

Astro 242

The Physics of Galaxies and the Universe: Lecture Notes

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Syllabus

- Text: An Introduction to Modern Astrophysics 2nd Ed., Carroll and Ostlie
- First class Wed Jan 4. Reading period Mar 8-9
- Jan 4: Milky Way Galaxy
- Jan 11: Nature of Galaxies
- Jan 18: Galactic Evolution
- Jan 25: Active Galaxies
- Feb 1: In class Midterm
- Feb 8: Structure of the Universe
- Feb 15, 22: Cosmology
- Mar 29, 7: Early Universe

Common Themes

- Mapping out the Universe marching out in distance from Earth
Start with closest system: Galaxy
End with furthest system: whole Universe
- Limitations imposed by the ability to measure only a handful of quantities, all from our vantage point in the Galaxy
Common tools: flux and surface brightness, angular mapping, number counts
- Inferences on the dynamical nature of the systems by using physical laws to interpret observations: e.g. distance from inverse square law, mass from Newtonian dynamics
- Astrophysical units, while bizarre to a physicist, teach you what is being measured and how inferences are made

Astrophysical units

- Length scales
- $1\text{AU} = 1.496 \times 10^{13}\text{cm}$ – Earth-sun distance – used for solar system scales
- $1\text{pc} = 3.09 \times 10^{18}\text{cm} = 2.06 \times 10^5\text{AU}$ – 1AU subtends 1arcsecond on the sky at 1pc – distances between nearby stars

Defined by measuring parallax of nearby stars to infer distance - change in angular position during Earth's orbit: par(allax arc)sec(ond)

$$\frac{1\text{AU}}{1\text{pc}} = \frac{1}{2.06 \times 10^5} = 4.85 \times 10^{-6} = \frac{\pi}{60 * 60 * 180} = 1''$$

- $1\text{kpc} = 10^3\text{pc}$ – distances in the Galaxy
- $1\text{Mpc} = 10^6\text{pc}$ - distances between galaxies
- $1\text{Gpc} = 10^9\text{pc}$ - scale of the observable universe

Astrophysical units

- Fundamental observables are the **flux** F (energy per unit time per unit area) or **brightness** (+ per unit solid angle) and **angular position** of objects in a given **frequency** band
- Related to the physical quantities, e.g. the **luminosity** of the object L if the distance to the object is known

$$F = \frac{L}{4\pi d^2}$$

- Solar luminosity

$$L_{\odot} = 3.839 \times 10^{26} \text{W} = 3.839 \times 10^{33} \text{erg/s}$$

- Frequency band defined by filters - in limit of infinitesimal bands, the whole **frequency spectrum** measured – “spectroscopy”

Astrophysical units

- **Relative flux** easy to measure - absolute flux requires calibration of filter: (apparent) **magnitudes** (originally defined by eye as filter)

$$m_1 - m_2 = -2.5 \log(F_1/F_2)$$

- **Absolute magnitude**: apparent magnitude of object at $d = 10\text{pc}$

$$m - M = -2.5 \log(d/10\text{pc})^2 \rightarrow \frac{d(m - M)}{10\text{pc}} = 10^{(m-M)/5}$$

- If frequency spectrum has **lines**, **Doppler shift** gives relative **radial velocity** of object V_r aka **redshift** z

$$1 + z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{rest}}} = 1 + \frac{V_r}{c}$$

(where $V_r > 0$ denotes recession and redshift and $V_r \ll c$) used to measure velocity for dynamics of systems, including universe as whole

Astrophysical units

- Masses in units of solar mass $M_{\odot} = 1.989 \times 10^{33} \text{g}$
- Measurement of distance and angle gives physical size and Doppler shift gives velocity \rightarrow mass
- Mass measurement always boils down to inferring gravitational force necessary to keep test object in orbit
- For circular motion - centripetal force

$$\frac{mv^2}{r} \approx \frac{GmM}{r^2} \rightarrow M \approx \frac{v^2 r}{G}$$

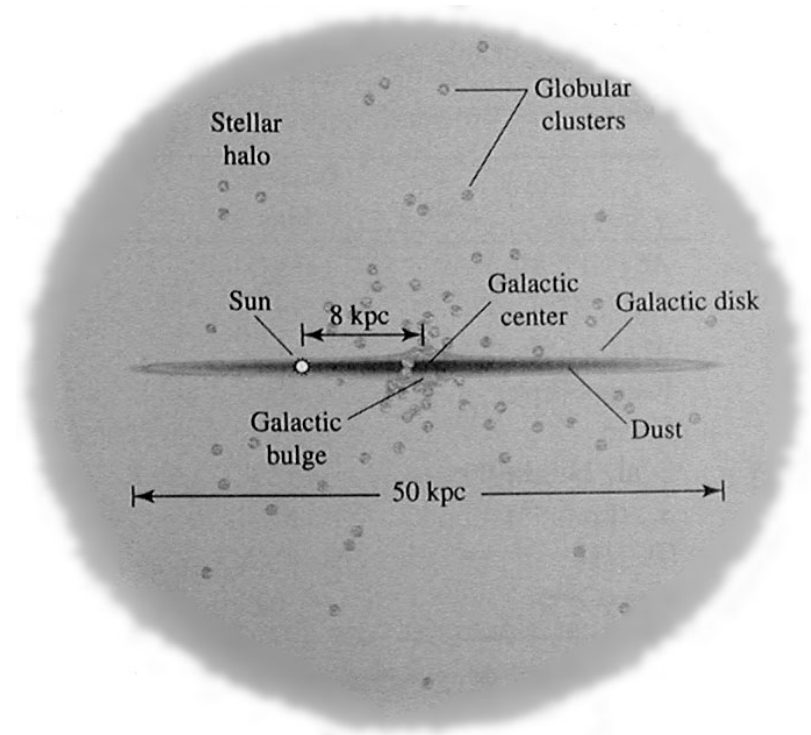
- Requires a measurement of velocity and a measurement or estimate of size
- Various systems will have order unity correction to this circular-motion based relation

Set 1:

Milky Way Galaxy

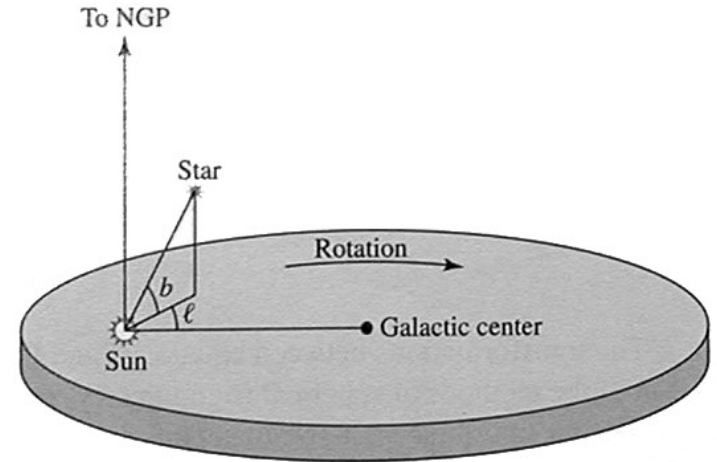
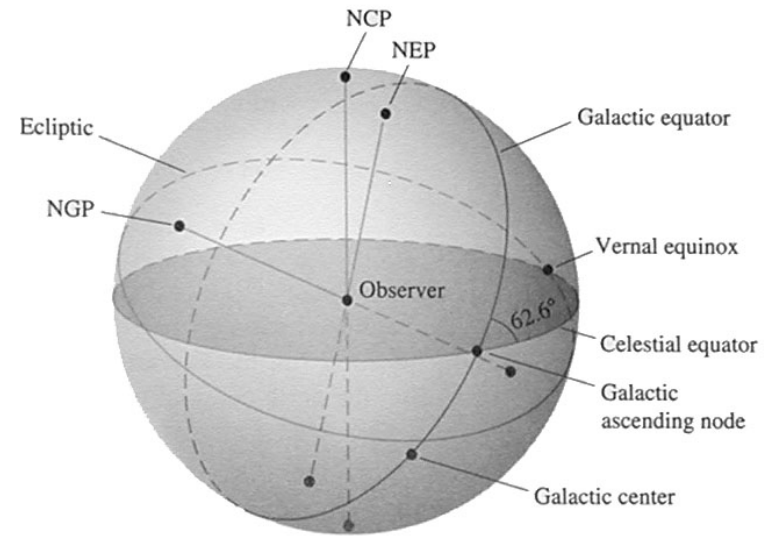
Galactic Census

