Astro 242: Problem Set 7

1 Problem 1: Conformal Time and Horizons

- Assume the universe today is flat with both matter (Ω_m) and a cosmological constant (Ω_Λ). (a) Compute the conformal age or horizon of the universe and plot your result for $H_0\eta_0$ as a function of Ω_m. [numerically integrate for a few values and sketch the behavior] (b) What is the current horizon size for a universe with $\Omega_m = 1/3$ and $h = 1/\sqrt{2}$? (c) What is the mass contained within the current horizon in solar masses. If all objects were $10^{13}h^{-1}\ M_\odot$ in mass, how many are in the observable universe.
- Evaluate the conformal age as a function of the scale factor in the above cosmology. What happens when $a \to \infty$. Comment on the implications for establishing causal contact between observers currently separated by much more than a Hubble length.

2 Problem 2: Friedmann Equation and Energy Conservation

- Take the energy conservation equation and the Friedmann equation

$$H^2 = \frac{8\pi G}{3}(\rho + \rho_K) \tag{1}$$

and derive the acceleration equation

$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{4\pi G}{3}(\rho + 3p) \tag{2}$$