1 Problem 1: Units that Define Cosmological Scales

Convert the following quantitites by inserting the appropriate factors of c, \hbar , k_B and unit conversions. Note that h in the formulae below is the reduced Hubble parameter not the Planck constant.

- $H_0 = 100h \text{km s}^{-1} \text{ Mpc}^{-1}$ into (a) eV, (b) Mpc⁻¹, (c) Gyr⁻¹. [Corresponds to upper limit on the mass of a dark energy particle, the inverse Hubble length, inverse approximate age of Universe. Keep h in your formulae.]
- $\rho_{\rm crit} = 3H_0^2/8\pi G$ into (a) g cm⁻³, (b) GeV⁴, (c) eV cm⁻³, (d) protons cm⁻³, (e) M_{\odot} Mpc⁻³. If the cosmological constant, has $\rho_{\Lambda} = 2\rho_{\rm crit}/3$, what is its energy scale in eV (i.e. $\rho_{\Lambda}^{1/4}$). Compare that to the Planck mass; that these numbers are so different is the cosmological constant problem.
- $T_{\rm CMB} = 2.728 {\rm K}$ to (a) eV. Assuming a black body distribution, convert this to number density n_{γ} in photons cm⁻³ and energy density ρ_{γ} in (b) eV cm⁻³ (c) g cm⁻³, and (d) $\Omega_{\gamma} = \rho_{\gamma}/\rho_{\rm crit}$.