- Sun is embedded in a **stellar disk**
~ 8 kpc from the galactic center
- Extent of disk
~ 25 kpc radius, **spiral structure**
- Thickness of **neutral gas**
disk < 0.1 kpc
- Thickness of **thin disk** of young stars ~ 0.35 kpc
- Thickness of **thick disk** ~ 1 kpc



Galactic Census

- Central **stellar bulge**
radius ~ 4 kpc, with **central bar**
- **Supermassive black hole**, inferred from large mass within 120AU (solar system scale) of center
- Extended spherical **stellar halo** with **globular clusters**, radius > 100 kpc
- Extended **dark matter halo**, radius > 200 kpc



Mass and Luminosity

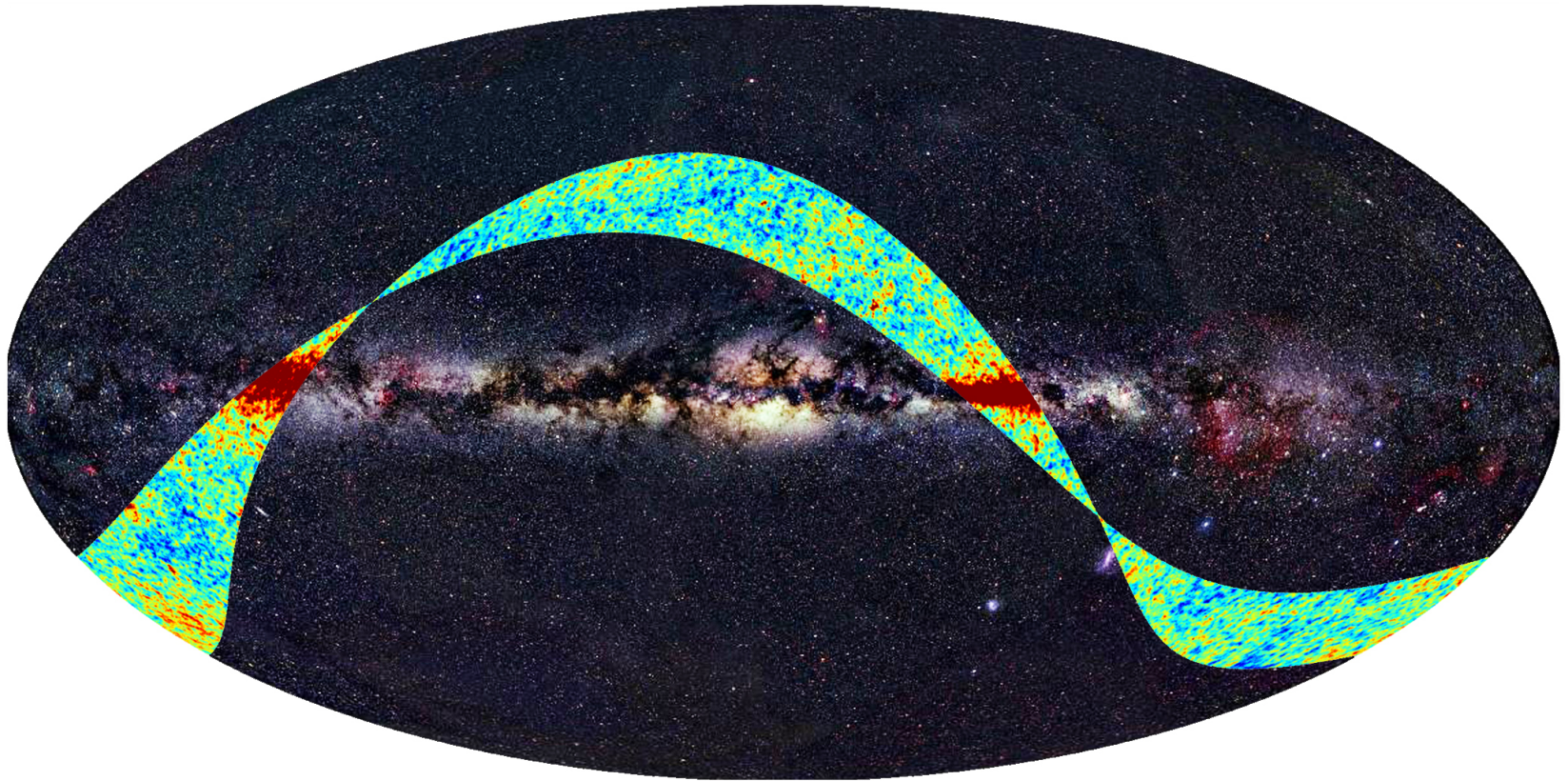
	Mass	Luminosity (L_B)
Neutral gas disk	$0.5 \times 10^{10} M_\odot$	
Thin disk	$6 \times 10^{10} M_\odot$	$1.8 \times 10^{10} L_\odot$
Thick disk	$0.2 - 0.4 \times 10^{10} M_\odot$	$0.02 \times 10^{10} L_\odot$
Bulge	$1 \times 10^{10} M_\odot$	$0.3 \times 10^{10} L_\odot$
Supermassive black hole	$3.7 \pm 0.2 \times 10^6 M_\odot$	
Stellar halo	$0.3 \times 10^{10} M_\odot$	$0.1 \times 10^{10} L_\odot$
Dark matter halo	$2 \times 10^{12} M_\odot$	
Total	$2 \times 10^{12} M_\odot$	$2.3 \times 10^{10} L_\odot$

How Do We Know?

- Infer this structure from the handful of observables that are directly accessible
- Convert intrinsically 2D information to 3D + dynamical model
- Flux and number of stars
- Angular positions of stars (as a function of season, time)
- Relative radial velocity from Doppler effect

Starlight: Optical Image

- Color overlay: microwave background



Star Counts

- One of the oldest methods for inferring the structure of the galaxy from 2D sky maps is from **star counts**
- History: **Kapteyn** (1922), building on early work by Herschel, used star counts to map out the structure of the galaxy
- Fundamental assumptions
 - Stars have a known (distribution in) **absolute magnitude**
 - No obscuration**
- Consider a star with known absolute magnitude M (magnitude at 10pc). Its **distance** can be inferred from the **inverse square law** from its **observed** m as

$$\frac{d(m - M)}{10\text{pc}} = 10^{(m-M)/5}$$

Star Counts

- Combined with the **angular position** on the sky, the **3d position** of the star can be measured - mapping the galaxy
- Use the star counts to determine **statistical properties**: number density of stars in each patch of sky
- A **fall off** in the number density in radial distance would determine the **edge** of the galaxy
- Suppose there is an **indicator** of absolute magnitude like spectral type that allows stars to be **selected** to within dM of M
- Describe the underlying quantity to be extracted as the **spatial number density** within dM of M : $n_M(M, \mathbf{r})dM$

Star Counts

- The observable is say the **total number** of stars **brighter** than a limiting **apparent magnitude** m in a **solid angle** $d\Omega$
- Stars at a given M can only be observed out to a distance $d(m - M)$ before their apparent magnitude falls below the limit
- So there is radial distance limit to the volume observed
- **Total number** observed out to in solid angle $d\Omega$ within dM of M is **integral** to that limit

$$N_M dM = \left[\int_0^{d(m-M)} n_M(M, r) r^2 dr \right] d\Omega dM$$

- **Differentiating** with respect to $d(m - M)$ provides a measurement of $n_M(M, r)$

