Set 3:

Galaxy Evolution
Environment

- Galaxies are clustered, found in groups like the local group up to large clusters of galaxies like the Coma cluster.
- Small satellite galaxies like the LMC and SMC are merging into the Milky way. Recent discovery of other satellites like the Sagitarius dwarf and tidal streams.
Environment

- cD galaxies in centers of rich galaxy clusters are the products of frequent mergers in the cluster environment.
- HST images of galaxies in the process of merging
- Theoretically, structure in the universe is thought to form bottom up from the merger of small objects into large objects
- Over the lifetime of the universe, galaxy evolution is a violent process
Antennae Galaxies
Cartwheel Galaxy

HST - WFPC2

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Gas in Magellanic Stream
N-body and Hydro Simulations

- To understand the physical processes behind the observations, $N$-body and hydrodynamic simulations are used.
- In an interaction between galaxies, stars and dark matter essentially never physically collide - act as collisionless point particles or “$N$-bodies” that interact gravitationally.
- Gas is more complicated and can shock, etc - use hydrodynamic techniques + cooling and star formation.
Interactions and Mergers

- $N$-body simulations reproduce the main features of mergers in terms of stars.
- As galaxies approach, tidal forces pull stars out into tidal streams much like tides on the Earth - features like the Antennae galaxies or the Magellenic stream.
- Similar to the spiral arm considerations, conservation of angular momentum says that bodies that are pulled inwards advance in their orbits, outwards trail.
Interactions and Mergers

- In a minor merger, a satellite galaxy can warp the disk of a larger galaxy in a major merger, two spirals may have their disks disrupted and become an elliptical.

- Eventually the merger completes - though collisionless the stars interact gravitationally and their motion dissipates through dynamical friction.
Dynamical Friction

• Consider a single encounter of an object of mass $M$ with a (smaller) mass $m$

• Two body encounter can be re-expressed as a single particle of the reduced mass in the potential of the combined mass: Newton’s third law

$$M\ddot{x}_M = F_{Mm} = -F_{mM} = -m\ddot{x}_m$$

• Center of mass $x_{cm} = (Mx_M + mx_m) / (M + m)$ has zero acceleration (uniform velocity)

$$\ddot{x}_{cm} = 0 = \frac{M}{M + m}\ddot{x}_M + \frac{m}{M + m}\ddot{x}_m$$

• Separation $\mathbf{R} = x_m - x_M$ obeys

$$\ddot{\mathbf{R}} = \ddot{x}_m - \ddot{x}_M$$
Dynamical Friction

- Eliminate $x_M$

$$\ddot{x}_M = -\frac{m}{M} \dot{x}_m$$

$$\ddot{R} = \left(1 + \frac{m}{M}\right) \dot{x}_m$$

- Gravitational acceleration

$$\ddot{x}_m = -\frac{GM}{R^2} \hat{r}$$

$$\ddot{R} = -\frac{G(M + m)}{R^2} \hat{r}$$

- Test particle moving in gravitational potential of combined mass (equivalently particle with reduced mass $\mu = mM/(m + M)$ with the force $GMm/R^2$) if $M \gg m$ then $m$ is essentially the test mass and center of mass frame is rest frame of $M$
Dynamical Friction

• Want to find the change in velocity of $M$ due to interactions with $m$ given kinematics of the reduced mass $V = \dot{R}$

$$\Delta v_m - \Delta v_M = \Delta V$$

$$m\Delta v_m + M\Delta v_M = 0$$

$$\Delta v_M = - \left( \frac{m}{m + M} \right) \Delta V$$

• Now determine $\Delta V$ from single particle kinematics. Consider an initial relative velocity $V$ and an impact parameter $b$, the initial separation transverse to $V$

• If the impact parameter is sufficiently large then the encounter is weak and the trajectory of the test particle is only slightly deflected
Dynamical Friction

