1 Problem 1: Units that Define Cosmological Scales

Convert the following quantities by inserting the appropriate factors of $c$, $h$, $k_B$ and unit conversions. Note that $h$ in the formulae below is the reduced Hubble parameter not the Planck constant.

- $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$ into (a) eV, (b) Mpc$^{-1}$, (c) Gyr$^{-1}$. [Corresponds to upper limit on the mass of a dark energy particle, the inverse Hubble length, inverse approximate age of Universe. Keep $h$ in your formulae.]

- $\rho_{\text{crit}} = 3H_0^2/8\pi G$ into (a) g cm$^{-3}$, (b) GeV$^4$, (c) eV cm$^{-3}$, (d) protons cm$^{-3}$, (e) $M_\odot$ Mpc$^{-3}$. If the cosmological constant, has $\rho_\Lambda = 2\rho_{\text{crit}}/3$, what is its energy scale in eV (i.e. $\rho_\Lambda^{1/4}$). Compare that to the Planck mass; that these numbers are so different is the cosmological constant problem.

- $T_{\text{CMB}} = 2.728$K to (a) eV. Assuming a black body distribution, convert this to number density $n_\gamma$ in photons cm$^{-3}$ and energy density $\rho_\gamma$ in (b) eV cm$^{-3}$ (c) g cm$^{-3}$, and (d) $\Omega_\gamma = \rho_\gamma/\rho_{\text{crit}}$. 