Star Counts

- So dependence of counts on the limiting magnitude m determines the number density and e.g. the edge of the system
- In fact, if there were no edge to system the total flux would diverge as $m \rightarrow 0$ - volume grows as d^3 flux decreases as d^{-2} : Olber's paradox
- Generalizations of the basic method:
- Selection criteria is not a perfect indicator of M and so dM is not infinitesimal and some stars in the range will be missed - $S(M)$ and M is integrated over - total number

$$N = \int_{-\infty}^{\infty} dM S(M) N_M$$

Star Counts

- Alternately use **all stars** [weak or no $S(M)$] but assume a **functional form** for $n_M(M, 0)$ e.g. derived from local estimates and assumed to be the same at larger $r > 0$

In this case, measurements determine the **normalization** of a distribution with **fixed shape**, i.e.

$$n(\mathbf{r}) = \int_{-\infty}^{\infty} n_M(M, \mathbf{r}) dM$$

- **Kapteyn** used all of the stars (assumed to have the same n_M shape in r)
- He inferred a **flattened spheroidal system** of $< 10\text{kpc}$ extent in plane and $< 2\text{kpc}$ out of plane: **too small**
- Missing: **interstellar extinction** dims stars dropping them out of the sample at a given limiting magnitude

Variable Stars

- With a good indicator of absolute magnitude or “standard candle” one can use individual objects to map out the structure of the Galaxy (and Universe)
- History: Shapley (1910-1920) used RR Lyrae and W Virginis variable stars - with a period-luminosity relation
Radial oscillations with a density dependent sound speed - luminosity and density related on the instability strip
Calibrated locally by moving cluster and other methods
- Measure the period of oscillation, infer a luminosity and hence an absolute magnitude, infer a distance from the observed apparent magnitude

Variable Stars

- Inferred a **100kpc** scale for the Galaxy - overestimate due to **differences** in types of **variable stars** and **interstellar extinction**
- **Apparent magnitude** is **dimmed** by extinction leading to the variable stars being **less distant** than they appear
- Both Kapteyn and Shapley off because of dust extinction: discrepancy between two independent methods indicates **systematic error**
- Caveat emptor: in astronomy always want to see a **cross check** with two or more **independent methods** before believing result you read in the NYT!

Common techniques

- Particular objects such as stars of a given type and Cepheid variables are both interesting in their own right and, once understood, useful for tracing out larger systems
- Star counts is an example of a general theme in astronomy: using a large survey (stars) to infer statistical properties
- Likewise, cepheids are an example of using a standard candle to map out a system
- Similar method applies to mapping out the Universe with galaxies e.g. baryon acoustic oscillations vs supernova

Interstellar Extinction

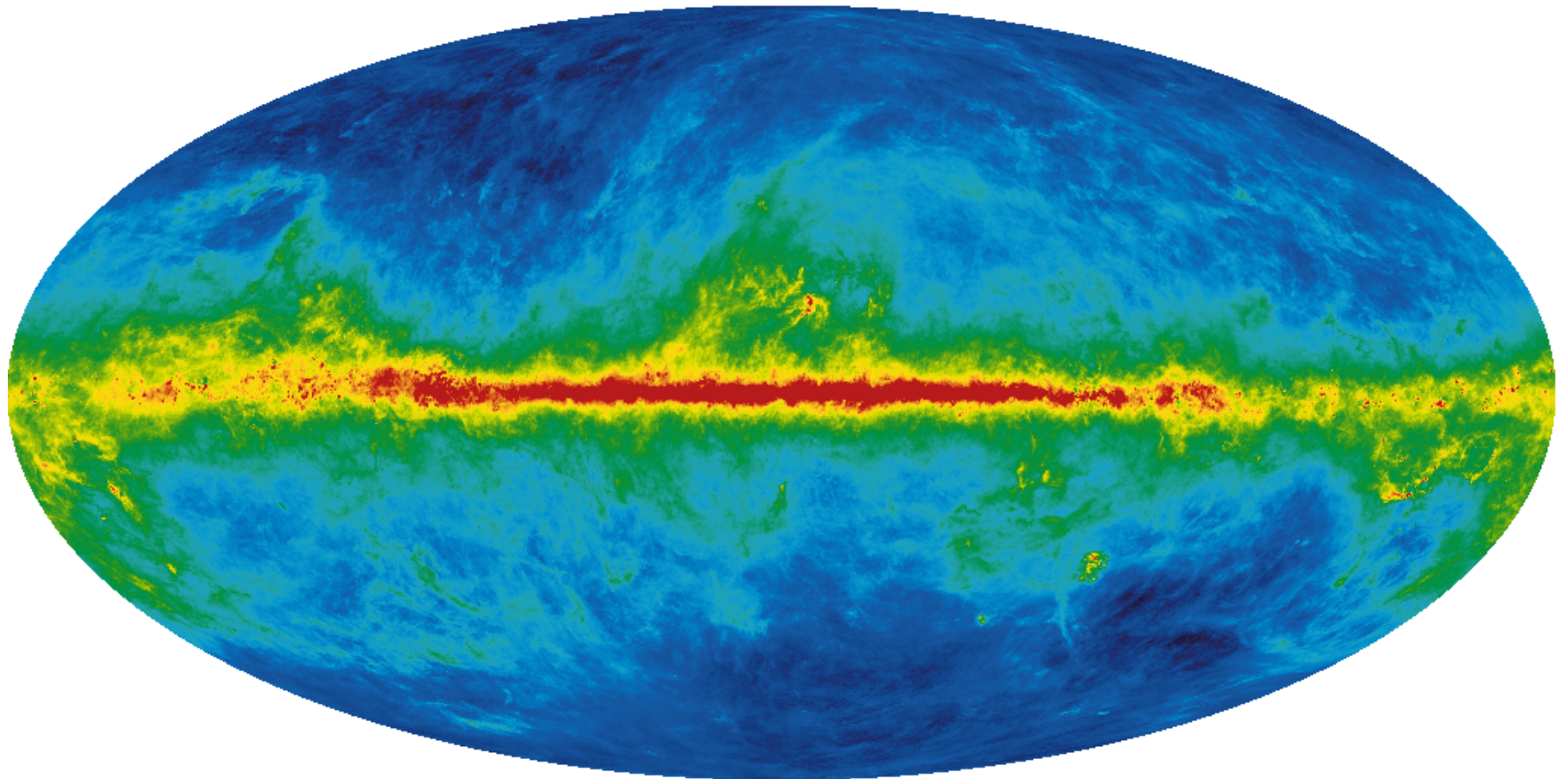
- Dust (silicates, graphite, hydrocarbons) in ISM (Chap 12) dims stars at visible wavelengths making true distance less than apparent
- Distance formula modified to be

