Astro 242

The Physics of Galaxies and the Universe: Lecture Notes

Wayne Hu

Syllabus

Text: An Introduction to Modern Astrophysics 2nd Ed., Carroll and Ostlie

- Milky Way Galaxy
- Nature of Galaxies
- Galactic Evolution
- Active Galaxies
- In class Midterm
- Structure of the Universe
- Cosmology
- Early Universe
- Final

Common Themes

- Mapping out the Universe marching out in distance from Earth

  Start with closest system: Galaxy

  End with furthest system: whole Universe

- Limitations imposed by the ability to measure only a handful of quantities, all from our vantage point in the Galaxy

  Common tools: flux and surface brightness, angular mapping, number counts

- Inferences on the dynamical nature of the systems by using physical laws to interpret observations: e.g. distance from inverse square law, mass from Newtonian dynamics

- Astrophysical units, while bizarre to a physicist, teach you what is being measured and how inferences are made

Astrophysical units

- Length scales
- 1AU = 1.496×10^{13}cm – Earth-sun distance – used for solar system scales
- 1pc = 3.09×10^{18}cm = 2.06×10^{5}AU – 1AU subtends 1arcsecond on the sky at 1pc – distances between nearby stars

  Defined by measuring parallax of nearby stars to infer distance - change in angular position during Earth's orbit: par(allax arc)sec(ond)

$$\frac{1\mathrm{AU}}{1\mathrm{pc}} = \frac{1}{2.06 \times 10^5} = 4.85 \times 10^{-6} = \frac{\pi}{60 * 60 * 180} = 1''$$

- 1kpc = 10^3 pc – distances in the Galaxy
- 1Mpc = 10^6 pc - distances between galaxies
- 1Gpc = 10^9 pc - scale of the observable universe

Astrophysical units

- Fundamental observables are the flux F (energy per unit time per unit area) or brightness (+ per unit solid angle) and angular position of objects in a given frequency band
- Related to the physical quantities, e.g. the luminosity of the object L if the distance to the object is known

$$F = \frac{L}{4\pi d^2}$$

- Solar luminosity

$$L_\odot = 3.839 \times 10^{26}\mathrm{W} = 3.839 \times 10^{33}\mathrm{erg/s}$$

- Frequency band defined by filters - in limit of infinitesimal bands, the whole frequency spectrum measured – “spectroscopy”

Astrophysical units

- Relative flux easy to measure - absolute flux requires calibration of filter: (apparent) magnitudes (originally defined by eye as filter)

$$m_1 - m_2 = -2.5 \log(F_1/F_2)$$

- Absolute magnitude: apparent magnitude of object at $d = $10pc

$$m - M = -2.5 \log(d/10\mathrm{pc})^{-2} \to \frac{d(m-M)}{10\mathrm{pc}} = 10^{(m-M)/5}$$

- If frequency spectrum has lines, Doppler shift gives relative or radial velocity of object V_r aka redshift z

$$1 + z = \frac{\lambda_{\rm obs}}{\lambda_{\rm rest}} = 1 + \frac{V_r}{c}$$

(where $V_r > 0$ denotes recession and redshift and $V_r \ll c$) used to measure velocity for dynamics of systems, including universe as whole

Astrophysical units

- Masses in units of solar mass $M_\odot = 1.989 \times 10^{33}$g
- Measurement of distance and angle gives physical size and Doppler shift gives velocity $\rightarrow$ mass
- Mass measurement always boils down to inferring gravitational force necessary to keep test object in orbit
- For circular motion - centripetal force

$$\frac{mv^2}{r} \approx \frac{GmM}{r^2} \rightarrow M \approx \frac{v^2 r}{G}$$

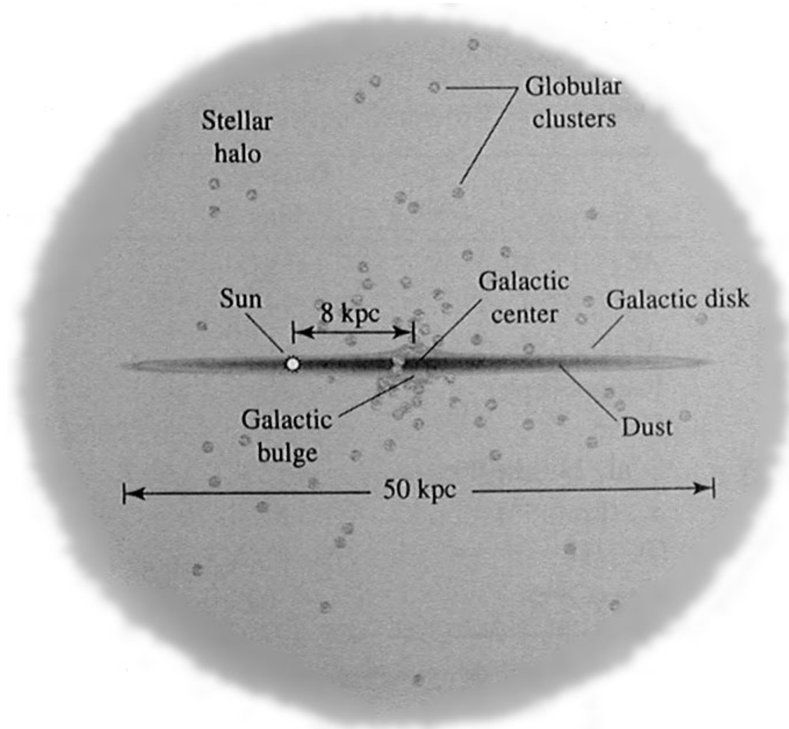
- Requires a measurement of velocity and a measurement or estimate of size
- Various systems will have order unity correction to this circular-motion based relation

Set 1:

Milky Way Galaxy

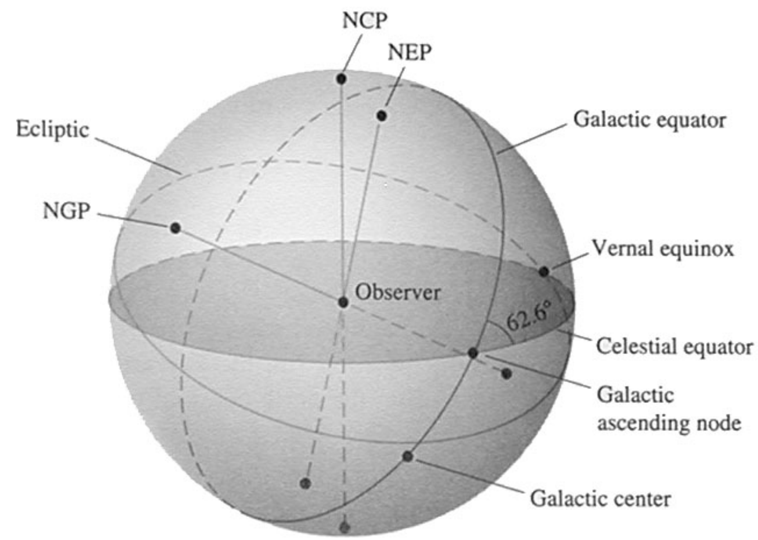
Galactic Census

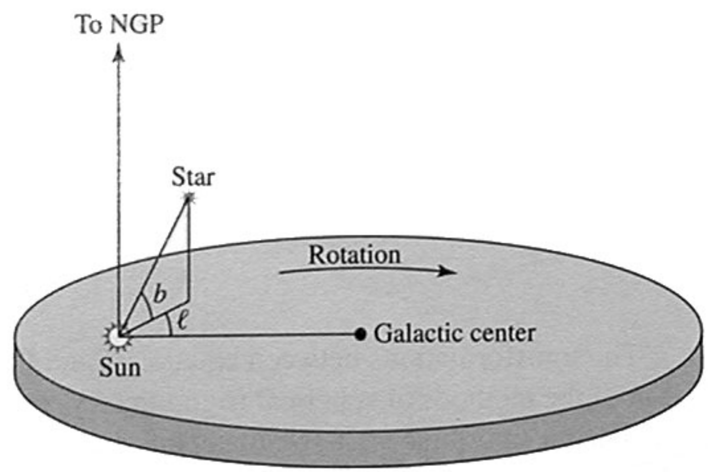
- Sun is embedded in a stellar disk ~ 8 kpc from the galactic center
- Extent of disk ~ 25 kpc radius, spiral structure
- Thickness of neutral gas disk < 0.1 kpc
- Thickness of thin disk of young stars ~ 0.35 kpc
- Thickness of thick disk ~ 1 kpc



