Astro 242: Problem Set 7

1 Problem 1: Energy Conservation

- Take the Liouville or collisionless Boltzmann equation in an expanding universe

$$\frac{\partial f}{\partial t} + \frac{dq}{dt}\frac{\partial f}{\partial q} = 0 \tag{1}$$

and derive the continuity or energy conservation equation

$$\frac{d\rho}{dt} = -3H(\rho + p) \tag{2}$$

- Take the energy conservation equation and the Friedmann equation

$$H^2 = \frac{8\pi G}{3}(\rho + \rho_K) \tag{3}$$

and derive the acceleration equation

$$\frac{1}{a}\frac{d^2 a}{dt^2} = -\frac{4\pi G}{3}(\rho + 3p) \tag{4}$$

2 Problem 2: Predict the CMB Temperature

- Assume that deuterium forms when the background temperature is $T = 10^9$K. Require that neutron capture be efficient enough to form light elements but not so efficient as to leave no deuterium so that $\langle \sigma v \rangle n_b / H \sim \langle \sigma v \rangle n_b t \sim 1$. (a) With $\langle \sigma v \rangle = 4.6 \times 10^{-20}$ cm^3 s^{-1}, and the age of the universe at $T = 10^9$K of $t =$ 3min (calculated from assuming the photons and neutrinos are the dominant contributors to the Friedmann equation) estimate the baryon density n_b under this condition. (b) Assuming a current baryon number density corresponding to $\Omega_b h^2 = 0.02$, what is the scale factor at $T = 10^9$K? (c) What is the temperature of the background today?