

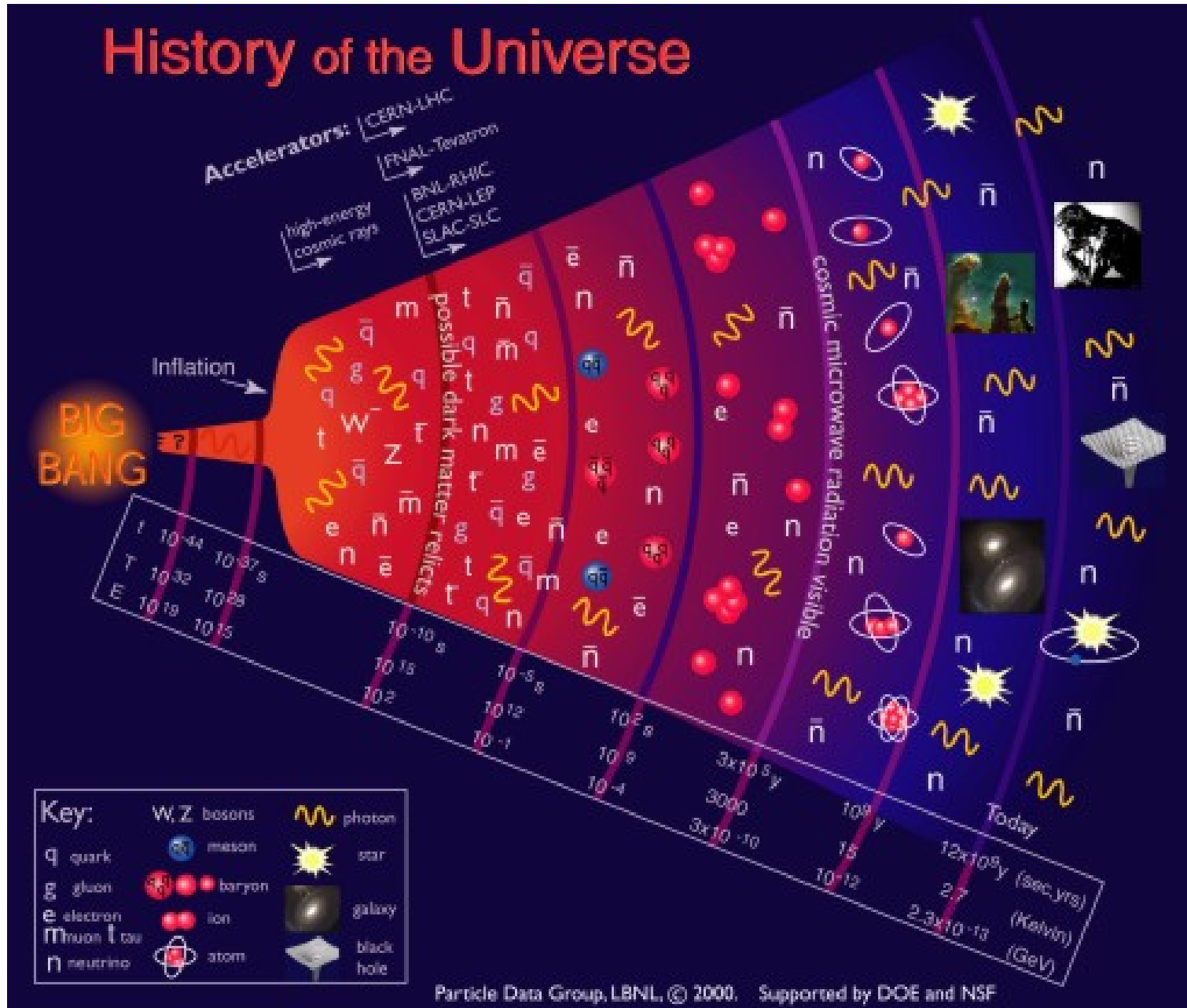
Set 7:

Thermal History

# Macro vs Micro Description

- In FRW cosmology notes, we used a **macroscopic** description.
- **Gravity** only cares about **bulk properties**: energy density, momentum density, pressure, anisotropic stress – **stress tensor**
- Matter and radiation is composed of particles whose properties can be described by their **phase space distribution** or occupation function
- Macroscopic properties are integrals or **moments** of the **phase space distribution**
- Particle **interactions** involve the evolution of the phase space distribution
- Rapid interactions drive distribution to **thermal equilibrium** but must compete with the expansion rate of universe
- **Freeze out**, the origin of species

# Brief Thermal History



# Origin Examples

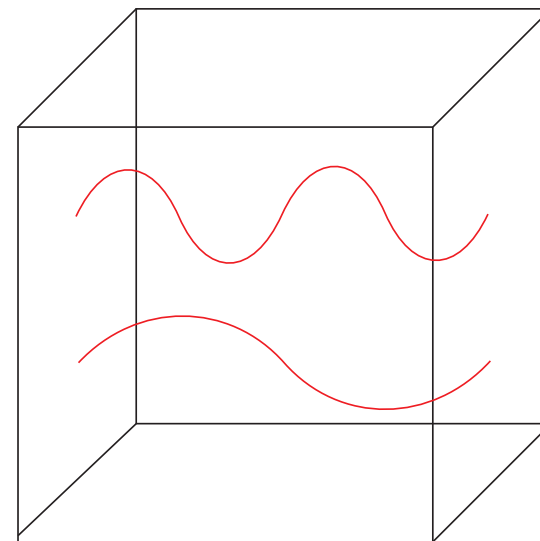
- Neutrino background (weak freezeout)
- CDM freezeout (annihilation freezeout)
- Light elements (nuclear statistical equilibrium freezeout)
- Baryogenesis
- Blackbody freezeout (thermalization)
- Atomic hydrogen (recombination; free electron freezeout)