Galactic Census

- Central stellar bulge
  radius ~ 4 kpc, with central bar
- Supermassive black hole, inferred from large mass within 120AU (solar system scale) of center
- Extended spherical stellar halo with globular clusters, radius > 100 kpc
- Extended dark matter halo, radius > 200 kpc





Mass and Luminosity

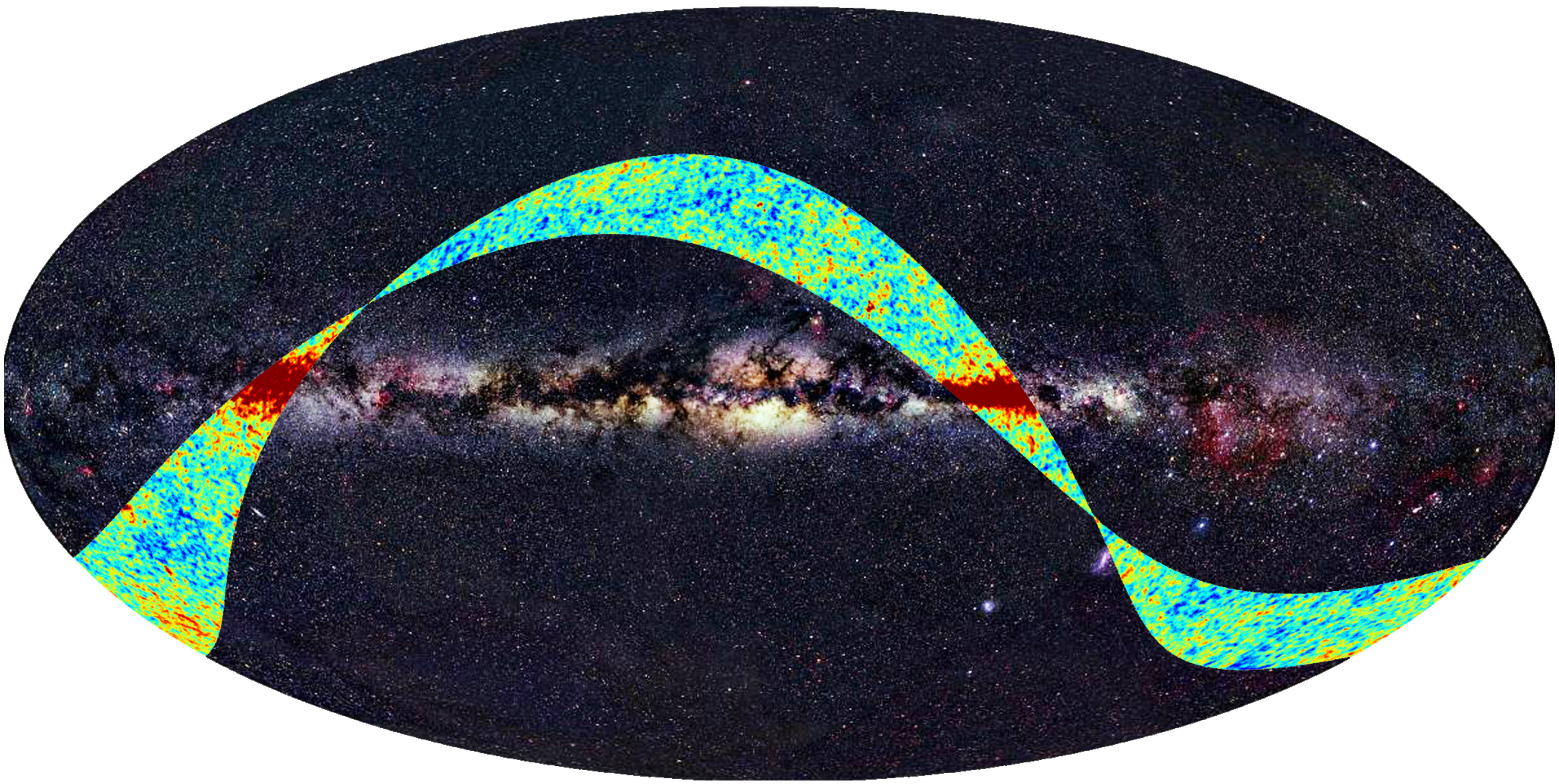
	Mass	Luminosity (L_B)
Neutral gas disk	$0.5 \times 10^{10} M_\odot$	
Thin disk	$6 \times 10^{10} M_\odot$	$1.8 \times 10^{10} L_\odot$
Thick disk	$0.2 - 0.4 \times 10^{10} M_\odot$	$0.02 \times 10^{10} L_\odot$
Bulge	$1 \times 10^{10} M_\odot$	$0.3 \times 10^{10} L_\odot$
Supermassive black hole	$3.7 \pm 0.2 \times 10^{6} M_\odot$	
Stellar halo	$0.3 \times 10^{10} M_\odot$	$0.1 \times 10^{10} L_\odot$
Dark matter halo	$2 \times 10^{12} M_\odot$	
Total	$2 \times 10^{12} M_\odot$	$2.3 \times 10^{10} L_\odot$

How Do We Know?

- Infer this structure from the handful of observables that are directly accessible
- Convert intrinsically 2D information to 3D + dynamical model
- Flux and number of stars
- Angular positions of stars (as a function of season, time)
- Relative radial velocity from Doppler effect

Starlight: Optical Image

- Color overlay: microwave background



Star Counts

- One of the oldest methods for inferring the structure of the galaxy from 2D sky maps is from star counts
- History: Kapteyn (1922), building on early work by Herschel, used star counts to map out the structure of the galaxy
- Fundamental assumptions

  Stars have a known (distribution in) absolute magnitude

  No obscuration
- Consider a star with known absolute magnitude M (magnitude at 10pc). Its distance can be inferred from the inverse square law from its observed m as

$$\frac{d(m-M)}{10\mathrm{pc}} = 10^{(m-M)/5}$$

Star Counts

- Combined with the angular position on the sky, the 3d position of the star can be measured - mapping the galaxy
- Use the star counts to determine statistical properities: number density of stars in each patch of sky
- A fall off in the number density in radial distance would determine the edge of the galaxy
- Suppose there is an indicator of absolute magnitude like spectral type that allows stars to be selected to within dM of M
- Describe the underlying quantity to be extracted as the spatial number density within dM of M: $n_M(M, \mathbf{r})dM$

Star Counts

- The observable is say the total number of stars brighter than a limiting apparent magnitude m in a solid angle $d\Omega$
- Stars at a given M can only be observed out to a distance $d(m-M)$ before their apparent magnitude falls below the limit
- So there is radial distance limit to the volume observed
- Total number observed in solid angle $d\Omega$ within dM of M is integral to that limit

$$N_M dM = \left[\int_0^{d(m-M)} n_M(M,r) r^2 dr \right] d\Omega dM$$

- Differentiating with respect to $d(m-M)$ provides a measurement of $n_M(M,r)$

Star Counts

- So dependence of counts on the limiting magnitude m determines the number density and e.g. the edge of the system
- In fact, if there were no edge to system the total flux, assuming point sources, would diverge as $m \to 0$ - volume grows as d^3 flux decreases as d^{-2}: Olber's paradox
- Generalizations of the basic method:
- Selection criteria is not a perfect indicator of M and so dM is not infinitesimal and some stars in the range will be missed - $S(M)$ and M is integrated over - total number

$$N = \int_{-\infty}^{\infty} dM S(M) N_M$$

Star Counts

- Alternately use all stars [weak or no $S(M)$] but assume a functional form for $n_M(M, r) = A(r)n_M(M, 0)$ e.g. derived from local estimates and assumed to be the same at larger $r > 0$

  In this case, measurements determine the normalization of a distribution with fixed shape, i.e.

$$n(\mathbf{r}) = \int_{-\infty}^{\infty} n_M(M, \mathbf{r}) dM$$

- Kapteyn used all of the stars (assumed to have the same n_M shape in r)
- He inferred a flattened spheroidal system of < 10kpc extent in plane and < 2kpc out of plane: too small
- Missing: interstellar extinction dims stars dropping them out of the sample at a given limiting magnitude

Variable Stars

- With a good indicator of absolute magnitude or “standard candle” one can use individual objects to map out the structure of the Galaxy (and Universe)
- History: Shapley (1910-1920) used RR Lyrae and W Virginis variable stars - with a period-luminosity relation

  Radial oscillations with a density dependent sound speed - luminosity and density related on the instability strip

  Calibrated locally by moving cluster and other methods
- Measure the period of oscillation, infer a luminosity and hence an absolute magnitude, infer a distance from the observed apparent magnitude

Variable Stars

- Inferred a 100kpc scale for the Galaxy - overestimate due to differences in types of variable stars and interstellar extinction
- Apparent magnitude is dimmed by extinction leading to the variable stars being less distant than they appear
- Both Kapteyn and Shapley off because of dust extinction: discrepancy between two independent methods indicates systematic error
- Caveat emptor: in astronomy always want to see a cross check with two or more independent methods before believing result you read in the NYT! – see recent example of gravitational wave vs dust polarization.