- The test particle then experiences the potential on the unperturbed trajectory: “Born approximation”. The force perpendicular to the velocity

\[
\dot{V}_\perp = -\frac{G(M + m)}{b^2 + x^2(t)} \frac{b}{\sqrt{b^2 + x^2}}
\]

where \(x(t) = V_0 t\) if \(t = 0\) and \(x = 0\) is set to be at the closest approach

\[
|\Delta V_\perp| = \int_{-\infty}^{\infty} dt \frac{G(M + m)b}{(b^2 + V_0^2 t^2)^{3/2}} = \frac{2G(M + m)}{bV_0}
\]

- Change in \(V_\perp\) has no net effect since there is an equal probability of an impact with \(-b\).
Dynamical Friction

- There is a coherent effect on $V_\parallel$. Energy conservation says that the speed $V_0$ is conserved so that $V_\parallel$ is reduced
  
  $$\theta_{\text{def}} \approx \sin \theta_{\text{defl}} = \frac{|\Delta V_\perp|}{V_0} = \frac{2G(M + m)}{bV_0^2}$$

  $$|\Delta V_\parallel| = V_0(1 - \cos \theta_{\text{defl}}) \approx \frac{1}{2}V_0\theta_{\text{defl}}^2 \approx \frac{2G^2(m + M)^2}{b^2V_0^3}$$

  with a direction opposite to $V_0$

- Back to the change in the velocity of the real mass $M$
  
  $$|\Delta v_{M\parallel}| = \frac{2G^2m(m + M)}{b^2V_0^3}$$

  with the same direction as $V_0$ - i.e. $M$ will get a kick in the direction of oncoming $m$ particles
Dynamical Friction

- Now consider the mass $M$ to be moving through a sea of particles $m$ with number density $n$ and mass density $\rho = mn$

- Rate of encounters at an impact parameter $db$ around $b$ will be $nV_0\sigma$ where $\sigma$ is the cross sectional area

\[ nV_0 \times 2\pi bdb \]

- Total rate of change of velocity is the integral over all allowed impact parameters

\[
\left| \frac{dv_M}{dt} \right| = \int_{b_{\text{min}}}^{b_{\text{max}}} V_0n|\Delta v_M||2\pi bdb
\]

\[
\left| \frac{dv_M}{dt} \right| = \frac{4\pi G^2 mn(m + M)}{V_0^2} \ln \frac{b_{\text{max}}}{b_{\text{min}}} = \frac{4\pi G^2 \rho(m + M)}{V_0^2} \ln \frac{b_{\text{max}}}{b_{\text{min}}}
\]
Dynamical Friction

- Rate depends weakly (logarithmically) on the limits for the impact parameter. $b_{\text{max}}$ is size of $m$ system. $b_{\text{min}}$ is set by the validity of the “Born approximation”

$$\Delta V_\perp = \frac{2G(M + m)}{b_{\text{min}}V_0} \approx V_0$$

$$b_{\text{min}} \approx \frac{2G(M + m)}{V_0^2}$$

- For $b_{\text{max}} < b_{\text{min}}$ this term must go to zero and a better calculation from Chandrashekar (see Binney & Tremaine) replaces the log “Gaunt” factor with

$$\ln \frac{b_{\text{max}}}{b_{\text{min}}} \to \ln \left[ 1 + \left( \frac{b_{\text{max}}V_0^2}{G(M + m)} \right)^2 \right]^{1/2} \equiv \ln \Lambda$$
Dynamical Friction

- Considering $M$ to be falling into a body of density $\rho$ whose particles $m \ll M$ have no net velocity $V_0 = -v_M$ there is a frictional force that will stop the body

$$M \frac{dv_M}{dt} \approx - \left[ \frac{4\pi G^2 \rho M^2}{v_M^2} \ln \Lambda \right] \hat{v}_M$$
Galaxy Formation

- The same process of merging but with smaller proto-Galactic objects of $10^6 - 10^8 \, M_\odot$ can eventually assemble the galaxies of $10^{12} \, M_\odot$ we see today. Both lower and upper range determined by cooling.

- Proto-galactic objects can form if cooling is sufficiently rapid that the heating of the gas during collapse (which would prevent collapse due to pressure, internal motions) can be overcome.