# Fitting in a Box

- Counting momentum states with momentum  $q$  and de Broglie wavelength

$$\lambda = \frac{h}{q} = \frac{2\pi\hbar}{q}$$

- In a discrete volume  $L^3$  there is a discrete set of states that satisfy periodic boundary conditions
- We will hereafter set  $\hbar = c = 1$
- As in Fourier analysis



$$e^{2\pi i x/\lambda} = e^{iqx} = e^{iq(x+L)} \rightarrow e^{iqL} = 1$$

# Fitting in a Box

- Periodicity yields a discrete set of allowed states

$$Lq = 2\pi m_i, \quad m_i = 1, 2, 3\dots$$

$$q_i = \frac{2\pi}{L} m_i$$

- In each of 3 directions

$$\sum_{m_{xi} m_{yj} m_{zk}} \rightarrow \int d^3 m$$

- The differential number of allowed momenta in the volume

$$d^3 m = \left( \frac{L}{2\pi} \right)^3 d^3 q$$

# Density of States

- The total number of states allows for a number of internal degrees of freedom, e.g. spin, quantified by the degeneracy factor  $g$
- Total density of states:

$$\frac{dN_s}{V} = \frac{g}{V} d^3 m = \frac{g}{(2\pi)^3} d^3 q$$

- If all states were occupied by a single particle, then particle density

$$n_s = \frac{N_s}{V} = \frac{1}{V} \int dN_s = \int \frac{g}{(2\pi)^3} d^3 q$$

# Distribution Function

- The distribution function  $f$  quantifies the occupation of the allowed momentum states

$$n = \frac{N}{V} = \frac{1}{V} \int f dN_s = \int \frac{g}{(2\pi)^3} f d^3q$$

- $f$ , aka phase space occupation number, also quantifies the density of particles per unit phase space  $dN/(\Delta x)^3(\Delta q)^3$
- For photons, the spin degeneracy  $g = 2$  accounting for the 2 polarization states
- Energy  $E(q) = (q^2 + m^2)^{1/2}$
- Momentum  $\rightarrow$  frequency  $q = 2\pi/\lambda = 2\pi\nu = \omega = E$  (where  $m = 0$  and  $\lambda\nu = c = 1$ )

# Bulk Properties

- Integrals over the distribution function define the bulk properties of the collection of particles
- Number density

$$n(\mathbf{x}, t) \equiv \frac{N}{V} = g \int \frac{d^3q}{(2\pi)^3} f$$

- Energy density

$$\rho(\mathbf{x}, t) = g \int \frac{d^3q}{(2\pi)^3} E(q) f$$

where  $E^2 = q^2 + m^2$

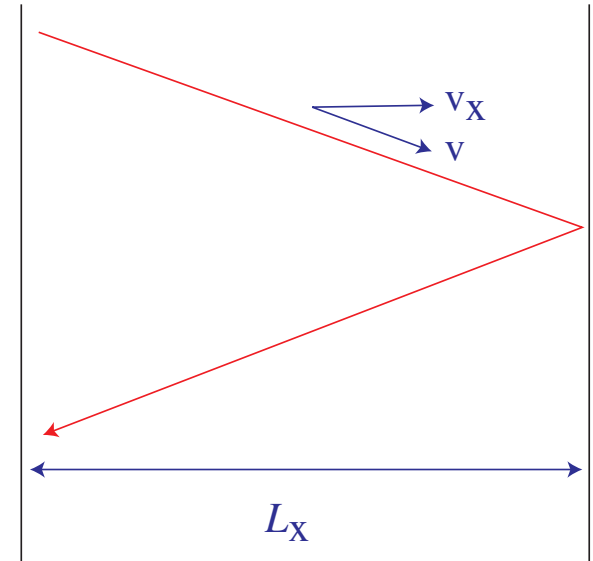
- Momentum density

$$(\rho + p)\mathbf{v}(\mathbf{x}, t) = g \int \frac{d^3q}{(2\pi)^3} \mathbf{q} f$$

# Bulk Properties

- Pressure: particles bouncing off a surface of area  $A$  in a volume spanned by  $L_x$ : per momentum state

$$\begin{aligned} p_q &= \frac{F}{A} = \frac{N_{\text{part}}}{A} \frac{\Delta q}{\Delta t} \\ &\quad (\Delta q = 2|q_x|, \quad \Delta t = 2L_x/v_x, ) \\ &= \frac{N_{\text{part}}}{V} |q_x| |v_x| = \frac{N_{\text{part}}}{V} \frac{|q||v|}{3} \\ &\quad (v = \gamma m v / \gamma m = q/E) \\ &= \frac{N_{\text{part}}}{V} \frac{q^2}{3E} \end{aligned}$$



# Bulk Properties

- So that summed over occupied momenta states

$$p(\mathbf{x}, t) = g \int \frac{d^3 q}{(2\pi)^3} \frac{|q|^2}{3E(q)} f$$

- Pressure is just one of the quadratic in  $q$  moments, in particular the isotropic one
- The remaining 5 components are the anisotropic stress (vanishes in the background)

$$\pi^i_j(\mathbf{x}, t) = g \int \frac{d^3 q}{(2\pi)^3} \frac{3q^i q_j - q^2 \delta^i_j}{3E(q)} f$$

- We shall see that these are related to the 5 quadrupole moments of the angular distribution

# Bulk Properties

- These are more generally the components of the stress-energy tensor

$$T^{\mu}_{\nu} = g \int \frac{d^3q}{(2\pi)^3} \frac{q^{\mu} q_{\nu}}{E(q)} f$$

- 0-0: energy density
- 0- $i$ : momentum density
- $i - i$ : pressure
- $i \neq j$ : anisotropic stress
- In the FRW background cosmology, isotropy requires that there be only a net energy density and pressure

# Liouville Equation

- Liouville theorem: phase space distribution function is conserved along a trajectory in the absence of particle interactions

$$\frac{Df}{Dt} = \left[ \frac{\partial}{\partial t} + \frac{d\mathbf{q}}{dt} \frac{\partial}{\partial \mathbf{q}} + \frac{d\mathbf{x}}{dt} \frac{\partial}{\partial \mathbf{x}} \right] f = 0$$

Expanding universe: de Broglie wavelength of particles “stretches”

$$q \propto a^{-1}$$

- Homogeneous and isotropic limit

$$\frac{\partial f}{\partial t} + \frac{dq}{dt} \frac{\partial f}{\partial q} = \frac{\partial f}{\partial t} - H(a) \frac{\partial f}{\partial \ln q} = 0$$

- Implies energy conservation:  $d\rho/dt = -3H(\rho + p)$

# Boltzmann Equation

- Boltzmann equation says that Liouville theorem must be modified to account for collisions

$$\frac{Df}{Dt} = C[f]$$

- Heuristically

$$C[f] = \text{particle sources} - \text{sinks}$$

- Collision term: integrate over phase space of incoming particles, connect to outgoing state with some interaction strength

# Poor Man's Boltzmann Equation

- Non expanding medium

$$\frac{\partial f}{\partial t} = \Gamma (f - f_{\text{eq}})$$

where  $\Gamma$  is some rate for collisions

- Add in expansion in a homogeneous medium

$$\frac{\partial f}{\partial t} + \frac{dq}{dt} \frac{\partial f}{\partial q} = \Gamma (f - f_{\text{eq}})$$

$$(q \propto a^{-1} \rightarrow \frac{1}{q} \frac{dq}{dt} = -\frac{1}{a} \frac{da}{dt} = H)$$

$$\frac{\partial f}{\partial t} - H \frac{\partial f}{\partial \ln q} = \Gamma (f - f_{\text{eq}})$$