Common techniques

- Particular objects such as stars of a given type and Cepheid variables are both interesting in their own right and, once understood, useful for tracing out larger systems
- Star counts is an example of a general theme in astronomy: using a large survey (stars) to infer statistical properties
- Likewise, cepheids are an example of using a standard candle to map out a system
- Similar method applies to mapping out the Universe with galaxies e.g. baryon acoustic oscillations vs supernova

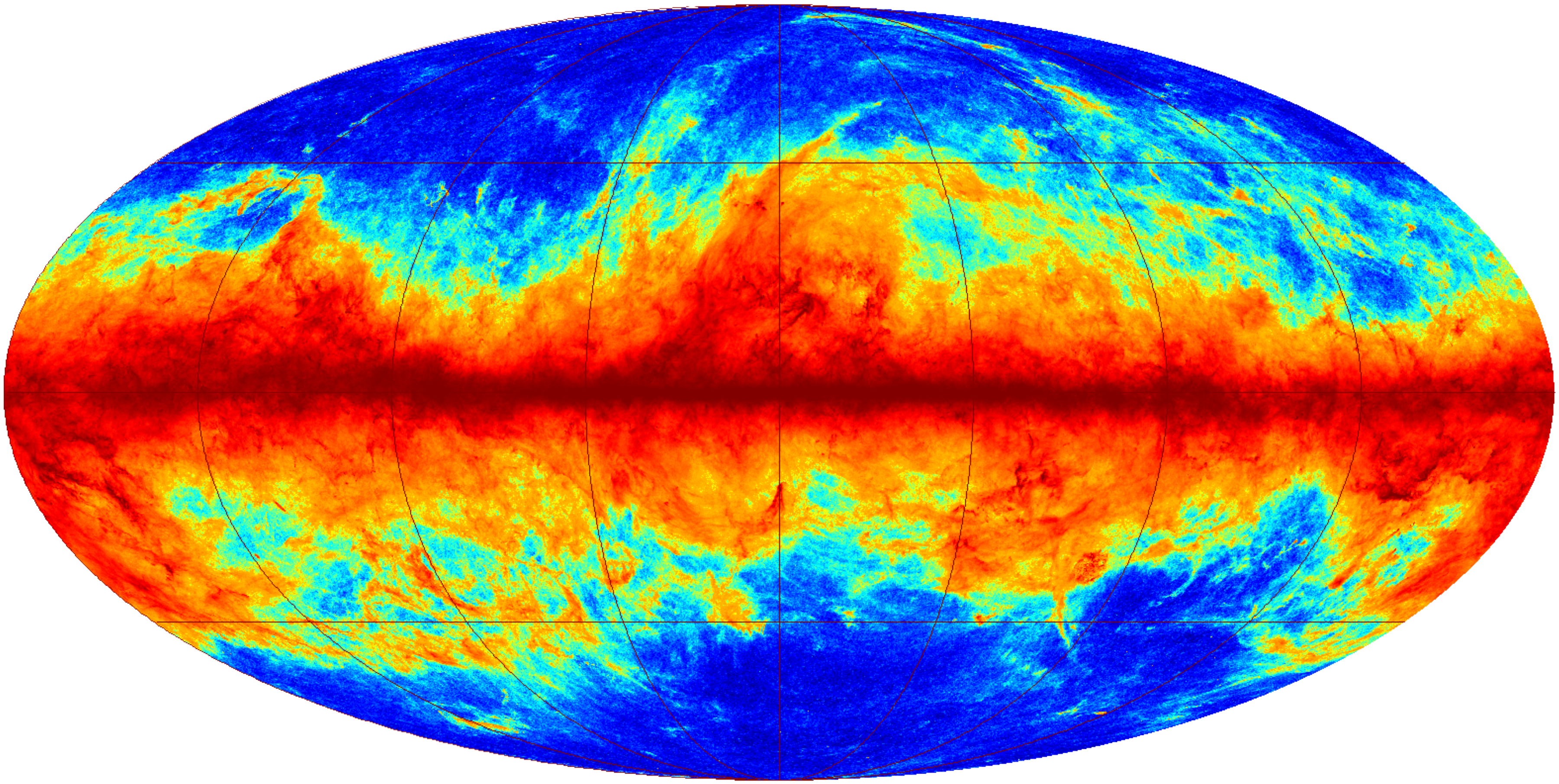
Interstellar Extinction

- Dust (silicates, graphite, hydrocarbons) in ISM (Chap 12) dims stars at visible wavelengths making true distance less than apparent
- Distance formula modified to be

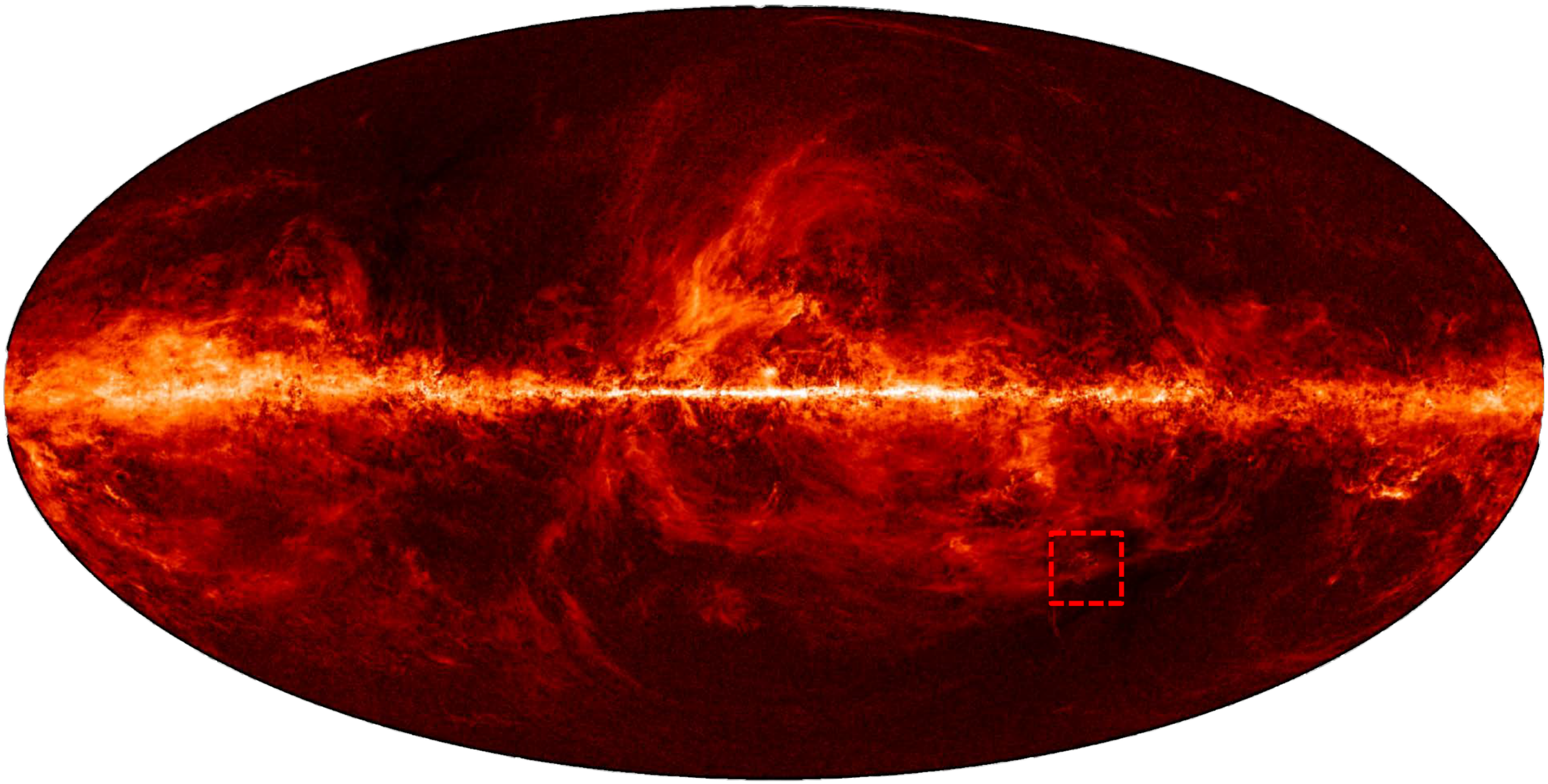
$$\frac{d}{10\mathrm{pc}} = 10^{(m_\lambda - M_\lambda - A_\lambda)/5}$$