$$\frac{d}{10\text{pc}} = 10^{(m_\lambda - M_\lambda - A_\lambda)/5}$$

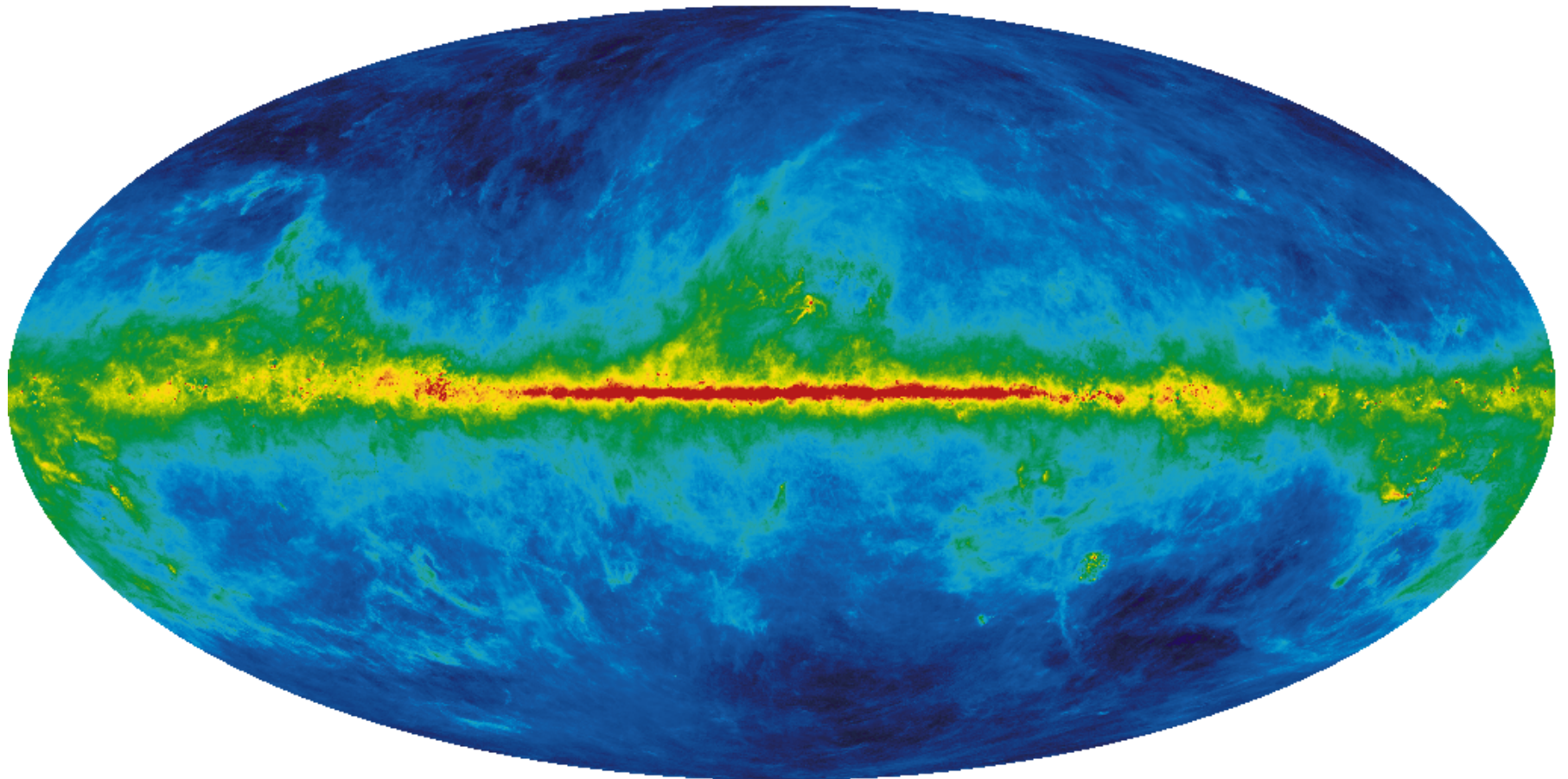
where the extinction coefficient $A_\lambda \geq 0$ depends on wavelength λ

- Extinction also depends on direction, e.g. through the disk, through a giant molecular cloud, etc. Typical value at visible wavelengths and in the disk is 1 mag/kpc
- Dust emits or reradiates starlight in the infrared - maps from these frequencies [IRAS, DIRBE] can be used to calibrate extinction

Dust Emission



Extinction Correction



Kinematic Distances to Stars

- Only nearby stars have their distance measured by parallax - further than a parsec the change in angle is < 1 arcsec:

$$p(\text{arcsec}) = 1\text{pc}/d$$

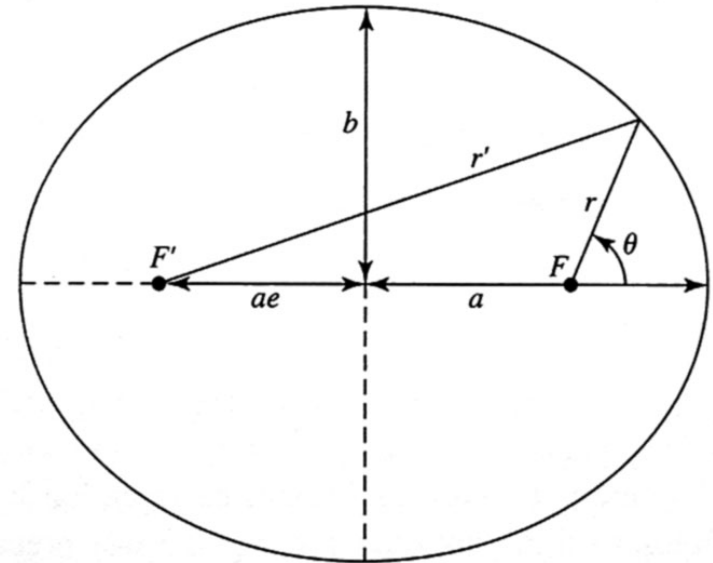
- If proper motion across the sky can be measured from the change in angular position μ in rad/s

$$v_t = \mu d$$

- Often v_t can be inferred from the radial velocity and a comparison with μ gives distance d given assumption of the dynamics
- Example: Keplerian orbits of stars around galactic center
 $R_0 = 7.6 \pm 0.3\text{kpc}$
- Example: Stars in a moving cluster share a single total velocity whose direction can be inferred from apparent convergent motion (see Fig 24.30)

Stellar Kinematics

- Can infer more than just distance: SMBH
- Galactic center: follow orbits of stars close to galactic center
- One star: orbital period 15.2yrs, eccentricity $e = 0.87$, perigalacticon distance (closest point on orbit to F) 120 AU= 1.8×10^{13} m
- Estimate mass: $a = ae - r_p$ so semimajor axis



$$a = \frac{r_p}{1 - e} = 1.4 \times 10^{14} \text{m}$$

Stellar Kinematics

- Kepler's 3rd law

$$M = \frac{4\pi^2 a^3}{GP^2} = 7 \times 10^{36} \text{kg} = 3.5 \times 10^6 M_{\odot}$$

- That much mass in that small a radius can plausibly only be a (supermassive) black hole
- Note that this is an example of the general statement that masses are estimated by taking

$$M \approx \frac{v^2 r}{G} = \frac{(2\pi a)^2 a}{GP^2} = \frac{4\pi^2 a^3}{GP^2}$$

Stellar Kinematics

- Apply these stellar kinematics techniques to the **galactic disk** of stars
- Direct observables are the **kinematics** of **neighboring stars**
 - ℓ, b : angular position of star in galactic coordinates
 - v_r : relative motion radial to line of sight
 - v_t : relative motion tangential to line of sight
- These stars have their parallax distance d measured
- The distance to center of galaxy R_0 is known
- Infer the **rotation** of stars in the **disk** around galactic center

Stellar Kinematics

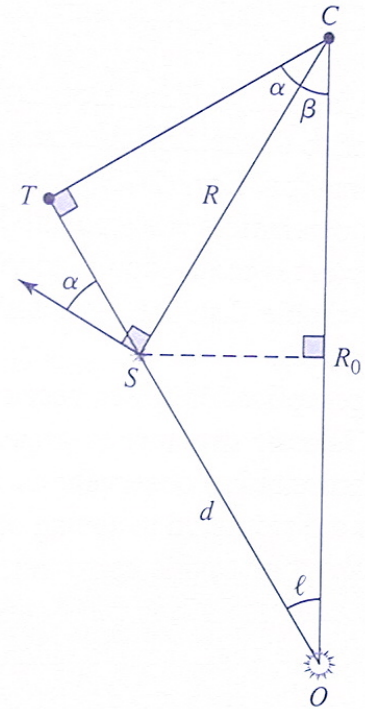
- Differential rotation $\Theta(R) = R\Omega(R)$
where $\Omega(R)$ is the angular velocity curve—
observables are radial and tangential motion
with respect to LSR

$$v_r = R\Omega \cos \alpha - R_0\Omega_0 \sin \ell$$

$$v_t = R\Omega \sin \alpha - R_0\Omega_0 \cos \ell$$

where Θ_0 is the angular velocity of the LSR

- Technical point: defined through local standard of rest (LSR)
rather than sun's rest frame due to small differences between solar
motion and the average star around us