- So equilibrium will be maintained if collision rate exceeds expansion rate  $\Gamma = n \langle \sigma v \rangle > H$

# Equilibrium

- Thermal physics describes the equilibrium distribution of particles for a medium at temperature  $T$
- Expect that the typical energy of a particle by equipartition is  $E \sim T$ , so that  $f_{\text{eq}}(E/T, ?)$  in equilibrium
- Must be a second variable of import. Number density

$$n = g \int \frac{d^3 q}{(2\pi\hbar)^3} f_{\text{eq}}(E/T) =? \quad n(T)$$

- If particles are conserved then  $n$  cannot simply be a function of temperature.
- The integration constant that concerns particle conservation is called the chemical potential. Relevant for photons when creation and annihilation processes are ineffective

# Temperature and Chemical Potential

- Fundamental assumption of statistical mechanics is that all accessible states have an equal probability of being populated. The number of states  $G$  defines the entropy  $S(U, N, V) = \ln G$  where  $U$  is the energy,  $N$  is the number of particles and  $V$  is the volume
- When two systems are placed in thermal contact they may exchange energy, leading to a wider range of accessible states

$$G(U, N, V) = \sum_{U_1} G_1(U_1, N_1, V_1) G_2(U - U_1, N - N_1, V - V_1)$$

- The most likely distribution of  $U_1$  and  $U_2$  is given for the maximum  $dG/dU_1 = 0$

$$\left( \frac{\partial G_1}{\partial U_1} \right)_{N_1, V_1} G_2 dU_1 + G_1 \left( \frac{\partial G_2}{\partial U_2} \right)_{N_2, V_2} dU_2 = 0 \quad dU_1 + dU_2 = 0$$

# Temperature and Chemical Potential

- Or equilibrium requires

$$\left( \frac{\partial \ln G_1}{\partial U_1} \right)_{N_1, V_1} = \left( \frac{\partial \ln G_2}{\partial U_2} \right)_{N_2, V_2} \equiv \frac{1}{T}$$

which is the definition of the temperature (equal for systems in thermal contact)

- Likewise define a chemical potential  $\mu$  for a system in diffusive equilibrium

$$\left( \frac{\partial \ln G_1}{\partial N_1} \right)_{U_1, V_1} = \left( \frac{\partial \ln G_2}{\partial N_2} \right)_{U_2, V_2} \equiv -\frac{\mu}{T}$$

defines the most likely distribution of particle numbers as a system with equal chemical potentials: generalize to multiple types of particles undergoing “chemical” reaction  $\rightarrow$  law of mass action

$$\sum_i \mu_i dN_i = 0$$

# Temperature and Chemical Potential

- Equivalent definition: the chemical potential is the free energy cost associated with adding a particle at fixed temperature and volume

$$\mu = \left. \frac{\partial F}{\partial N} \right|_{T,V}, \quad F = U - TS$$

free energy: balance between minimizing energy and maximizing entropy  $S$

- Temperature and chemical potential determine the probability of a state being occupied if the system is in thermal and diffusive contact with a large reservoir at temperature  $T$

# Gibbs or Boltzmann Factor

- Suppose the system has two states unoccupied  $N_1 = 0, U_1 = 0$  and occupied  $N_1 = 1, U_1 = E$  then the ratio of probabilities in the occupied to unoccupied states is given by

$$P = \frac{\exp[\ln G_{\text{res}}(U - E, N - 1, V)]}{\exp[\ln G_{\text{res}}(U, N, V)]}$$

- Taylor expand

$$\ln G_{\text{res}}(U - E, N - 1, V) \approx \ln G_{\text{res}}(U, N, V) - \frac{E}{T} + \frac{\mu}{T}$$

$$P \approx \exp[-(E - \mu)/T]$$

- This is the Gibbs factor.

# Gibbs or Boltzmann Factor

- More generally the probability of a system being in a state of energy  $E_i$  and particle number  $N_i$  is given by the Gibbs factor

$$P(E_i, N_i) \propto \exp[-(E_i - \mu N_i)/T]$$

- Unlikely to be in an energy state  $E_i \gg T$  mitigated by the number of particles
- Dropping the diffusive contact, this is the Boltzmann factor

# Thermal & Diffusive Equilibrium

- A gas in thermal & diffusive contact with a reservoir at temperature  $T$
- Probability of system being in state of energy  $E_i$  and number  $N_i$  (Gibbs Factor)

$$P(E_i, N_i) \propto \exp[-(E_i - \mu N_i)/T]$$

where  $\mu$  is the chemical potential (defines the free energy “cost” for adding a particle at fixed temperature and volume)

- Chemical potential appears when particles are conserved
- CMB photons can carry chemical potential if creation and annihilation processes inefficient, as they are after  $t \sim 1\text{yr}$ .

# Distribution Function

- Mean occupation of the state in thermal equilibrium

$$f \equiv \frac{\sum N_i P(E_i, N_i)}{\sum P(E_i, N_i)}$$

where the total energy is related to the particle energy as

$$E_i = N_i E \text{ (ignoring zero pt)}$$

- Density of (energy) states in phase space makes the net spatial density of particles

$$n = g \int \frac{d^3 p}{(2\pi)^3} f$$

where  $g$  is the number of spin states

# Fermi-Dirac Distribution

- For fermions, the occupancy can only be  $N_i = 0, 1$

$$\begin{aligned} f &= \frac{P(E, 1)}{P(0, 0) + P(E, 1)} \\ &= \frac{e^{-(E-\mu)/T}}{1 + e^{-(E-\mu)/T}} \\ &= \frac{1}{e^{(E-\mu)/T} + 1} \end{aligned}$$

- In the non-relativistic limit

$$E = (p^2 + m^2)^{1/2} \approx m + \frac{1}{2} \frac{q^2}{m}$$

and  $m \gg T$  so the distribution is Maxwell-Boltzmann

$$f = e^{-(m-\mu)/T} e^{-q^2/2mT} = e^{-(m-\mu)/T} e^{-mv^2/2T}$$

# Bose-Einstein Distribution

- For bosons each state can have multiple occupation,

$$f = \frac{\frac{d}{d\mu/T} \sum_{N=0}^{\infty} (e^{-(E-\mu)/T})^N}{\sum_{N=0}^{\infty} (e^{-(E-\mu)/T})^N} \quad \text{with} \quad \sum_{N=0}^{\infty} x^N = \frac{1}{1-x}$$
$$= \frac{1}{e^{(E-\mu)/T} - 1}$$