  where the extinction coefficient $A_\lambda \geq 0$ depends on wavelength λ
- Extinction also depends on direction, e.g. through the disk, through a giant molecular cloud, etc. Typical value at visible wavelengths and in the disk is 1 mag/kpc
- Dust emits or reradiates starlight in the infrared - maps from these frequencies [IRAS, DIRBE, now Planck] can be used to calibrate extinction

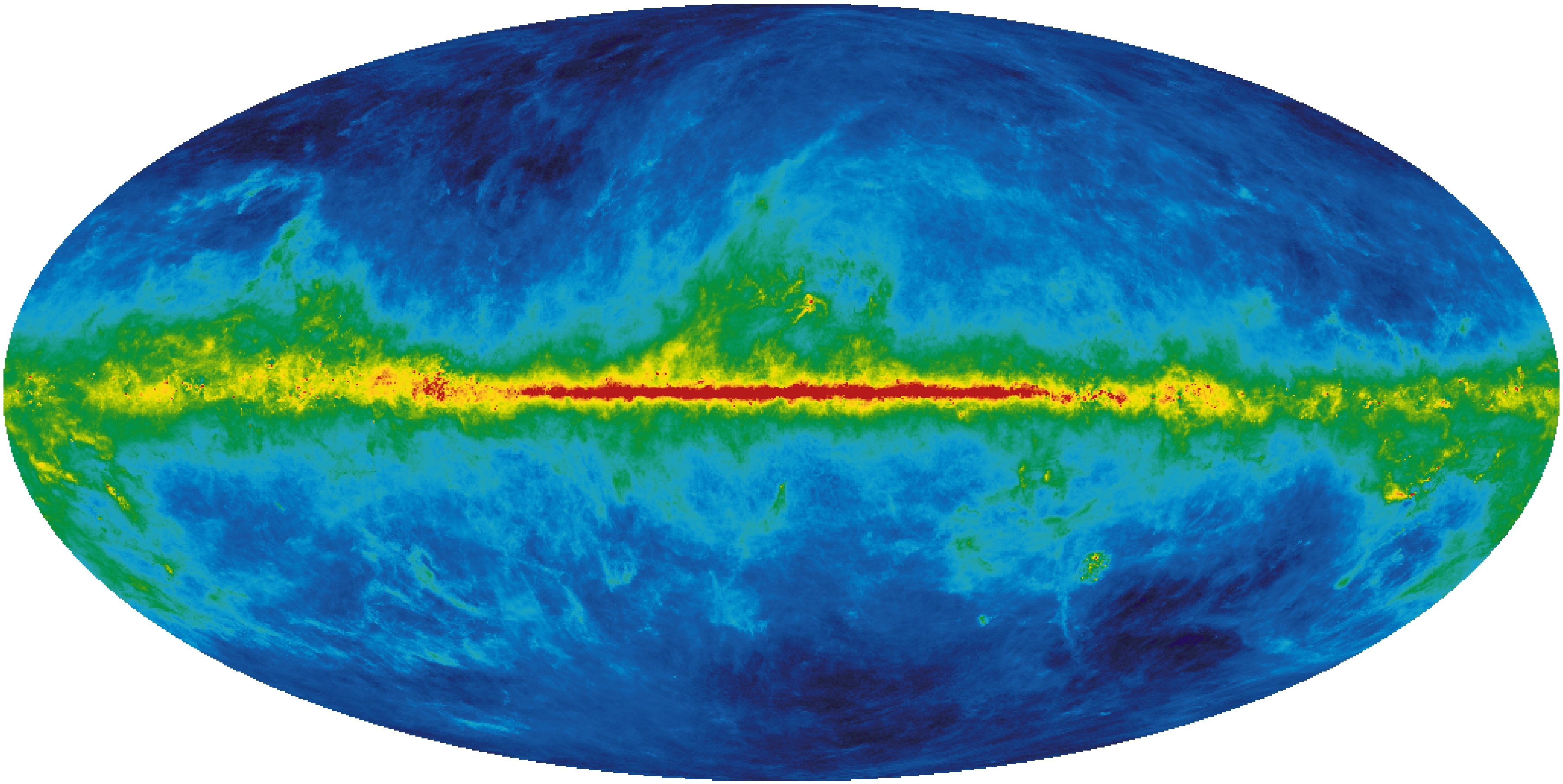
Planck Dust Emission



Planck Dust Polarization



Extinction Correction



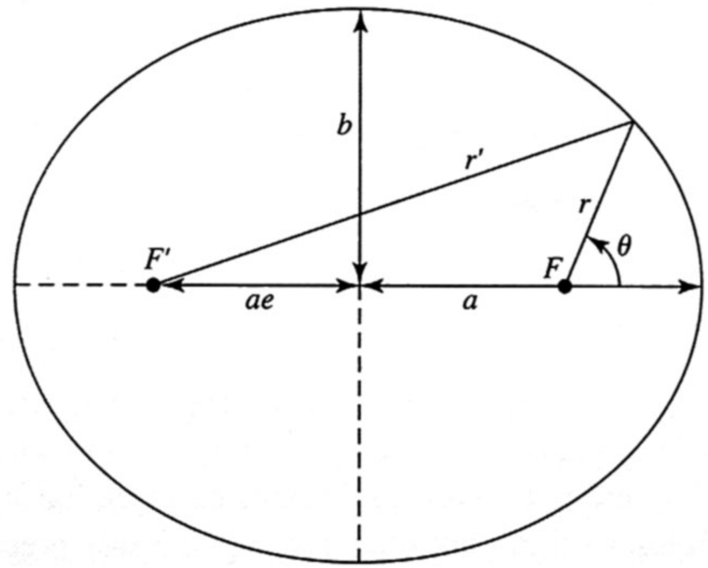
Kinematic Distances to Stars

- If proper motion across the sky can be measured from the change in angular position μ in rad/s

$$v_t = \mu d$$

- Often v_t can be inferred from the radial velocity and a comparison with μ gives distance d given assumption of the dynamics
- Example: Keplerian orbits of stars around galactic center $R_0 = 7.6 \pm 0.3\text{kpc}$
- Example: Stars in a moving cluster share a single total velocity whose direction can be inferred from apparent convergent motion (see Fig 24.30)

Stellar Kinematics



- Can infer more than just distance: SMBH
- Galactic center: follow orbits of stars close to galactic center
- One star: orbital period 15.2yrs, eccentricity $e = 0.87$, perigalacticon distance (closest point on orbit to F 120 AU=1.8×10^{13}m
- Estimate mass: $a = ae - r_p$ so semimajor axis

$$a = \frac{r_p}{1 - e} = 1.4 \times 10^{14}\mathrm{m}$$

Stellar Kinematics

- Kepler's 3rd law

$$M = \frac{4\pi^2 a^3}{GP^2} = 7 \times 10^{36}\mathrm{kg} = 3.5 \times 10^6 M_\odot$$