- Recall virial theorem supplies estimate of thermal kinetic energy

\[-2\langle K \rangle = \langle U \rangle\]

\[-2N \frac{1}{2} \mu m_H \sigma^2 = -\frac{3}{5} \frac{G M N \mu m_H}{R}\]

where $\mu m_H$ is the average mass of particles in the gas, $M$ is the total mass and $\sigma$ is the rms velocity.
Galaxy Formation

- Solve for velocity dispersion for a self gravitating system

\[ \sigma = \left( \frac{3 \, GM}{5 \, R} \right)^{1/2} \]

- Associate the average kinetic energy with a temperature, called the virial temperature

\[ \frac{1}{2} \mu m_H \sigma^2 = \frac{3}{2} k T_{\text{virial}} \]

where \( \mu \) is the mean molecular weight. Solve for virial temperature

\[ T_{\text{virial}} = \frac{\mu m_H \sigma^2}{3k} = \frac{\mu m_H}{5k} \frac{GM}{R} \approx \frac{\mu m_H}{5k} GM^{2/3} \left( \frac{4\pi \rho}{3} \right)^{1/3} \]

- Cooling is a function of the gas temperature through the cooling function.
Galaxy Formation

- Cooling rate (luminosity) per volume

\[ r_{\text{cool}} = n^2 \Lambda(T) \]

\( n^2 \) (number density squared) comes from the fact that cooling is usually a 2 body process - for \( T > 10^6 \text{K} \) thermal bremsstrahlung and Compton scattering, for \( T \sim 10^4 - 10^5 \text{K} \) from the collisional excitation of atomic lines of hydrogen and helium

- Galaxy formation only starts when dark matter mass makes the virial temperature exceed \( T \sim 10^4 \text{K} \) when cooling becomes efficient \( M \sim 10^8 M_\odot \) -first objects and current dwarf ellipticals
Galaxy Formation

- Cooling time is the time required to radiate away all of the thermal energy of the gas

\[ r_{\text{cool}} V t_{\text{cool}} = \frac{3}{2} N k T_{\text{virial}} \]

\[ t_{\text{cool}} = \frac{3}{2} \frac{k T_{\text{virial}}}{n \Lambda} \]

- Compared with the free fall time - from our dimensional relation

\[ G \rho R^3 \propto G M \sim R v^2 \sim R (R^2 / t_{\text{ff}}^2) \]

we get \( t_{\text{ff}} \propto (G \rho)^{-1/2} \) with the proportionality given for the time of collapse for a homogeneous sphere of initial density \( \rho \)

\[ t_{\text{ff}} = \left( \frac{3 \pi}{32} \frac{1}{G \rho} \right)^{1/2} \]
Galaxy Formation

• If $t_{\text{cool}} < t_{\text{ff}}$ then the object will collapse essentially in free fall - fragment and form stars. If opposite, then gravitational potential energy heats the gas making it stabilized by pressure establishing virial equilibrium

$$
\left( \frac{t_{\text{ff}}}{t_{\text{cool}}} \right) > \left( \frac{3\pi}{32} \frac{1}{G\rho} \right)^{1/2} \frac{2}{3} \frac{n\Lambda}{kT_{\text{virial}}}
$$

• Taking typical numbers $T \sim 10^6 \text{K}$ and $n \sim 5 \times 10^4 \text{m}^{-3}$ and with the density of the collapsing medium being associated with the gas $\rho = \mu m_H n$ gives an upper limit on the gas mass that can cool of $10^{12} M_\odot$ comparable to a large galaxy.
Disk Formation

- Proto-galactic gas fragment and collide retaining initial angular momentum provided from torques from other proto-galactic systems
- Rotationally supported gas disk, cooling in dense regions until HI clouds form from which star formation occurs - thick disk
- Cool molecular gas settles to midplane of thick disk efficiently forming stars - thinness is self regulating - if disk continued to get thinner then density and star formation goes up heating the material and re-puffing out the disk