- Again, non relativistic distribution is Maxwell-Boltzmann

$$f = e^{-(m-\mu)/T} e^{-q^2/2mT} = e^{-(m-\mu)/T} e^{-mv^2/2T}$$

with a spatial number density

$$n = g e^{-(m-\mu)/T} \int \frac{d^3q}{(2\pi)^3} e^{-q^2/2mT}$$
$$= g e^{-(m-\mu)/T} \left( \frac{mT}{2\pi} \right)^{3/2}$$

# Ultra-Relativistic Bulk Properties

- Chemical potential  $\mu = 0$ ,  $\zeta(3) \approx 1.202$
- Number density

$$n_{\text{boson}} = gT^3 \frac{\zeta(3)}{\pi^2} \quad \zeta(n+1) \equiv \frac{1}{n!} \int_0^\infty dx \frac{x^n}{e^x - 1}$$

$$n_{\text{fermion}} = \frac{3}{4} gT^3 \frac{\zeta(3)}{\pi^2}$$

- Energy density

$$\rho_{\text{boson}} = gT^4 \frac{3}{\pi^2} \zeta(4) = gT^4 \frac{\pi^2}{30}$$

$$\rho_{\text{fermion}} = \frac{7}{8} gT^4 \frac{3}{\pi^2} \zeta(4) = \frac{7}{8} gT^4 \frac{\pi^2}{30}$$

- Pressure  $q^2/3E = E/3 \rightarrow p = \rho/3$ ,  $w_r = 1/3$

# Entropy Density

- Second law of thermodynamics

$$dS = \frac{1}{T}(d\rho(T)V + p(T)dV)$$

so that

$$\left. \frac{\partial S}{\partial V} \right|_T = \frac{1}{T}[\rho(T) + p(T)]$$
$$\left. \frac{\partial S}{\partial T} \right|_V = \frac{V}{T} \frac{d\rho}{dT}$$

- Since  $S(V, T) \propto V$  is extensive

$$S = \frac{V}{T}[\rho(T) + p(T)] \quad \sigma = S/V = \frac{1}{T}[\rho(T) + p(T)]$$

# Entropy Density

- Integrability condition  $dS/dV dT = dS/dT dV$  relates the evolution of entropy density

$$\begin{aligned}\frac{d\sigma}{dT} &= \frac{1}{T} \frac{d\rho}{dT} \\ \frac{d\sigma}{dt} &= \frac{1}{T} \frac{d\rho}{dt} = \frac{1}{T} [-3(\rho + p)] \frac{d \ln a}{dt} \\ \frac{d \ln \sigma}{dt} &= -3 \frac{d \ln a}{dt} \quad \sigma \propto a^{-3}\end{aligned}$$

comoving entropy density is conserved in thermal equilibrium

- For ultra relativistic bosons  $\sigma_{\text{boson}} = 3.602 n_{\text{boson}}$ ; for fermions factor of 7/8 from energy density.

$$g_* = \sum_{\text{bosons}} g_b + \frac{7}{8} \sum g_f$$

# Neutrino Freezeout

- Neutrino equilibrium maintained by weak interactions, e.g.  
 $e^+ + e^- \leftrightarrow \nu + \bar{\nu}$
- Weak interaction cross section  $T_{10} = T/10^{10} K \sim T/1\text{MeV}$

$$\sigma_w \sim G_F^2 E_\nu^2 \approx 4 \times 10^{-44} T_{10}^2 \text{cm}^2$$

- Rate  $\Gamma = n_\nu \sigma_w = H$  at  $T_{10} \sim 3$  or  $t \sim 0.2\text{s}$
- After neutrino freezeout, electrons and positrons annihilate dumping their entropy into the photons
- Before  $g_*$ :  $\gamma, e^+, e^- = 2 + \frac{7}{8}(2 + 2) = \frac{11}{2}$
- After  $g_*$ :  $\gamma = 2$ ; so conservation of entropy gives

$$g_* T^3 \Big|_{\text{initial}} = g_* T^3 \Big|_{\text{final}} \quad T_\nu = \left( \frac{4}{11} \right)^{1/3} T_\gamma$$

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# Relic Neutrinos

- Relic number density (zero chemical potential; now required by oscillations & BBN)

$$n_\nu = n_\gamma \frac{3}{4} \frac{4}{11} = 112 \text{cm}^{-3}$$

- Relic energy density assuming one species with finite  $m_\nu$ :

$$\rho_\nu = m_\nu n_\nu$$

$$\rho_\nu = 112 \frac{m_\nu}{\text{eV}} \text{eV cm}^{-3} \quad \rho_c = 1.05 \times 10^4 h^2 \text{eV cm}^{-3}$$

$$\Omega_\nu h^2 = \frac{m_\nu}{93.7 \text{eV}}$$

- Candidate for dark matter? an eV mass neutrino goes non relativistic around  $z \sim 1000$  and retains a substantial velocity dispersion  $\sigma_\nu$ .

# Hot Dark Matter

- Momenta for a nonrelativistic species redshifts like temperature for a relativistic one, so average momentum is still given by

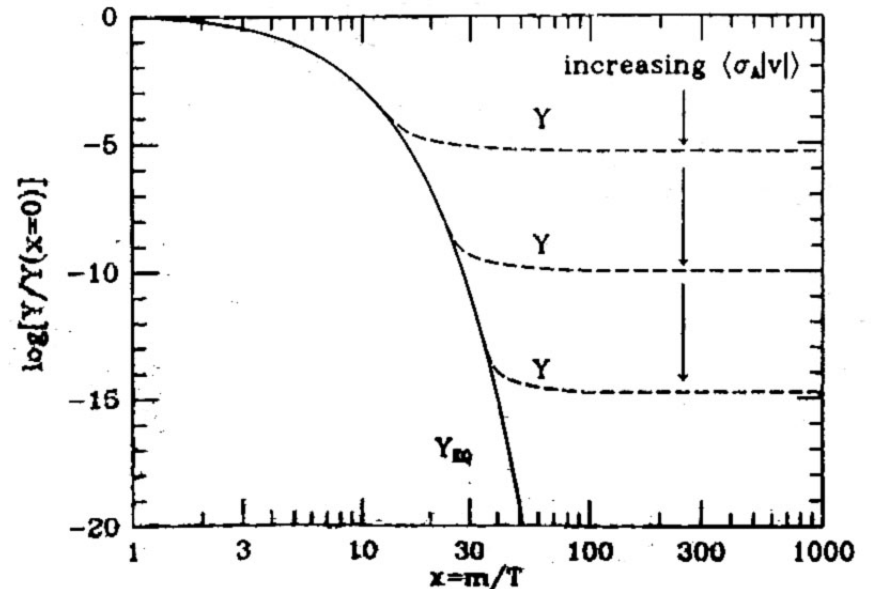
$$\langle q \rangle = 3T_\nu = m\sigma_\nu$$