- That much mass in that small a radius can plausibly only be a (supermassive) black hole
- Note that this is an example of the general statement that masses are estimated by taking

$$M \approx \frac{v^2 r}{G} = \frac{(2\pi a)^2 a}{GP^2} = \frac{4\pi^2 a^3}{GP^2}$$

Stellar Kinematics

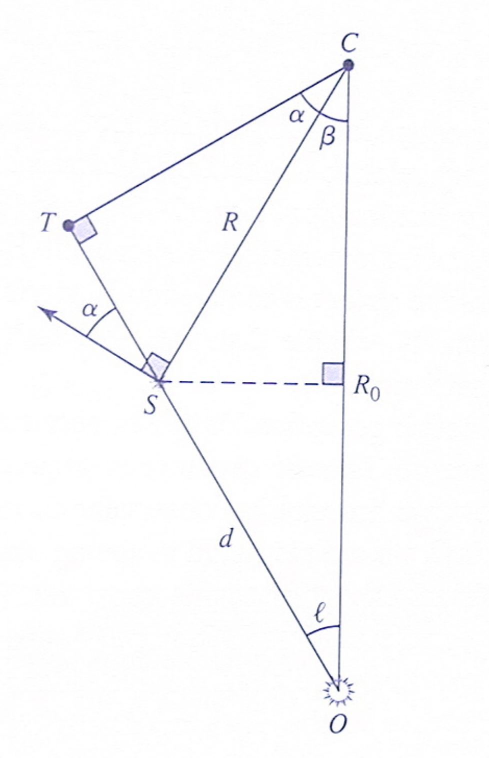
- Apply these stellar kinematics techniques to the galactic disk of stars
- Direct observables are the kinematics of neighboring stars

  ℓ, b: angular position of star in galactic coordinates

  v_r: relative motion radial to line of sight

  v_t: relative motion tangential to line of sight
- These stars have their parallax distance d measured
- The distance to center of galaxy R_0 is known
- Infer the rotation of stars in the disk around galactic center

Stellar Kinematics



- Differential rotation $\Theta(R) = R\Omega(R)$
  where $\Omega(R)$ is the angular velocity curve–
  observables are radial and tangential motion
  with respect to LSR

$$v_r = R\Omega \cos\alpha - R_0\Omega_0 \sin\ell$$
$$v_t = R\Omega \sin\alpha - R_0\Omega_0 \cos\ell$$

  where Θ_0 is the angular velocity of the LSR

- Technical point: defined through local standard of rest (LSR) rather than sun's rest frame due to small differences between solar motion and the average star around us

Stellar Kinematics

- d (parallax) and R_0 are known observables, R is not - eliminate with trig relations

$$R\cos\alpha = R_0 \sin\ell \qquad R\sin\alpha = R_0\cos\ell - d$$

- Eliminate R and solve for (Ω, Ω_0)

$$\begin{aligned} v_r &= (\Omega - \Omega_0)R_0 \sin\ell \\ v_t &= (\Omega - \Omega_0)R_0\cos\ell - \Omega d \end{aligned}$$

- Historical context: solve for $\Omega(R)$ locally where

$$\begin{aligned} \Omega - \Omega_0 &\approx \frac{d\Omega}{dR}(R - R_0) \\ &\approx \frac{1}{R_0}\left(\frac{d\Theta}{dR} - \frac{\Theta_0}{R_0}\right)(R - R_0) \quad [\Omega = \Theta/R] \end{aligned}$$

and $d \ll R_0$, $\cos\beta \approx 1$

Stellar Kinematics

- Reduce with trig identities

$$R_0 = d\cos\ell + R\cos\beta \approx d\cos\ell + R$$

$$R - R_0 \approx -d\cos\ell$$

$$\cos\ell\sin\ell = \frac{1}{2}\sin 2\ell$$

$$\cos^2\ell = \frac{1}{2}\left(\cos 2\ell + 1\right)$$

to obtain

$$v_r \approx Ad\sin 2\ell$$

$$v_t \approx Ad\cos 2\ell + Bd$$

Stellar Kinematics

- Oort constants

$$A = -\frac{1}{2}\left[\frac{d\Theta}{dR} - \frac{\Theta_0}{R_0}\right] = -\frac{R_0}{2}\frac{d\Omega}{dR}$$

$$B = -\frac{1}{2}\left[\frac{d\Theta}{dR} + \frac{\Theta_0}{R_0}\right]$$

- Observables v_r, v_t, ℓ, d: solve for Oort's constants. From Hipparcos

$$A = 14.8 \pm 0.8 \mathrm{km/s/kpc}$$

$$B = -12.4 \pm 0.6 \mathrm{km/s/kpc}$$

- Angular velocity $\Omega = \Theta/R$ decreases with radius: differential rotation.

Stellar Kinematics

- Physical velocity $\Theta(R)$

$$\left.\frac{d\Theta}{dR}\right|_{R_0} = -(A+B) = -2.4\mathrm{km/s/kpc}$$

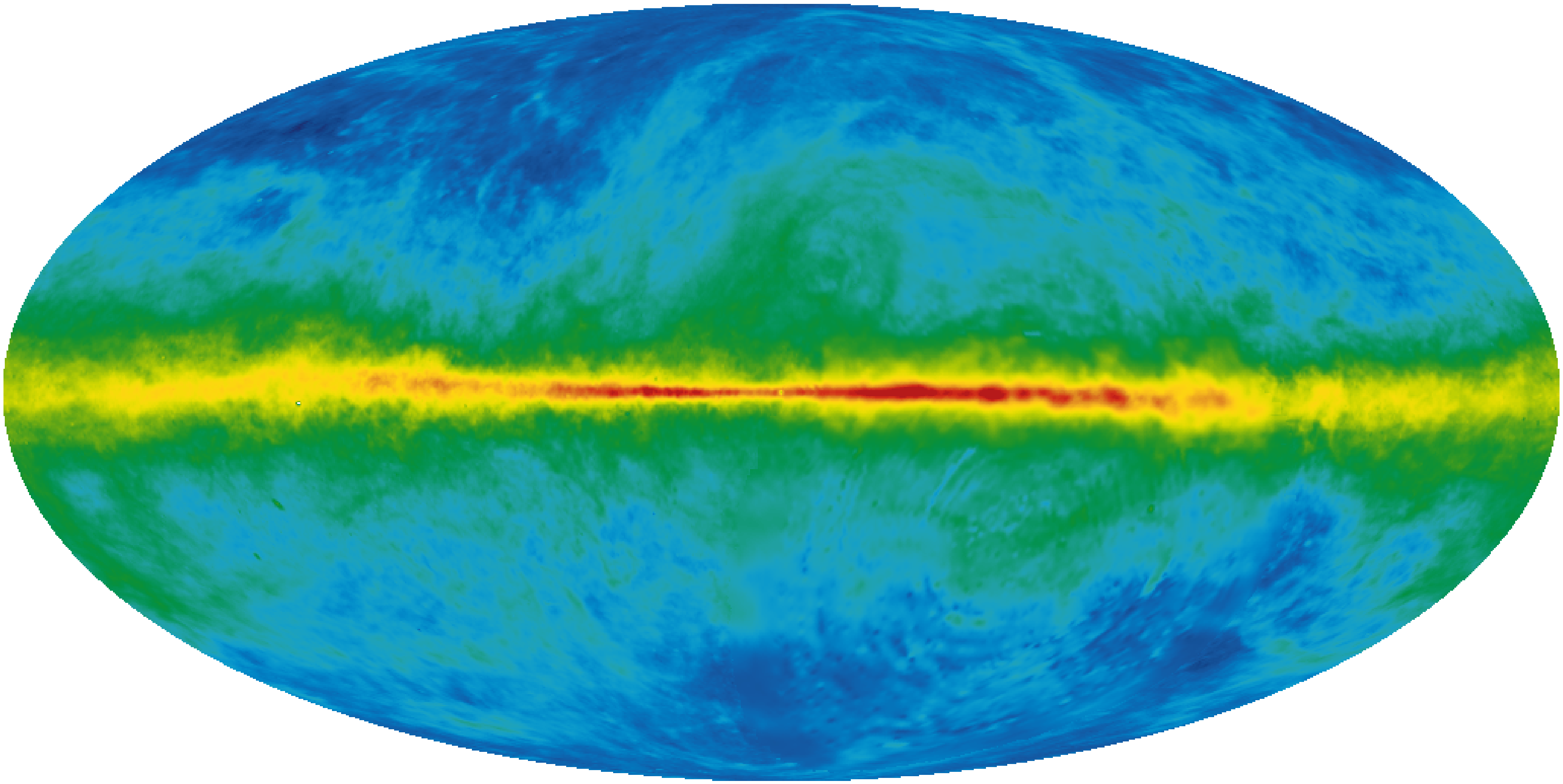
decreases slowly compared with

$$\frac{\Theta_0}{R_0} = A - B = 27.2\mathrm{km/s/kpc} \rightarrow \Theta_0 \approx 220\mathrm{km/s}$$

