \( d \ll R_0, \cos \beta \approx 1 \)
Methods: Stellar Kinematics

- Reduce with trig identities

\[ R_0 = d \cos \ell + R \cos \beta \approx d \cos \ell + R \]

\[ R - R_0 \approx -d \cos \ell \]

\[ \cos \ell \sin \ell = \frac{1}{2} \sin 2\ell \]

\[ \cos^2 \ell = \frac{1}{2} (\cos 2\ell + 1) \]

to obtain

\[ v_r \approx Ad \sin 2\ell \]

\[ v_t \approx Ad \cos 2\ell + Bd \]
Methods: Stellar Kinematics

- Oort constants

\[
A = -\frac{1}{2} \left[ \frac{d\Theta}{dR} - \frac{\Theta_0}{R_0} \right] = -\frac{R_0}{2} \frac{d\Omega}{dR}
\]

\[
B = -\frac{1}{2} \left[ \frac{d\Theta}{dR} + \frac{\Theta_0}{R_0} \right]
\]

- Observables \(v_r, v_t, \ell, d\): solve for Oort’s constants. From Hipparcos

\[
A = 14.8 \pm 0.8 \text{km/s/kpc}
\]

\[
B = -12.4 \pm 0.6 \text{km/s/kpc}
\]

- Angular velocity \(\Omega = v/r\) decreases with radius: differential rotation. Physical velocity \(\Theta(R)\):

\[
\left. \frac{d\Theta}{dR} \right|_{R_0} = -(A + B) = -2.4 \text{ km/s/kpc decreases slowly compared with 220km/s - flat rotation curve}
\]
Methods: 21 cm

- Spin interaction of the electron and proton leads to a spin flip transition in neutral hydrogen with wavelength 21 cm.
- Line does not suffer substantial extinction and can be used to probe the neutral gas and its radial velocity from the Doppler shift throughout the galaxy.
- No intrinsic distance measure.
- Neutral gas is distributed inhomogeneously in clouds leading to distinct peaks in emission along each sight line.
Methods: 21 cm

- Due to projection of velocities along the line of sight and differential rotation, the highest velocity occurs at the closest approach to the galactic center or tangent point.
- Build up a rotation curve interior to the solar circle $R < R_0$.
- Rotation curve steeply rises in the interior $R < 1\text{kpc}$, consistent with near rigid body rotation and then remains flat out through the solar circle.
Neutral Gas: 21cm Emission
Ionized Gas: Hα Line Emission
Cosmic Rays in $B$ Field: Synchrotron
Gamma Rays
Methods: Rotation Curves

- Extending the rotation curve beyond the solar circle with objects like Cepheids whose distances are known reveals a flat curve out to $\sim 20\text{kpc}$
- Mass required to keep rotation curves flat much larger than implied by stars and gas. Consider a test mass $m$ orbiting at a radius $r$ around an enclosed mass $M(r)$
Methods: Rotation Curves

- Setting the centripetal force to the gravitational force

\[
\frac{mv^2(r)}{r} = \frac{GM(r)m}{r^2}
\]

\[
v(r) = \left( \frac{GM(r)}{r} \right)^{1/2}
\]

Side note: this is the fundamental way masses are measured - balance internal motions of luminous matter with gravitational force - other examples: virial theorem with velocity dispersion, hydrostatic equilibrium with thermal motions

- Measuring the rotation curve \(v(r)\) is equivalent to measuring the mass profile \(M(r)\) or density profile \(\rho(r) \propto M(r)/r^3\)
Methods: Rotation Curves

- Flat rotation curve $v(r) = \text{const}$ implies $M \propto r$ - a mass linearly increasing with radius.

- Rigid rotation implies $\Omega = v/r = \text{const}$. $v \propto r$ or $M \propto r^3$ or $\rho = \text{const}$.

- Rotation curves in other galaxies show the same behavior: evidence that “dark matter” is ubiquitous in galaxies.
Methods: Rotation Curves

- Consistent with dark matter density given by

\[ \rho(r) = \frac{\rho_0}{1 + (r/a)^2} \]

- Also consistent with the NFW profile predicted by cold dark matter (e.g. weakly interacting massive particles or WIMPs)

\[ \rho(r) = \frac{\rho_0}{(r/a)(1 + r/a)^2} \]
Methods: Gravitational Lensing

- Rotation curves leave open the question of what dark matter is.
- Alternate hypothesis: dead stars or black holes - massive astrophysical compact halo object “MACHO”
- MACHOs have their mass concentrated into objects with mass comparable to the sun or large planet.
- A MACHO at an angular distance $u = \theta / \theta_E$ from the line of sight to the star will gravitationally lens or magnify the star by a factor of

$A(u) = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}}$

where $\theta_E$ is the Einstein ring radius in projection

$\theta_E = \sqrt{\frac{4GM}{c^2}} \frac{d_S - d_L}{d_S d_L}$
Methods: Gravitational Lensing

- A MACHO would move at a velocity typical of the disk and halo $v \sim 200\text{km/s}$ and so the star behind it would brighten as it crossed the line of sight to a background star. With $u_{\text{min}}$ as the distance of closest approach at $t = 0$

$$u^2(t) = u_{\text{min}}^2 + \left(\frac{vt}{d_L \theta_E}\right)^2$$

- Monitor a large number of stars for this characteristic brightening. Rate of events says how much of the dark matter could be in MACHOs.
Methods: Gravitational Lensing

- In the 1990’s large searches measured the rate of microlensing in the halo and bulge and determined that only a small fraction of its mass could be in MACHOs.
Methods: Gravitational Lensing

- Current searches (toward the bulge) are used to find planets.
- Enhanced microlensing by planet around star leads to a blip in the brightening.