$$\begin{aligned}\sigma_\nu &= 3 \left( \frac{m_\nu}{1\text{eV}} \right)^{-1} \left( \frac{T_\nu}{1\text{eV}} \right) = 3 \left( \frac{m_\nu}{1\text{eV}} \right)^{-1} \left( \frac{T_\nu}{10^4\text{K}} \right) \\ &= 6 \times 10^{-4} \left( \frac{m_\nu}{1\text{eV}} \right)^{-1} = 200\text{km/s} \left( \frac{m_\nu}{1\text{eV}} \right)^{-1}\end{aligned}$$

- Of order the rotation velocity of galactic halos and higher at higher redshift - small objects can't form: top down structure formation – not observed – must not constitute the bulk of the dark matter

# Cold Dark Matter

- Problem with neutrinos is they decouple while relativistic and hence have a comparable number density to photons - for a reasonable energy density, the mass must be small



- The equilibrium distribution for a non-relativistic species declines exponentially beyond the mass threshold

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$$

- Exponential will eventually win soon after  $T < m$ , suppressing annihilation rates

# WIMP Miracle

- Freezeout when annihilation rate equal expansion rate  $\Gamma \propto \sigma_A$ , increasing annihilation cross section decreases abundance

$$\Gamma = n \langle \sigma_A v \rangle = H$$

$$H \propto T^2 \sim m^2$$

$$\rho_{\text{freeze}} = mn \propto \frac{m^3}{\langle \sigma_A v \rangle}$$

$$\rho_c = \rho_{\text{freeze}} (T/T_0)^{-3} \propto \frac{1}{\langle \sigma_A v \rangle}$$

independently of the mass of the CDM particle

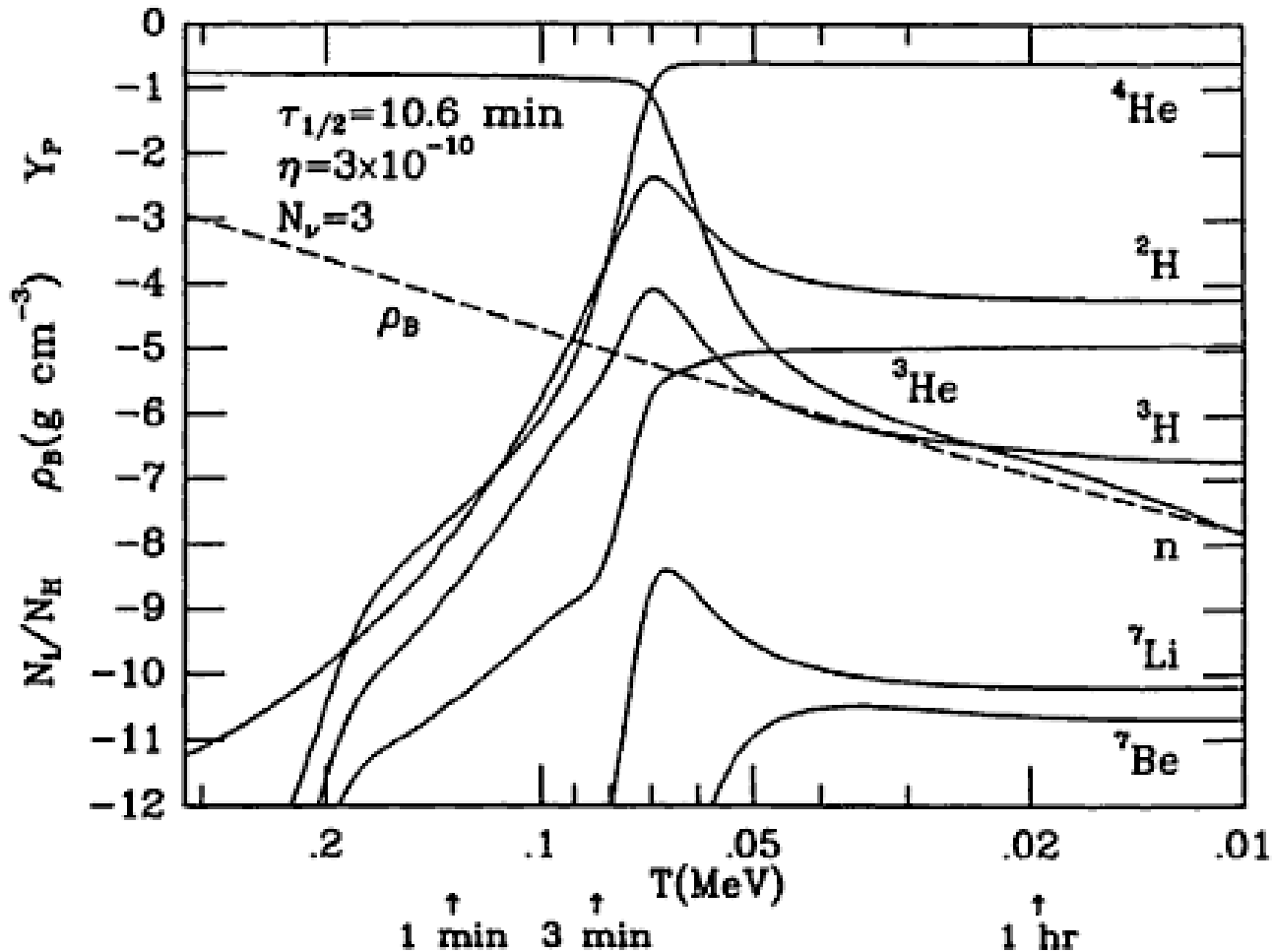
- Plug in some typical numbers for supersymmetric candidates or WIMPs (weakly interacting massive particles) of  $\langle \sigma_A v \rangle \approx 10^{-36} \text{ cm}^2$  and restore the proportionality constant  $\Omega_c h^2$  is of the right order of magnitude ( $\sim 0.1$ )!