a nearly flat rotation curve – at least locally

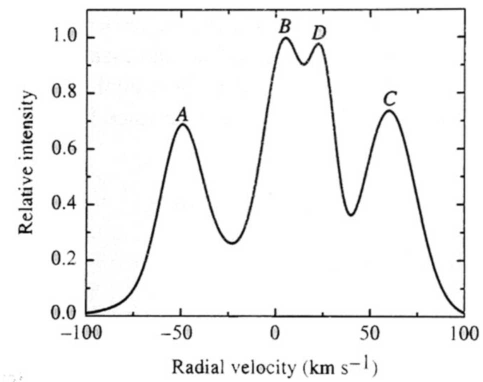
- Does this flat rotation curve extend out in the disk away from the sun?

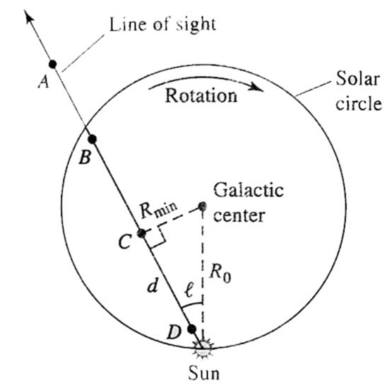
Neutral Gas: 21cm Emission



21 cm

- Spin interaction of the electron and proton leads to a spin flip transition in neutral hydrogen with wavelength 21cm



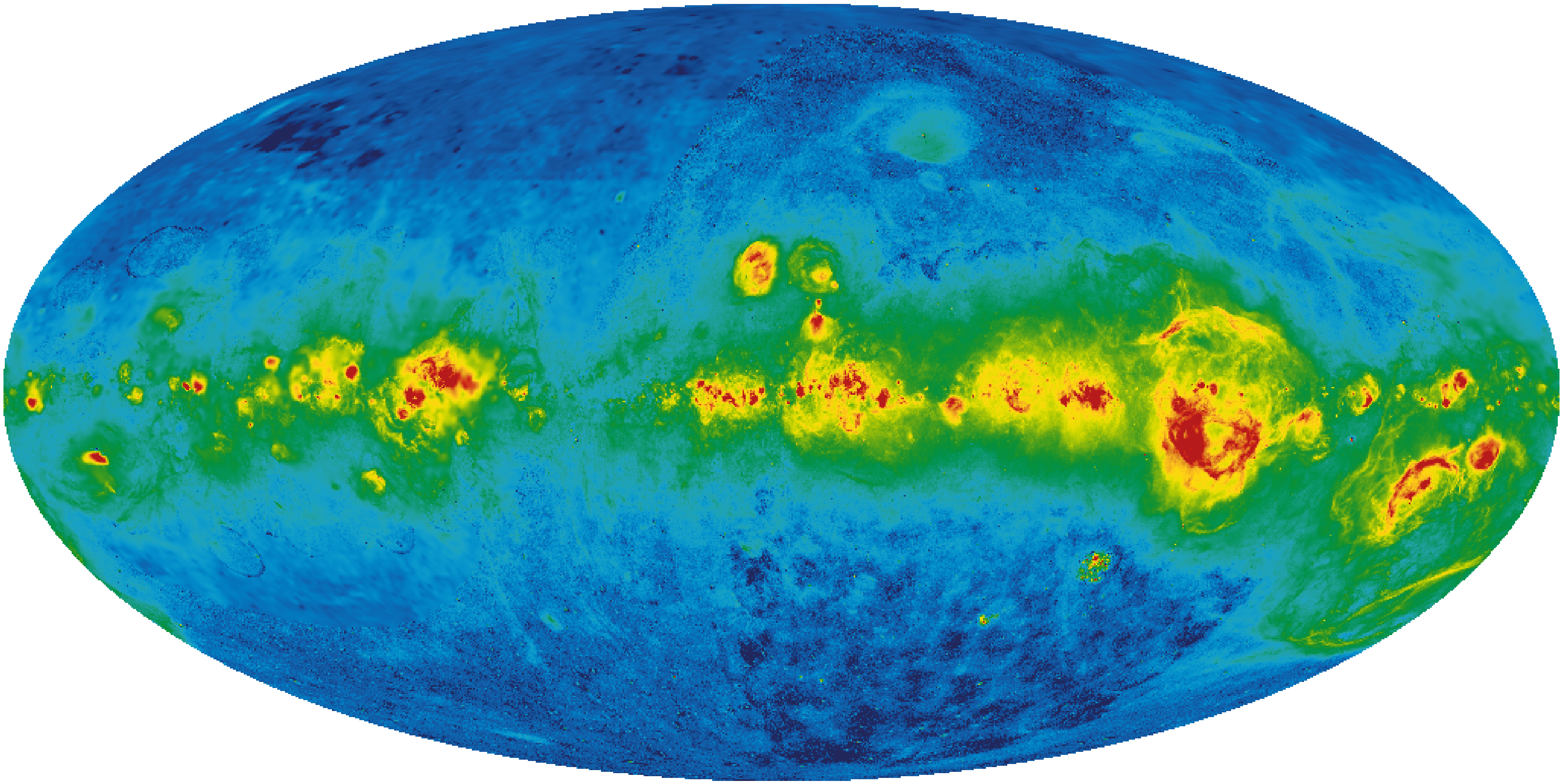


- Line does not suffer substantial extinction and can be used to probe the neutral gas and its radial velocity from the Doppler shift throughout the galaxy
- No intrinsic distance measure
- Neutral gas is distributed inhomogeneously in clouds leading to distinct peaks in emission along each sight line