Stellar Kinematics

- d (parallax) and R_0 are known observables, R is not - eliminate with trig relations

$$R \cos \alpha = R_0 \sin \ell \quad R \sin \alpha = R_0 \cos \ell - d$$

- Eliminate R and solve for (Ω, Ω_0)

$$v_r = (\Omega - \Omega_0) R_0 \sin \ell$$

$$v_t = (\Omega - \Omega_0) R_0 \cos \ell - \Omega d$$

- Historical context: solve for $\Omega(R)$ locally where

$$\begin{aligned} \Omega - \Omega_0 &\approx \frac{d\Omega}{dR} (R - R_0) \\ &\approx \frac{1}{R_0} \left(\frac{d\Theta}{dR} - \frac{\Theta_0}{R_0} \right) (R - R_0) \quad [\Omega = \Theta/R] \end{aligned}$$

and $d \ll R_0$, $\cos \beta \approx 1$

Stellar Kinematics

- Reduce with trig identities

$$R_0 = d \cos \ell + R \cos \beta \approx d \cos \ell + R$$

$$R - R_0 \approx -d \cos \ell$$

$$\cos \ell \sin \ell = \frac{1}{2} \sin 2\ell$$

$$\cos^2 \ell = \frac{1}{2} (\cos 2\ell + 1)$$

to obtain

$$v_r \approx Ad \sin 2\ell$$

$$v_t \approx Ad \cos 2\ell + Bd$$

Stellar Kinematics

- Oort constants

$$A = -\frac{1}{2} \left[\frac{d\Theta}{dR} - \frac{\Theta_0}{R_0} \right] = -\frac{R_0}{2} \frac{d\Omega}{dR}$$
$$B = -\frac{1}{2} \left[\frac{d\Theta}{dR} + \frac{\Theta_0}{R_0} \right]$$

- Observables v_r , v_t , ℓ , d : solve for Oort's constants. From Hipparcos

$$A = 14.8 \pm 0.8 \text{ km/s/kpc}$$

$$B = -12.4 \pm 0.6 \text{ km/s/kpc}$$

- Angular velocity $\Omega = \Theta/R$ decreases with radius: differential rotation.

Stellar Kinematics

- Physical velocity $\Theta(R)$

$$\left. \frac{d\Theta}{dR} \right|_{R_0} = -(A + B) = -2.4 \text{ km/s/kpc}$$

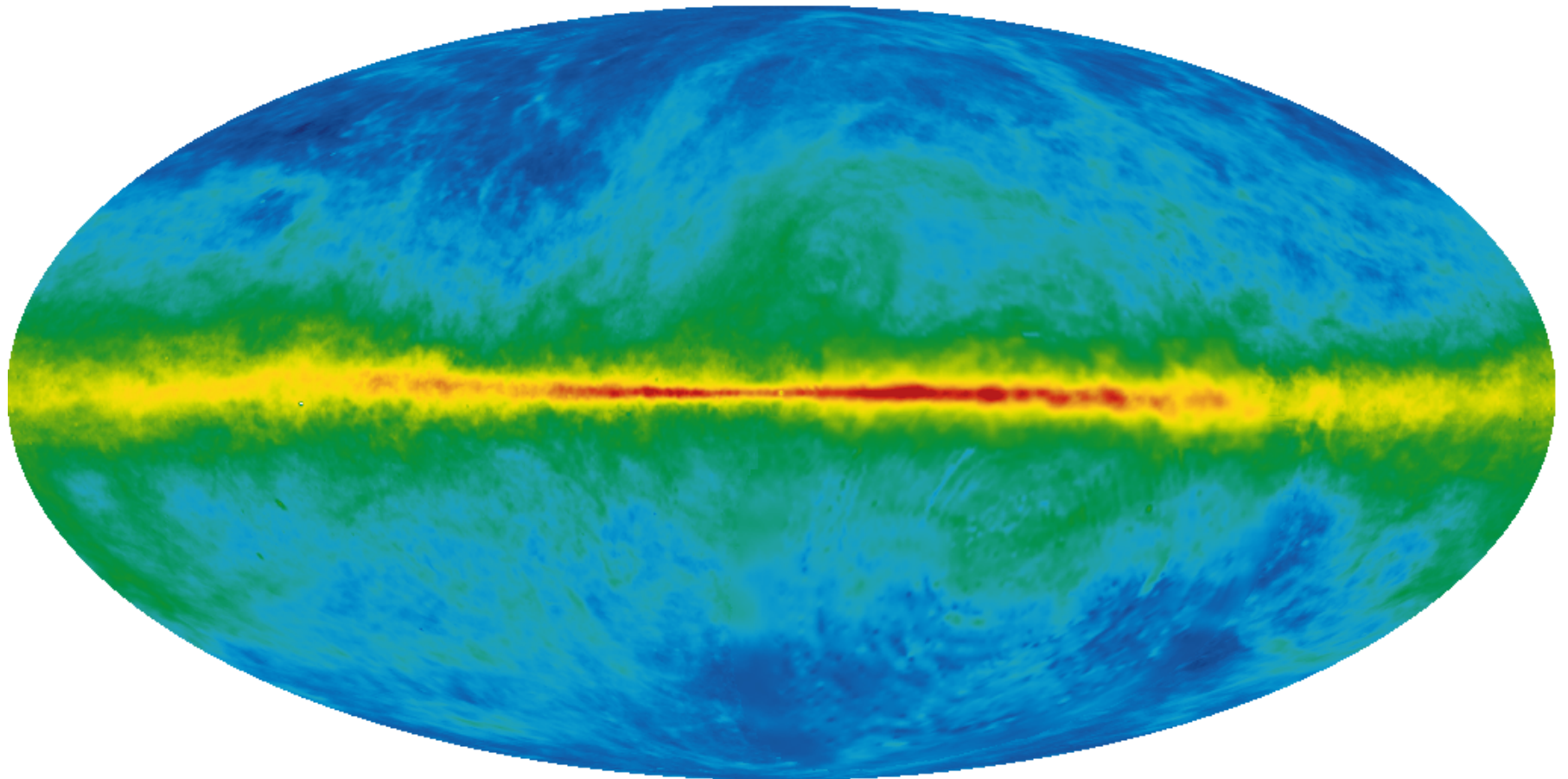
decreases slowly compared with

$$\frac{\Theta_0}{R_0} = A - B = 27.2 \text{ km/s/kpc} \rightarrow \Theta_0 \approx 220 \text{ km/s}$$

a nearly flat rotation curve – at least locally

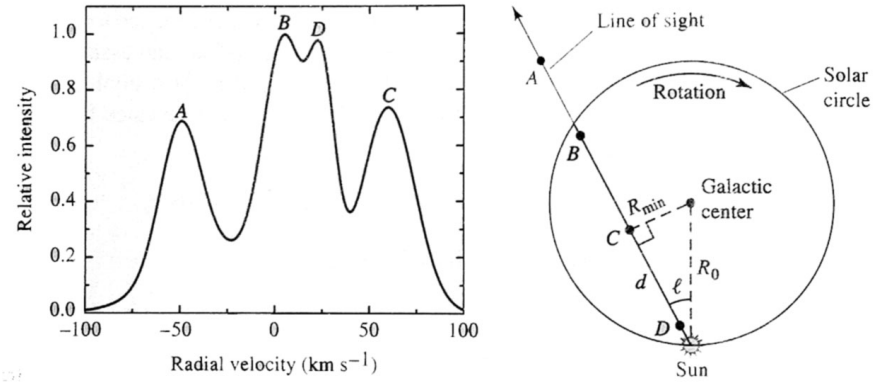
- Does this flat rotation curve extend out in the disk away from the sun?