# Axions

- Alternate solution: keep light particle but not created in thermal equilibrium
- Example: axion dark matter - particle that solves the strong CP problem
- Inflation sets initial conditions, fluctuation from potential minimum
- Once Hubble scale smaller than the mass scale, field unfreezes
- Coherent oscillations of the axion field - condensate state. Can be very light  $m \ll 1\text{eV}$  and yet remain cold.
- Same reason a quintessence dark energy candidate must be lighter than the Hubble scale today

# Big Bang Nucleosynthesis

- Integrating the Boltzmann equation for nuclear processes during first few minutes leads to synthesis and freezeout of light elements



# Big Bang Nucleosynthesis

- Most of light element synthesis can be understood through nuclear statistical equilibrium and reaction rates
- Equilibrium abundance of species with mass number  $A$  and charge  $Z$  ( $Z$  protons and  $A - Z$  neutrons)

$$n_A = g_A \left( \frac{m_A T}{2\pi} \right)^{3/2} e^{(\mu_A - m_A)/T}$$

- In chemical equilibrium with protons and neutrons

$$\mu_A = Z\mu_p + (A - Z)\mu_n$$

$$n_A = g_A \left( \frac{m_A T}{2\pi} \right)^{3/2} e^{-m_A/T} e^{(Z\mu_p + (A - Z)\mu_n)/T}$$

# Big Bang Nucleosynthesis

- Eliminate chemical potentials with  $n_p, n_n$

$$e^{\mu_p/T} = \frac{n_p}{g_p} \left( \frac{2\pi}{m_p T} \right)^{3/2} e^{m_p/T}$$

$$e^{\mu_n/T} = \frac{n_n}{g_n} \left( \frac{2\pi}{m_n T} \right)^{3/2} e^{m_n/T}$$

$$n_A = g_A g_p^{-Z} g_n^{Z-A} \left( \frac{m_A T}{2\pi} \right)^{3/2} \left( \frac{2\pi}{m_p T} \right)^{3Z/2} \left( \frac{2\pi}{m_n T} \right)^{3(A-Z)/2} \\ \times e^{-m_A/T} e^{(Z\mu_p + (A-Z)\mu_n)/T} n_p^Z n_n^{A-Z}$$

$$(g_p = g_n = 2; m_p \approx m_n = m_b = m_A/A)$$

$$(B_A = Zm_p + (A - Z)m_n - m_A)$$

$$= g_A 2^{-A} \left( \frac{2\pi}{m_b T} \right)^{3(A-1)/2} A^{3/2} n_p^Z n_n^{A-Z} e^{B_A/T}$$

# Big Bang Nucleosynthesis

- Convenient to define abundance fraction

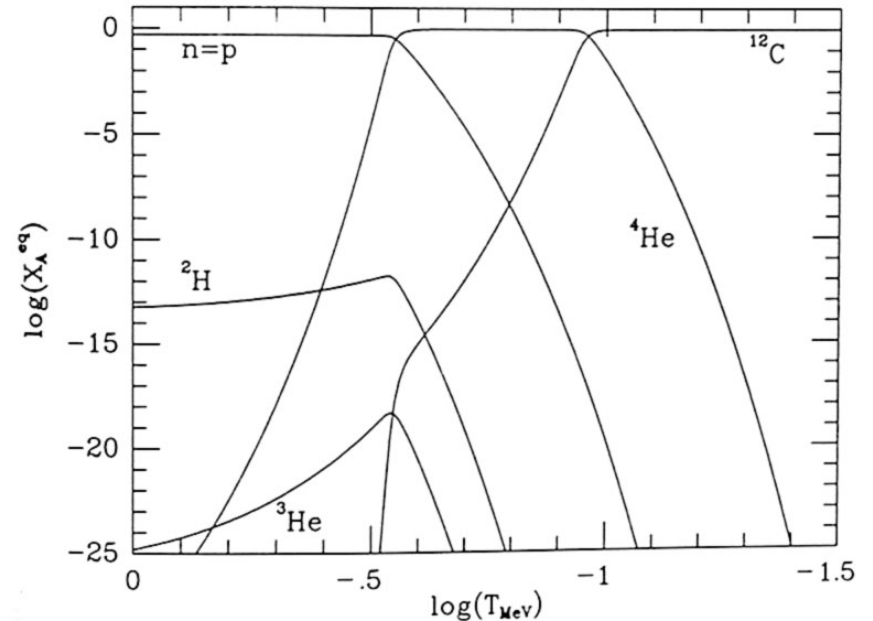
$$\begin{aligned}
 X_A &\equiv A \frac{n_A}{n_b} = A g_A 2^{-A} \left( \frac{2\pi}{m_b T} \right)^{3(A-1)/2} A^{3/2} n_p^Z n_n^{A-Z} n_b^{-1} e^{B_A/T} \\
 &= A g_A 2^{-A} \left( \frac{2\pi n_b^{2/3}}{m_b T} \right)^{3(A-1)/2} A^{3/2} e^{B_A/T} X_p^Z X_n^{A-Z} \\
 &\quad \left( n_\gamma = \frac{2}{\pi^2} T^3 \zeta(3) \quad \eta_{b\gamma} \equiv n_b / n_\gamma \right) \\
 &= A^{5/2} g_A 2^{-A} \left[ \left( \frac{2\pi T}{m_b} \right)^{3/2} \frac{2\zeta(3)\eta_{b\gamma}}{\pi^2} \right]^{A-1} e^{B_A/T} X_p^Z X_n^{A-Z}
 \end{aligned}$$

# Deuterium

- Deuterium  $A = 2$ ,  $Z = 1$ ,  $g_2 = 3$ ,  $B_2 = 2.225$  MeV

$$X_2 = \frac{3}{\pi^2} \left( \frac{4\pi T}{m_b} \right)^{3/2} \eta_{b\gamma} \zeta(3) e^{B_2/T} X_p X_n$$

- Deuterium  
“bottleneck” is mainly  
due to the low baryon-photon  
number of the universe  
 $\eta_{b\gamma} \sim 10^{-9}$ , secondarily due  
to the low binding energy  $B_2$