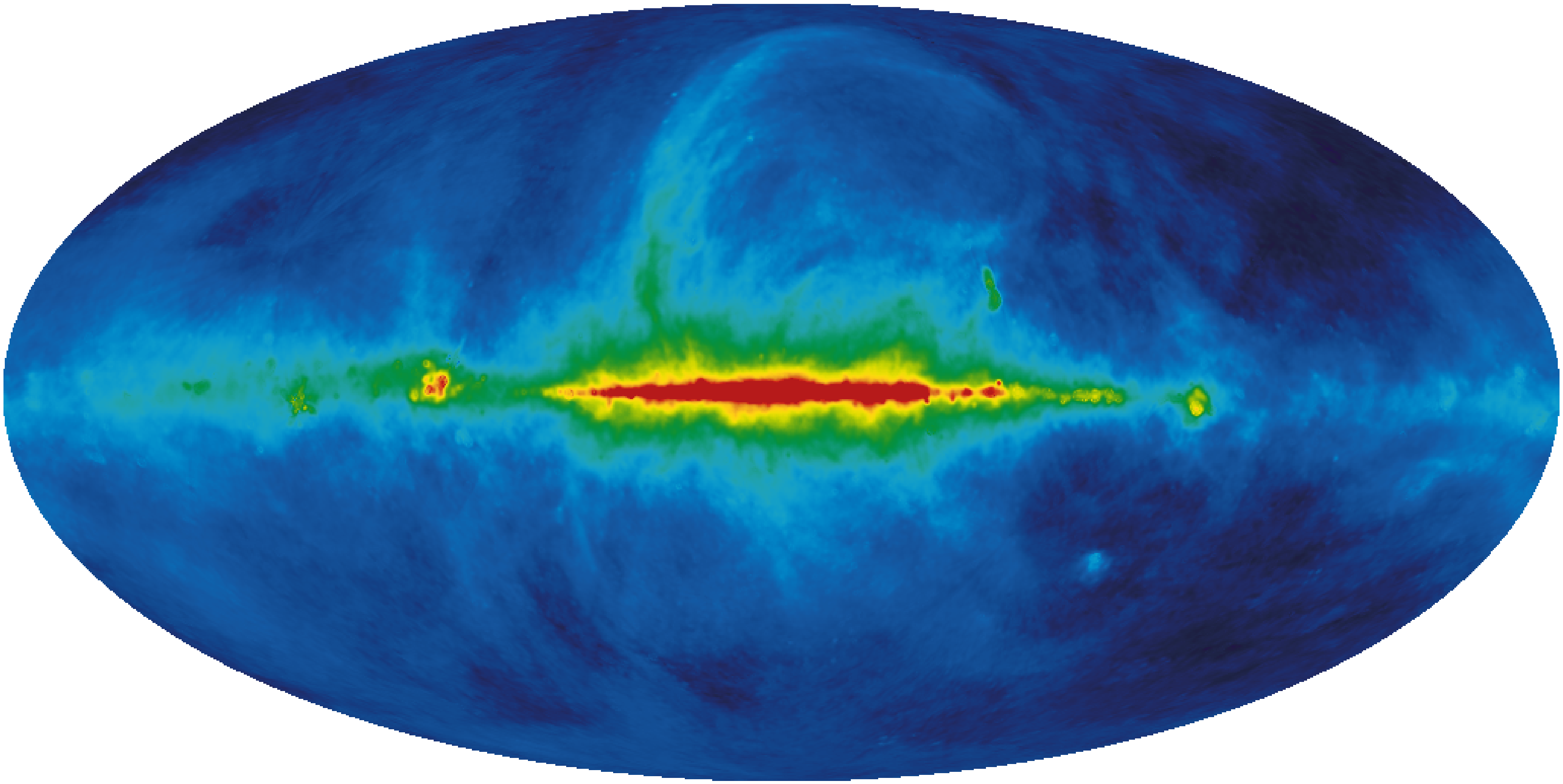
21 cm

- Due to projection of velocities along the line of sight and differential rotation, the highest velocity occurs at the closest approach to the galactic center or tangent point
- Build up a rotation curve interior to the solar circle $R < R_0$
- Rotation curve steeply rises in the interior $R < 1\text{kpc}$, consistent with near rigid body rotation and then remains flat out through the solar circle

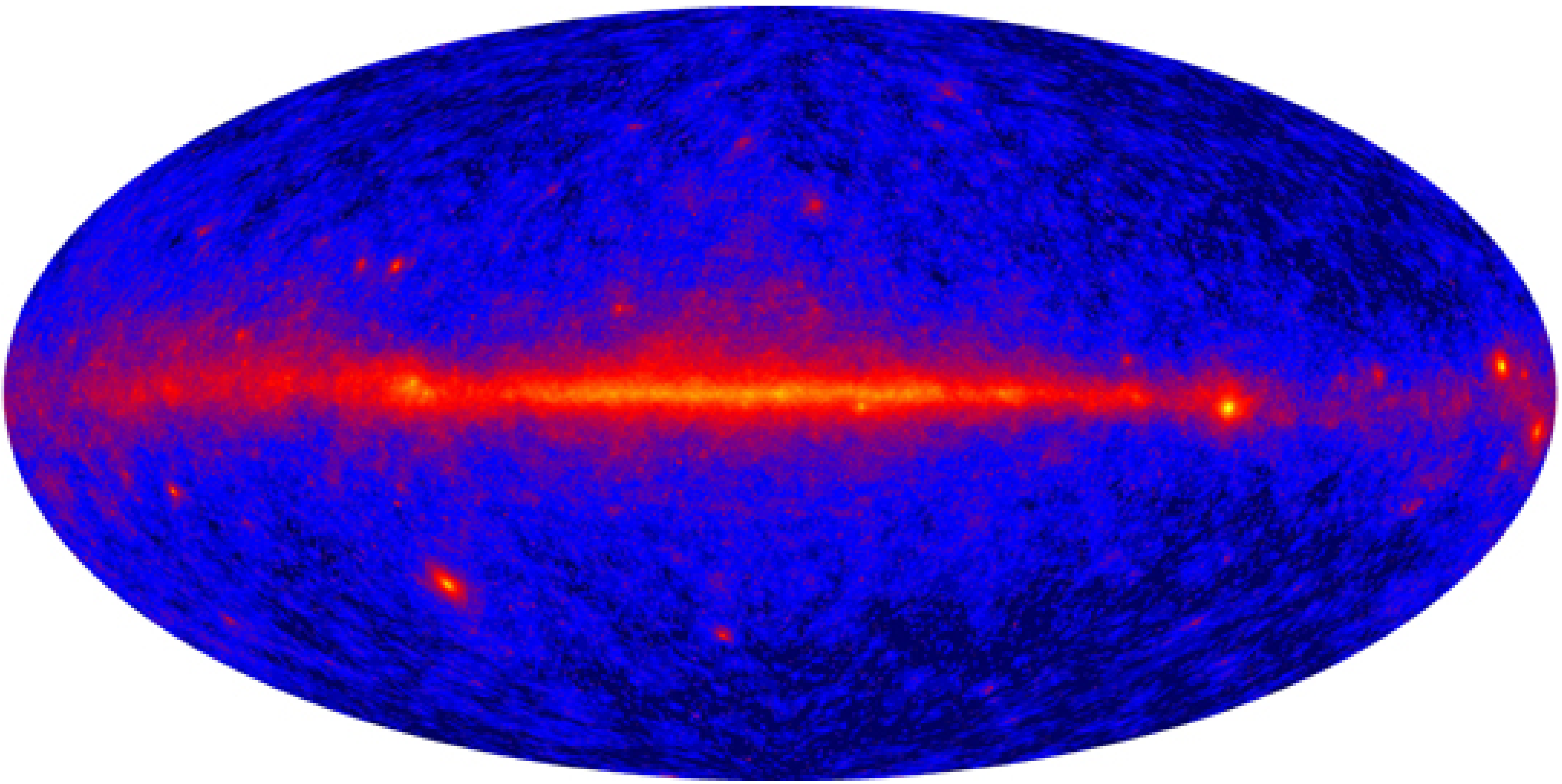
Ionized Gas: Hα Line Emission



Cosmic Rays in B Field: Synchrotron

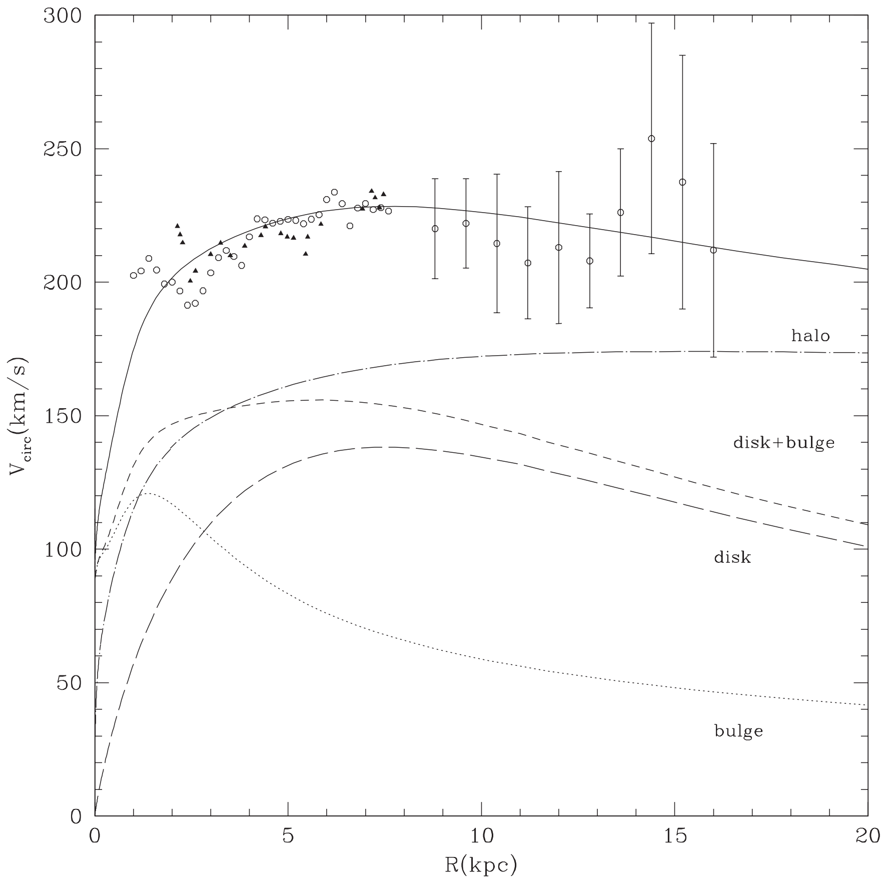


Gamma Rays



Rotation Curves

- Extending the rotation curve beyond the solar circle with objects like Cepheids whose distances are known reveals a flat curve out to $\sim$ 20kpc
- Mass required to keep rotation curves flat much larger than implied by stars and gas. Consider a test mass m orbiting at a radius r around an enclosed mass $M(r)$