Neutral Gas: 21cm Emission



21 cm

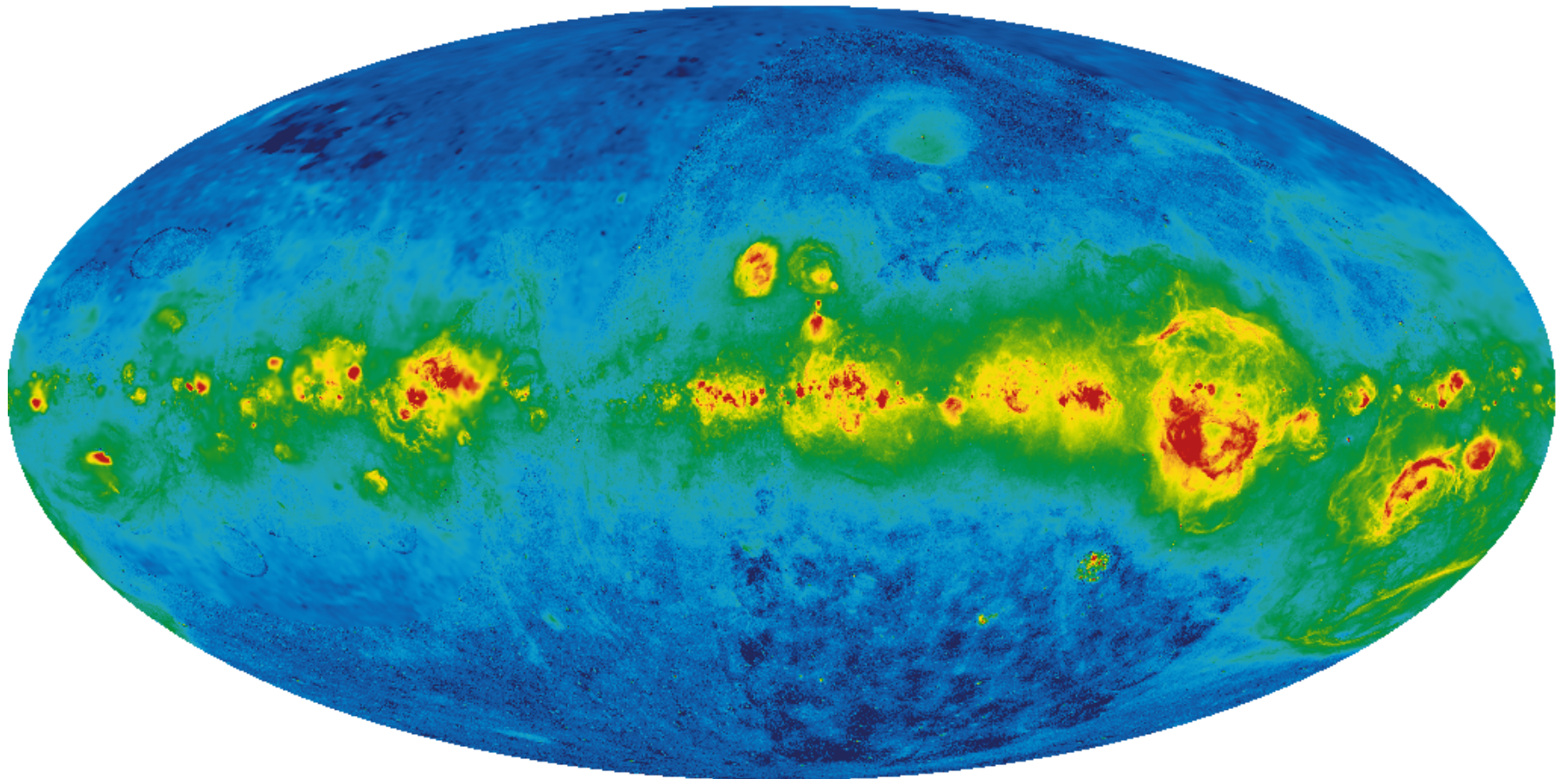
- Spin interaction of the electron and proton leads to a spin flip transition in neutral hydrogen with wavelength 21cm
- Line does not suffer substantial extinction and can be used to probe the neutral gas and its radial velocity from the Doppler shift throughout the galaxy
- No intrinsic distance measure
- Neutral gas is distributed inhomogeneously in clouds leading to distinct peaks in emission along each sight line



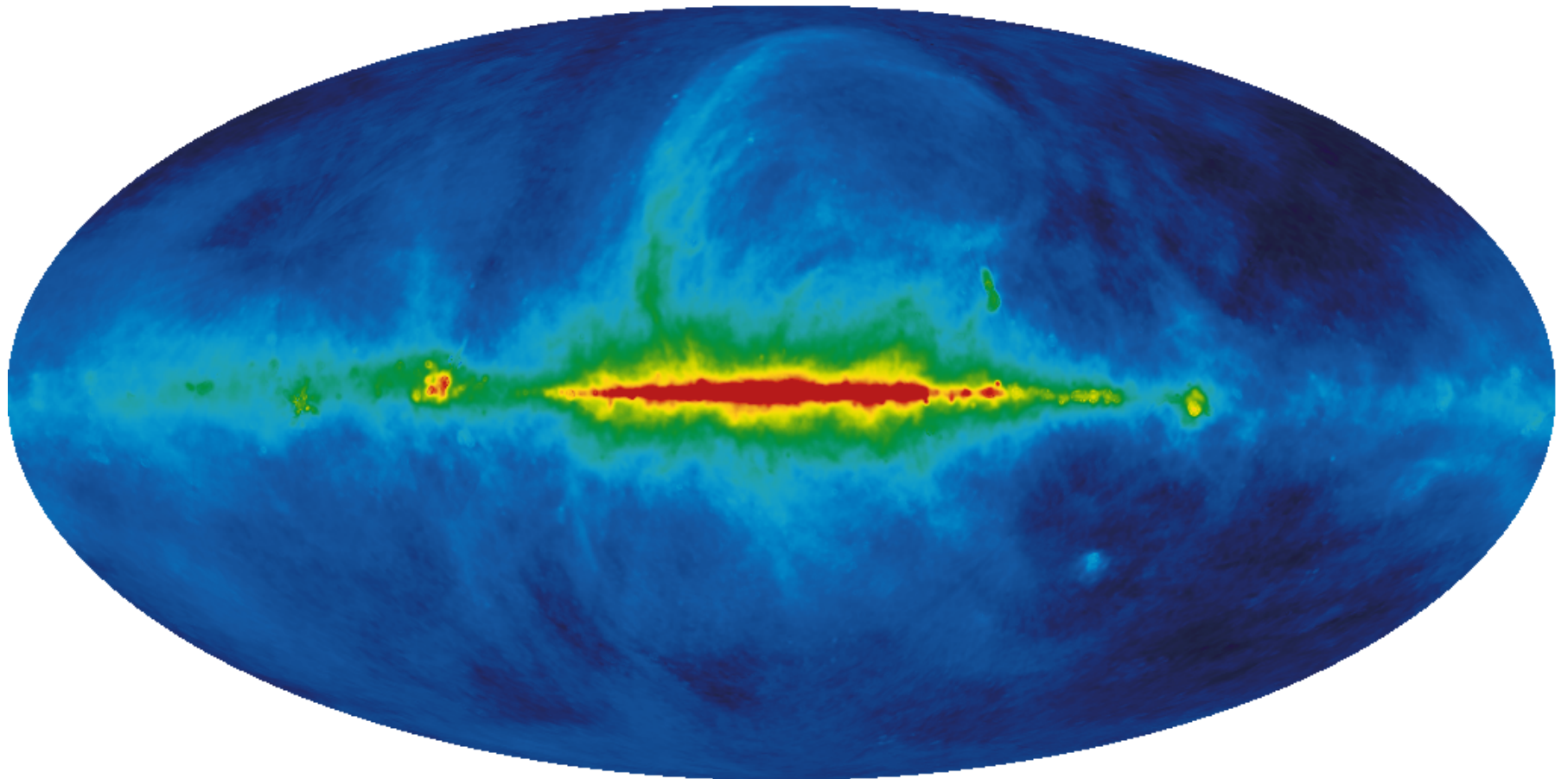
21 cm

- Due to projection of velocities along the line of sight and differential rotation, the highest velocity occurs at the closest approach to the galactic center or tangent point
- Build up a rotation curve interior to the solar circle $R < R_0$
- Rotation curve steeply rises in the interior $R < 1\text{kpc}$, consistent with near rigid body rotation and then remains flat out through the solar circle