# Deuterium

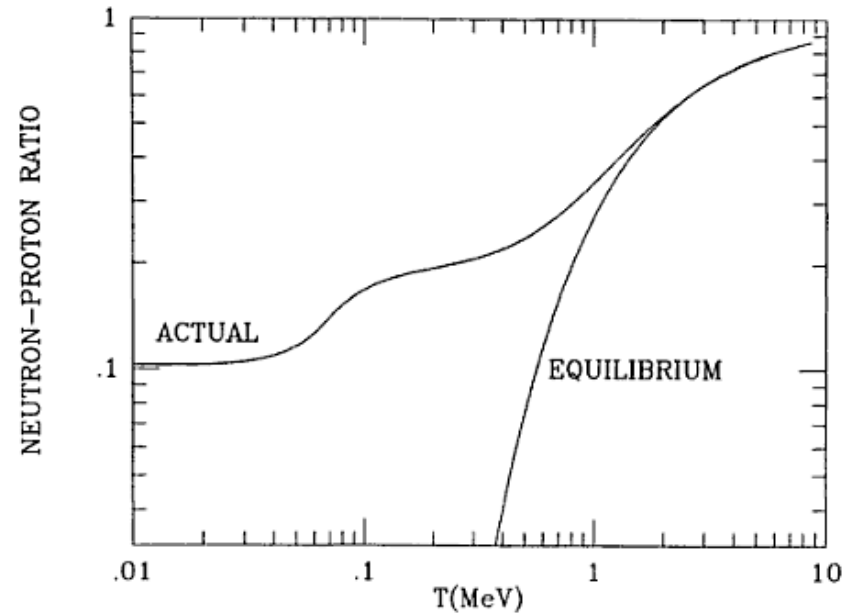
- $X_2/X_p X_n \approx \mathcal{O}(1)$  at  $T \approx 100\text{keV}$  or  $10^9\text{ K}$ , much lower than the binding energy  $B_2$
- Most of the deuterium formed then goes through to helium via  
 $D + D \rightarrow {}^3\text{He} + p$ ,  ${}^3\text{He} + D \rightarrow {}^4\text{He} + n$
- Deuterium freezes out as number abundance becomes too small to maintain reactions  $n_D = \text{const.}$  independent of  $n_b$
- The deuterium freezeout fraction  $n_D/n_b \propto \eta_{b\gamma}^{-1} \propto (\Omega_b h^2)^{-1}$  and so is fairly sensitive to the baryon density.
- Observations of the ratio in quasar absorption systems give  $\Omega_b h^2 \approx 0.02$

# Helium

- Essentially all neutrons around during nucleosynthesis end up in Helium
- In equilibrium, the neutron-to-proton ratio is determined by the mass difference

$$Q = m_n - m_p = 1.293 \text{ MeV}$$

$$\frac{n_n}{n_p} = \exp[-Q/T]$$



# Helium

- Equilibrium is maintained through weak interactions, e.g.  
 $n \leftrightarrow p + e^- + \bar{\nu}, \nu + n \leftrightarrow p + e^-, e^+ + n \leftrightarrow p + \bar{\nu}$  with rate

$$\frac{\Gamma}{H} \approx \frac{T}{0.8\text{MeV}}$$

- Freezeout fraction

$$\frac{n_n}{n_p} = \exp[-1.293/0.8] \approx 0.2$$

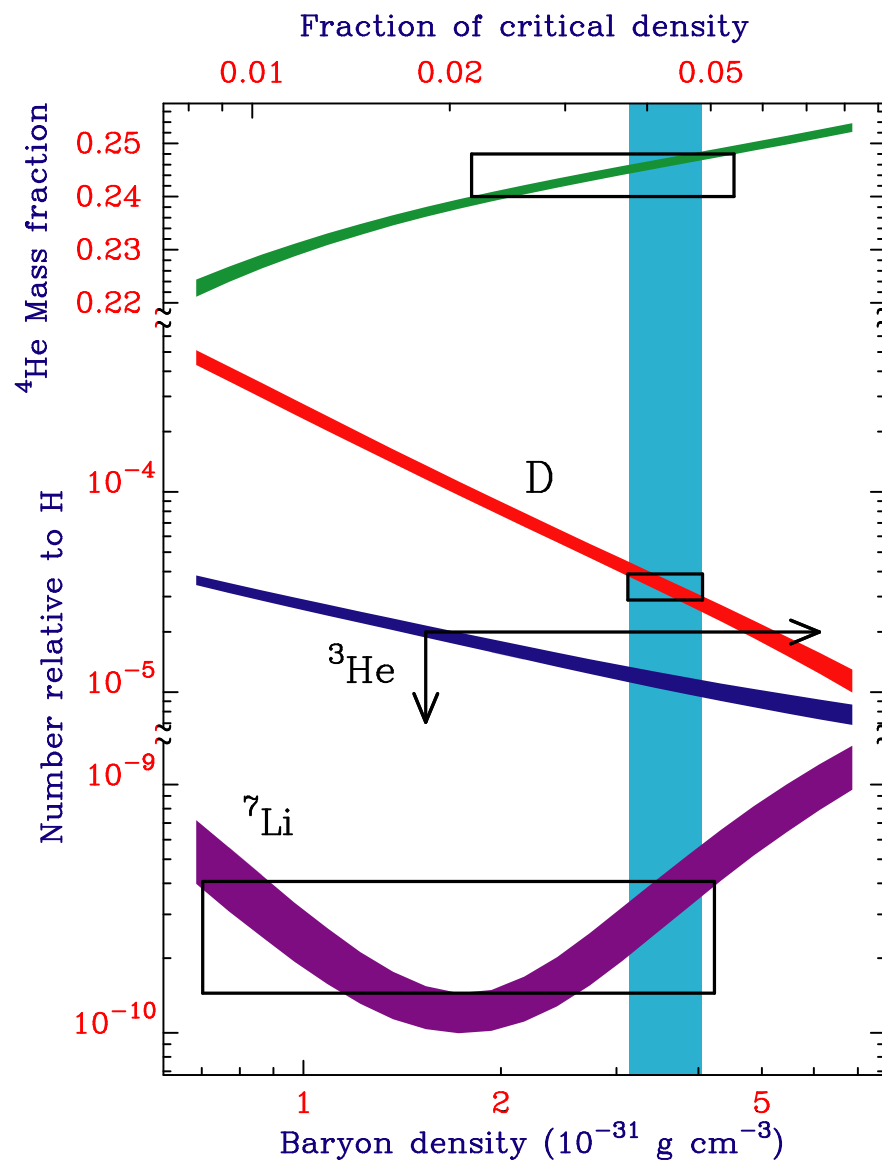
- Finite lifetime of neutrons brings this to  $\sim 1/7$  by  $10^9\text{K}$
- Helium mass fraction

$$\begin{aligned} Y_{\text{He}} &= \frac{4n_{\text{He}}}{n_b} = \frac{4(n_n/2)}{n_n + n_p} \\ &= \frac{2n_n/n_p}{1 + n_n/n_p} \approx \frac{2/7}{8/7} \approx \frac{1}{4} \end{aligned}$$

