

## 1 Problem 1: Units that Define Cosmological Scales

Convert the following quantities by inserting the appropriate factors of  $c$ ,  $\hbar$ ,  $k_B$  and unit conversions. Note that  $h$  in the formulae below is the reduced Hubble parameter not the Planck constant.

- $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$  into (a) eV, (b)  $\text{Mpc}^{-1}$ , (c)  $\text{Gyr}^{-1}$ . [Corresponds to upper limit on the mass of a dark energy particle, the inverse Hubble length, inverse approximate age of Universe. Keep  $h$  in your formulae.]
- $\rho_{\text{crit}} = 3H_0^2/8\pi G$  into (a)  $\text{g cm}^{-3}$ , (b)  $\text{GeV}^4$ , (c)  $\text{eV cm}^{-3}$ , (d) protons  $\text{cm}^{-3}$ , (e)  $M_\odot \text{ Mpc}^{-3}$ . If the cosmological constant, has  $\rho_\Lambda = 2\rho_{\text{crit}}/3$ , what is its energy scale in eV (i.e.  $\rho_\Lambda^{1/4}$ ). Compare that to the Planck mass; that these numbers are so different is the cosmological constant problem.
- $T_{\text{CMB}} = 2.728\text{K}$  to (a) eV. Assuming a black body distribution, convert this to number density  $n_\gamma$  in photons  $\text{cm}^{-3}$  and energy density  $\rho_\gamma$  in (b)  $\text{eV cm}^{-3}$  (c)  $\text{g cm}^{-3}$ , and (d)  $\Omega_\gamma = \rho_\gamma/\rho_{\text{crit}}$ .