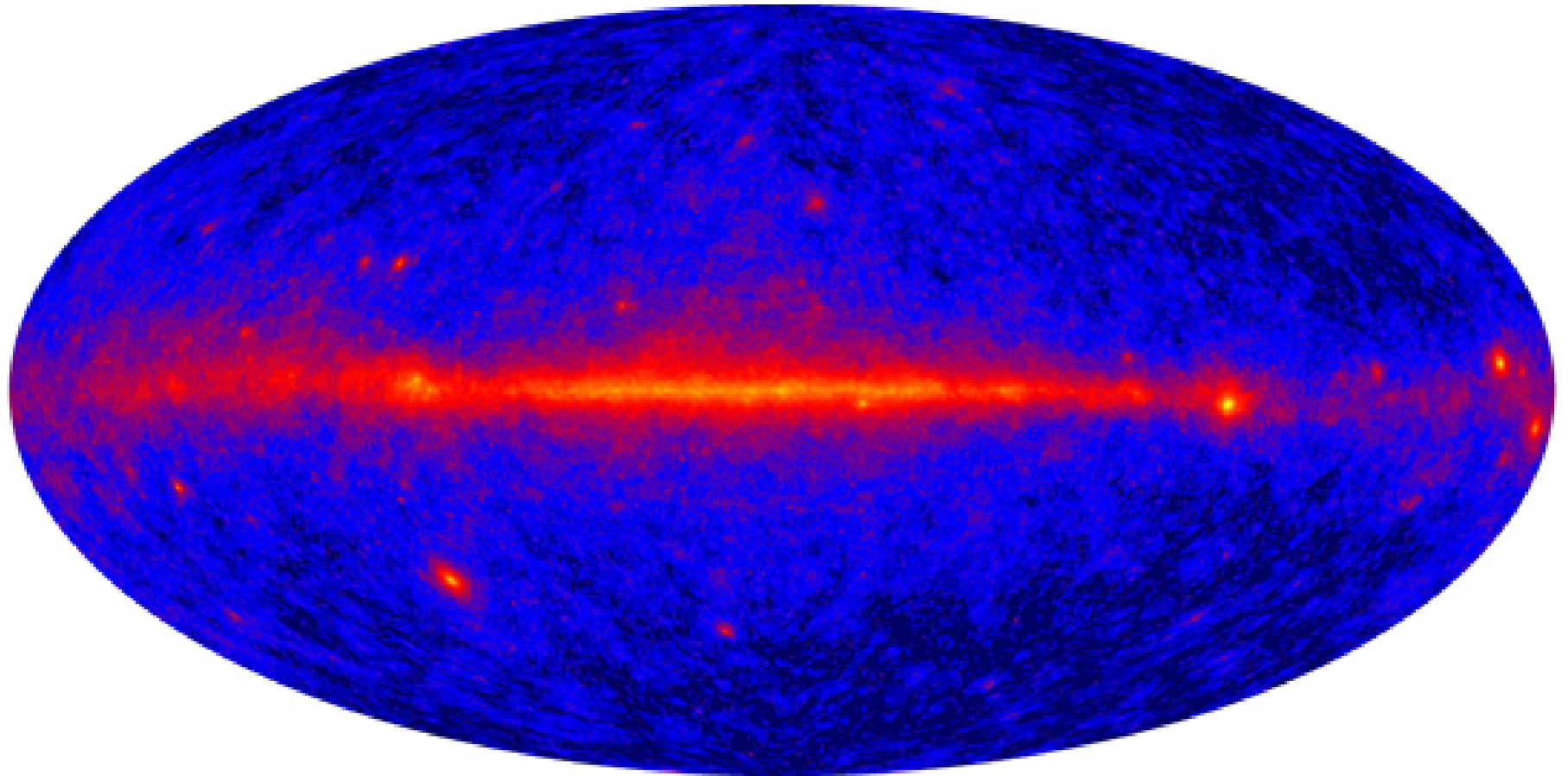
Ionized Gas: H α Line Emission



Cosmic Rays in B Field: Synchrotron

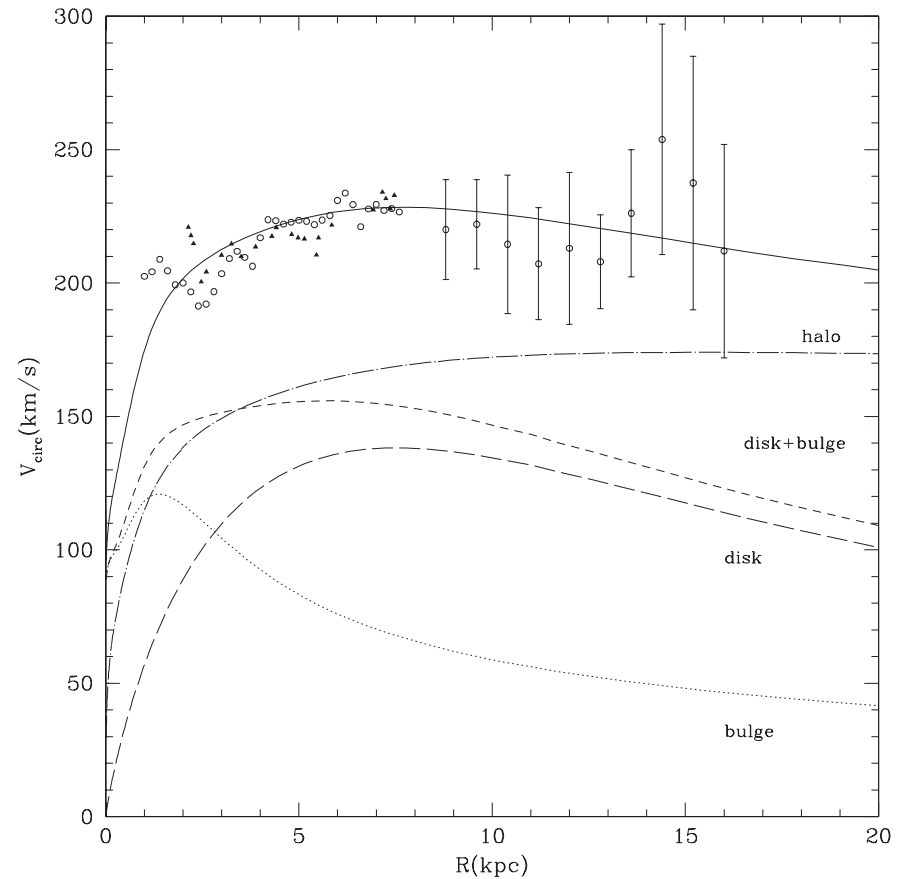


Gamma Rays



Rotation Curves

- Extending the rotation curve beyond the solar circle with objects like Cepheids whose distances are known reveals a flat curve out to $\sim 20\text{kpc}$
- Mass required to keep rotation curves flat much larger than implied by stars and gas. Consider a test mass m orbiting at a radius r around an enclosed mass $M(r)$



Rotation Curves

- Setting the centripetal force to the gravitational force

$$\frac{mv^2(r)}{r} = \frac{GM(r)m}{r^2}$$

$$M(r) = \frac{v(r)^2 r}{G}$$

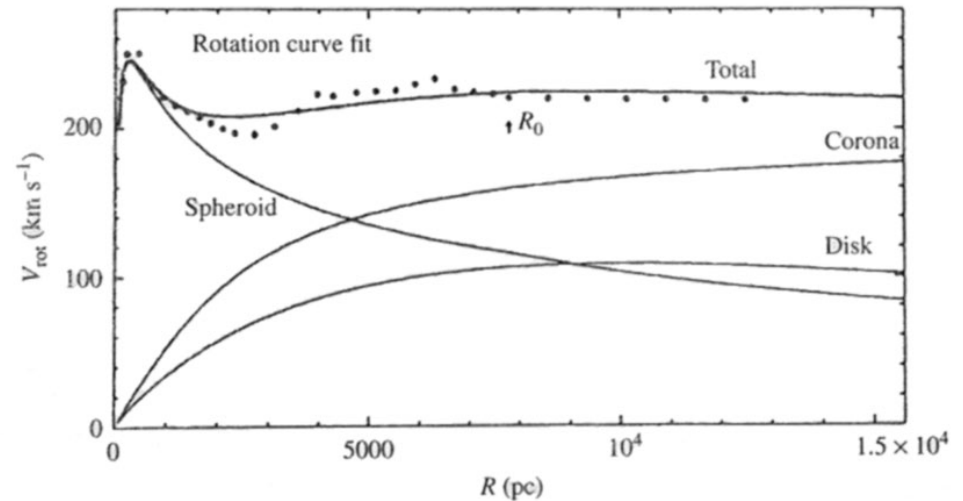
Again this combination of velocity and distance is the fundamental way masses are measured:

gravitational force binds object against the motion of the luminous objects - other examples: virial theorem with velocity dispersion, hydrostatic equilibrium with thermal motions