Rotation Curves

- Setting the centripetal force to the gravitational force

$$\frac{mv^2(r)}{r} = \frac{GM(r)m}{r^2}$$

$$M(r) = \frac{v(r)^2 r}{G}$$

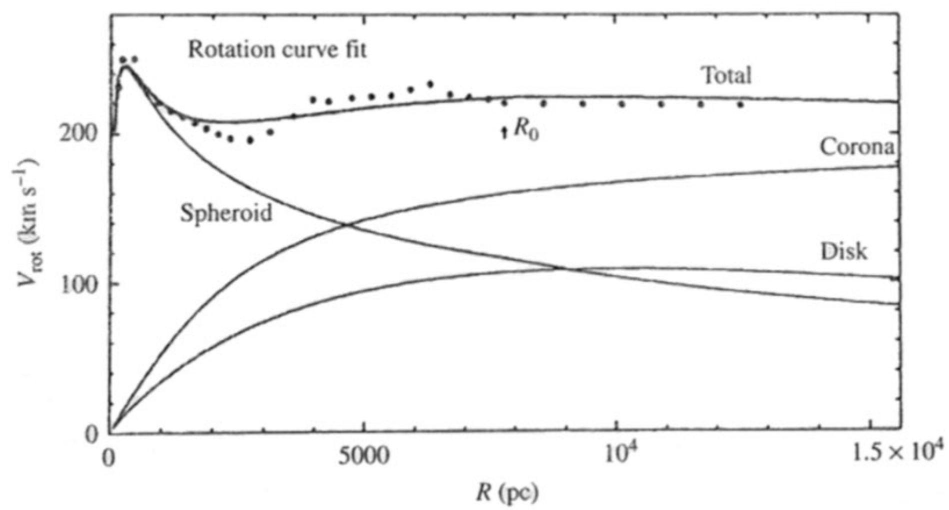
  Again this combination of velocity and distance is the fundamental way masses are measured:

  gravitational force binds object against the motion of the luminous objects - other examples: virial theorem with velocity dispersion, hydrostatic equilibrium with thermal motions

- Measuring the rotation curve $v(r)$ is equivalent to measuring the mass profile $M(r)$ or density profile $\rho(r) \propto M(r)/r^3$

Rotation Curves

- Flat rotation curve $v(r) = \text{const}$ implies $M \propto r$ - a mass linearly increasing with radius
- Rigid rotation implies $\Omega = v/r$=const. $v \propto r$ or $M \propto r^3$ or ρ =const



- Rotation curves in other galaxies show the same behavior: evidence that "dark matter" is ubiquitous in galaxies

Rotation Curves

- Consistent with dark matter density given by

$$\rho(r) = \frac{\rho_0}{1 + (r/a)^2}$$

- Also consistent with the NFW profile predicted by cold dark matter (e.g. weakly interacting massive particles or WIMPs)

$$\rho(r) = \frac{\rho_0}{(r/a)(1 + r/a)^2}$$

Gravitational Lensing

- Rotation curves leave open the question of what dark matter is
- Alternate hypothesis: dead stars or black holes - massive astrophysical compact halo object "MACHO"
- MACHOs have their mass concentrated into objects with mass comparable to the sun or large planet
- A MACHO at an angular distance $u = \theta/\theta_E$ from the line of sight to the star will gravitationally lens or magnify the star by a factor of

$$A(u) = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}}$$

where θ_E is the Einstein ring radius in projection

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{d_S - d_L}{d_S d_L}}$$

Gravitational Lensing

- Again masses related to a physical scale $r_E = \theta_E d_L$ and speed c

$$M \sim \frac{d_S}{4(d_S - d_L)} \frac{c^2 r_E}{G}$$

e.g. for a typical lens half way to the source the prefactor is $1/2$

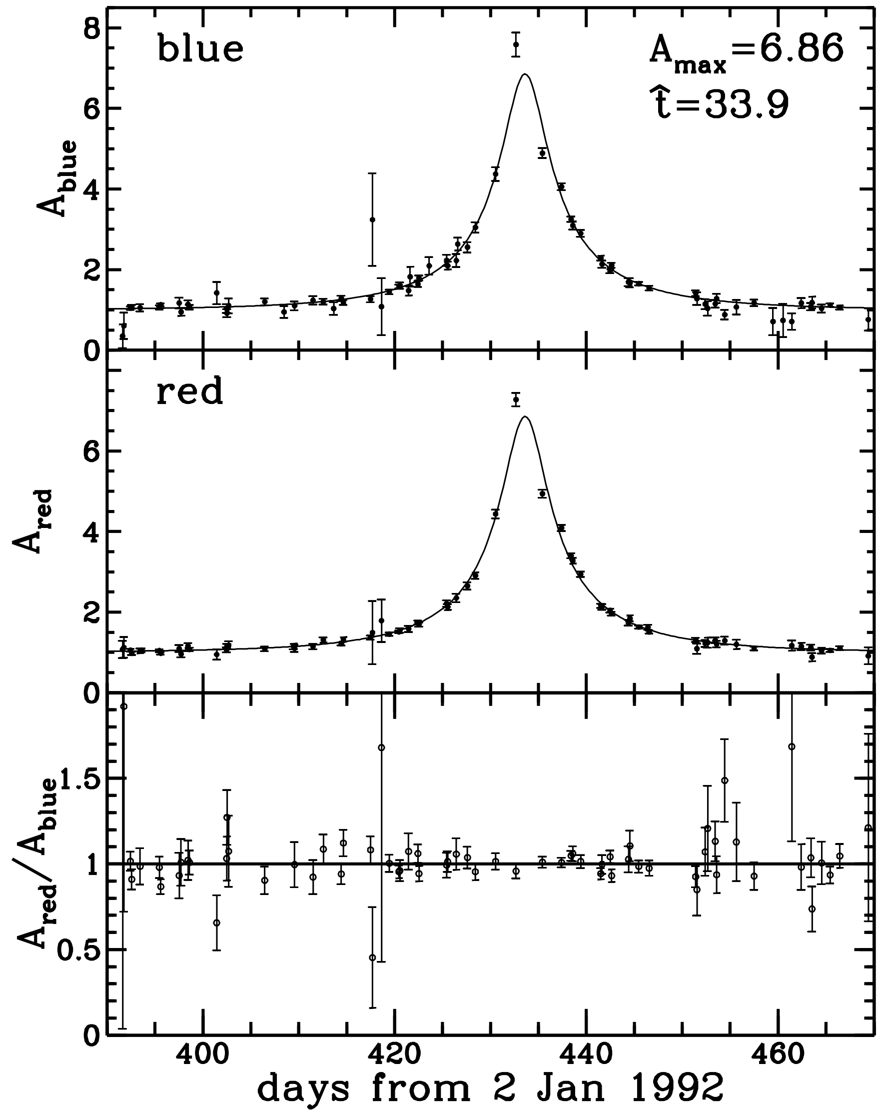
- A MACHO would move at a velocity typical of the disk and halo $v \sim 200$km/s and so the star behind it would brighten as it crossed the line of sight to a background star. With $u_{\rm min}$ as the distance of closest approach at $t = 0$

$$u^2(t) = u_{\rm min}^2 + \left(\frac{vt}{d_L \theta_E} \right)^2$$

- Monitor a large number of stars for this characteristic brightening. Rate of events says how much of the dark matter could be in MACHOs.

Gravitational Lensing

- In the 1990's
  large searches measured
  the rate of microlensing
  in the halo and
  bulge and determined that
  only a small fraction of its
  mass could be in MACHOs



Gravitational Lensing

- Current searches (toward the bulge) are used to find planets
- Enhanced microlensing by planet around star leads to a blip in the brightening.