Astro 242: Problem Set 7

1 Problem 1: Energy Conservation

- Take the Liouville or collisionless Boltzmann equation in an expanding universe

$$\frac{\partial f}{\partial t} + \frac{dq}{dt}\frac{\partial f}{\partial q} = 0 \tag{1}$$

and derive the continuity or energy conservation equation

$$\frac{d\rho}{dt} = -3H(\rho + p) \tag{2}$$

- Take the energy conservation equation and the Friedmann equation

$$H^2 = \frac{8\pi G}{3}(\rho + \rho_K) \tag{3}$$

and derive the acceleration equation

$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{4\pi G}{3}(\rho + 3p) \tag{4}$$

2 Problem 2: Ideal Gas Law

- Derive the ideal gas law $p = nT$ from the Maxwell Boltzmann distribution

$$f = e^{-(E-\mu)/kT} \tag{5}$$