# Helium

- Depends mainly on the expansion rate during BBN - measure number of relativistic species
- Traces of  ${}^7\text{Li}$  as well. Measured abundances in reasonable agreement with deuterium measure  $\Omega_b h^2 = 0.02$  but the detailed interpretation is still up for debate

# Light Elements



Burles, Nollett, Turner (1999)

# Baryogenesis

- What explains the small, but non-zero, baryon-to-photon ratio?

$$\eta_{b\gamma} = n_b/n_\gamma \approx 3 \times 10^{-8} \Omega_b h^2 \approx 6 \times 10^{-10}$$

- Must be a slight excess of baryons  $b$  to anti-baryons  $\bar{b}$  that remains after annihilation
- Sakharov conditions
  - Baryon number violation: some process must change the net baryon number
  - CP violation: process which produces  $b$  and  $\bar{b}$  must differ in rate
  - Out of equilibrium: else equilibrium distribution with vanishing chemical potential (processes exist which change baryon number) gives equal numbers for  $b$  and  $\bar{b}$
- Expanding universe provides 3; physics must provide 1,2

# Baryogenesis

- Example: out of equilibrium decay of some heavy boson  $X$ ,  $\bar{X}$
- Suppose  $X$  decays through 2 channels with baryon number  $b_1$  and  $b_2$  with branching ratio  $r$  and  $1 - r$  leading to a change in the baryon number per decay of

$$rb_1 + (1 - r)b_2$$

- And  $\bar{X}$  to  $-b_1$  and  $-b_2$  with ratio  $\bar{r}$  and  $1 - \bar{r}$

$$-\bar{r}b_1 - (1 - \bar{r})b_2$$

- Net production

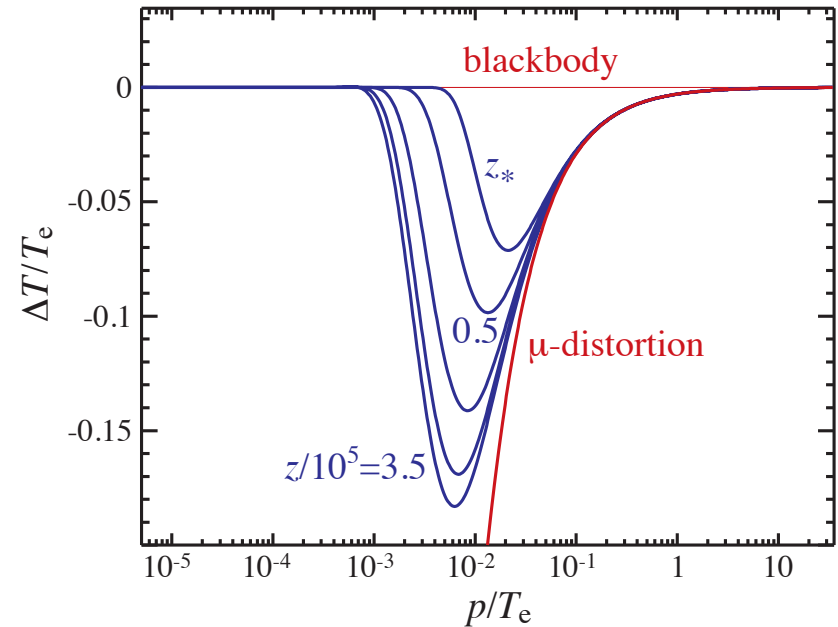
$$\Delta b = (r - \bar{r})(b_1 - b_2)$$

# Baryogenesis

- Condition 1:  $b_1 \neq 0, b_2 \neq 0$
- Condition 2:  $\bar{r} \neq r$
- Condition 3: out of equilibrium decay
- GUT and electroweak (instanton) baryogenesis mechanisms exist
- Active subject of research

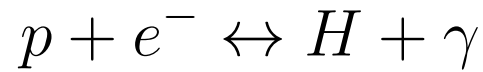
# Black Body Formation

- After  $z \sim 10^6$ , photon creating processes  $\gamma + e^- \leftrightarrow 2\gamma + e^-$  and bremsstrahlung  $e^- + p \leftrightarrow e^- + p + \gamma$  drop out of equilibrium for photon energies  $E \sim T$ .
- Compton scattering remains effective in redistributing energy via exchange with electrons
- Out of equilibrium processes like decays leave residual photon chemical potential imprint
- Observed black body spectrum places tight constraints on any that might dump energy into the CMB



# Recombination

- Maxwell-Boltzmann distribution determines the equilibrium distribution for reactions, e.g. big-bang nucleosynthesis, recombination:



$$\frac{n_p n_e}{n_H} \approx e^{-B/T} \left( \frac{m_e T}{2\pi} \right)^{3/2} e^{(\mu_p + \mu_e - \mu_H)/T}$$

where  $B = m_p + m_e - m_H = 13.6\text{eV}$  is the binding energy,  $g_p = g_e = \frac{1}{2}g_H = 2$ , and  $\mu_p + \mu_e = \mu_H$  in equilibrium

- Define ionization fraction

$$n_p = n_e = x_e n_b$$

$$n_H = n_b - n_e = (1 - x_e) n_b$$

# Recombination

- Saha Equation

$$\begin{aligned}\frac{n_e n_p}{n_H n_b} &= \frac{x_e^2}{1 - x_e} \\ &= \frac{1}{n_b} \left( \frac{m_e T}{2\pi} \right)^{3/2} e^{-B/T}\end{aligned}$$

- Naive guess of  $T_* = B$  wrong due to the low baryon-photon ratio  
–  $T_* \approx 0.3\text{eV}$  so recombination at  $z_* \approx 1000$
- But the photon-baryon ratio is very low

$$\eta_{b\gamma} \equiv n_b/n_\gamma \approx 3 \times 10^{-8} \Omega_b h^2$$

# Recombination

- Eliminate in favor of  $\eta_{b\gamma}$  and  $B/T$  through

$$n_\gamma = 0.244T^3, \quad \frac{m_e}{B} = 3.76 \times 10^4$$

- Big coefficient

$$\frac{x_e^2}{1 - x_e} = 3.16 \times 10^{15} \left( \frac{B}{T} \right)^{3/2} e^{-B/T}$$

$$T = 1/3\text{eV} \rightarrow x_e = 0.7, \quad T = 0.3\text{eV} \rightarrow x_e = 0.2$$

- Further delayed by inability to maintain equilibrium since net is through  $2\gamma$  process and redshifting out of line

# Recombination