- Measuring the rotation curve $v(r)$ is equivalent to measuring the mass profile $M(r)$ or density profile $\rho(r) \propto M(r)/r^3$

Rotation Curves

- Flat rotation curve $v(r) = \text{const}$ implies $M \propto r$ - a mass linearly increasing with radius
- Rigid rotation implies $\Omega = v/r = \text{const}$. $v \propto r$ or $M \propto r^3$ or $\rho = \text{const}$
- Rotation curves in other galaxies show the same behavior: evidence that “dark matter” is ubiquitous in galaxies



Rotation Curves

- Consistent with dark matter density given by

$$\rho(r) = \frac{\rho_0}{1 + (r/a)^2}$$

- Also consistent with the NFW profile predicted by cold dark matter (e.g. weakly interacting massive particles or WIMPs)

$$\rho(r) = \frac{\rho_0}{(r/a)(1 + r/a)^2}$$

Gravitational Lensing

- Rotation curves leave open the question of what dark matter is
- Alternate hypothesis: dead stars or black holes - massive astrophysical compact halo object “MACHO”
- MACHOs have their mass concentrated into objects with mass comparable to the sun or large planet
- A MACHO at an angular distance $u = \theta/\theta_E$ from the line of sight to the star will gravitationally lens or magnify the star by a factor of

$$A(u) = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}}$$

where θ_E is the Einstein ring radius in projection

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{d_S - d_L}{d_S d_L}}$$

Gravitational Lensing

- Again masses related to a physical scale $r_E = \theta_E d_L$ and speed c

$$M \sim \frac{d_S}{4(d_S - d_L)} \frac{c^2 r_E}{G}$$

e.g. for a typical lens half way to the source the prefactor is 1/2

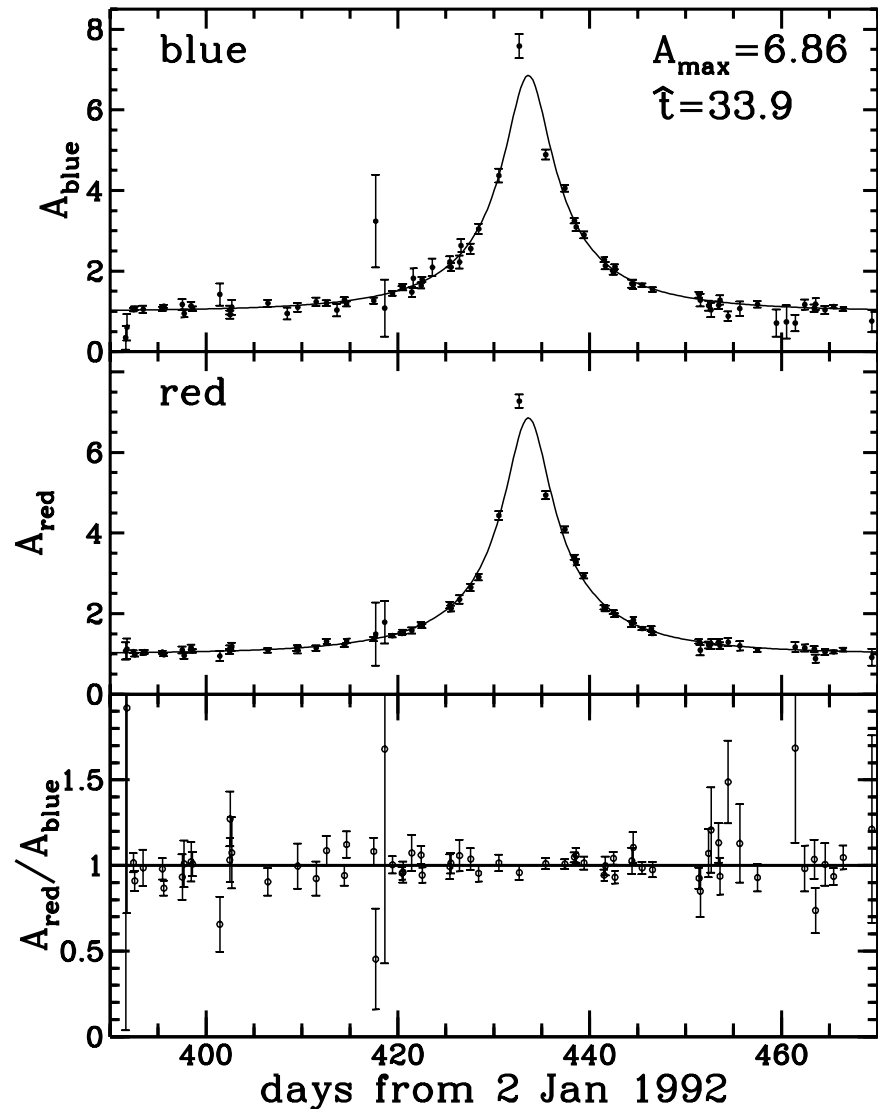
- A MACHO would move at a velocity typical of the disk and halo $v \sim 200\text{km/s}$ and so the star behind it would brighten as it crossed the line of sight to a background star. With u_{\min} as the distance of closest approach at $t = 0$

$$u^2(t) = u_{\min}^2 + \left(\frac{vt}{d_L \theta_E} \right)^2$$

- Monitor a large number of stars for this characteristic brightening. Rate of events says how much of the dark matter could be in MACHOs.

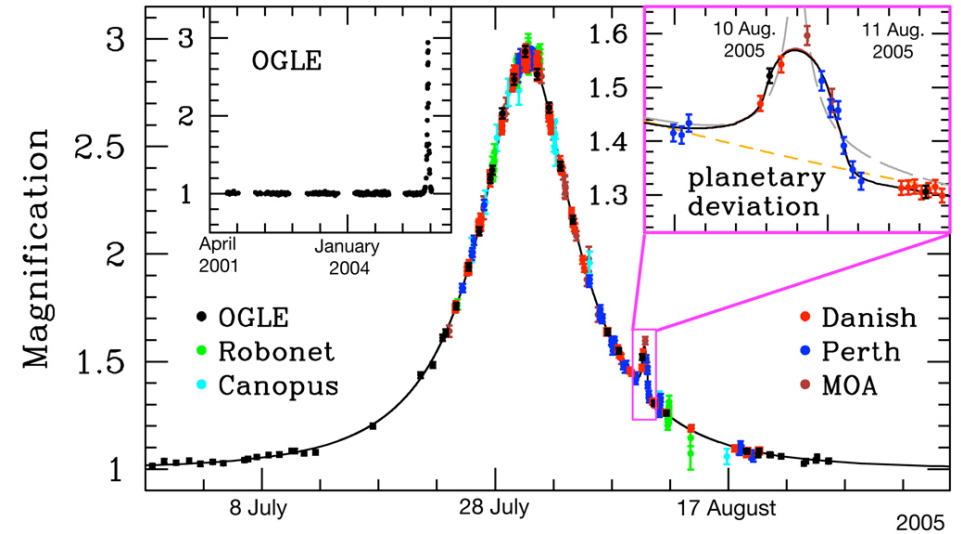
Gravitational Lensing

- In the 1990's large searches measured the rate of microlensing in the halo and bulge and determined that only a small fraction of its mass could be in MACHOs



Gravitational Lensing

- Current searches (toward the bulge) are used to find planets
- Enhanced microlensing by planet around star leads to a blip in the brightening.



Light Curve of OGLE-2005-BLG-390