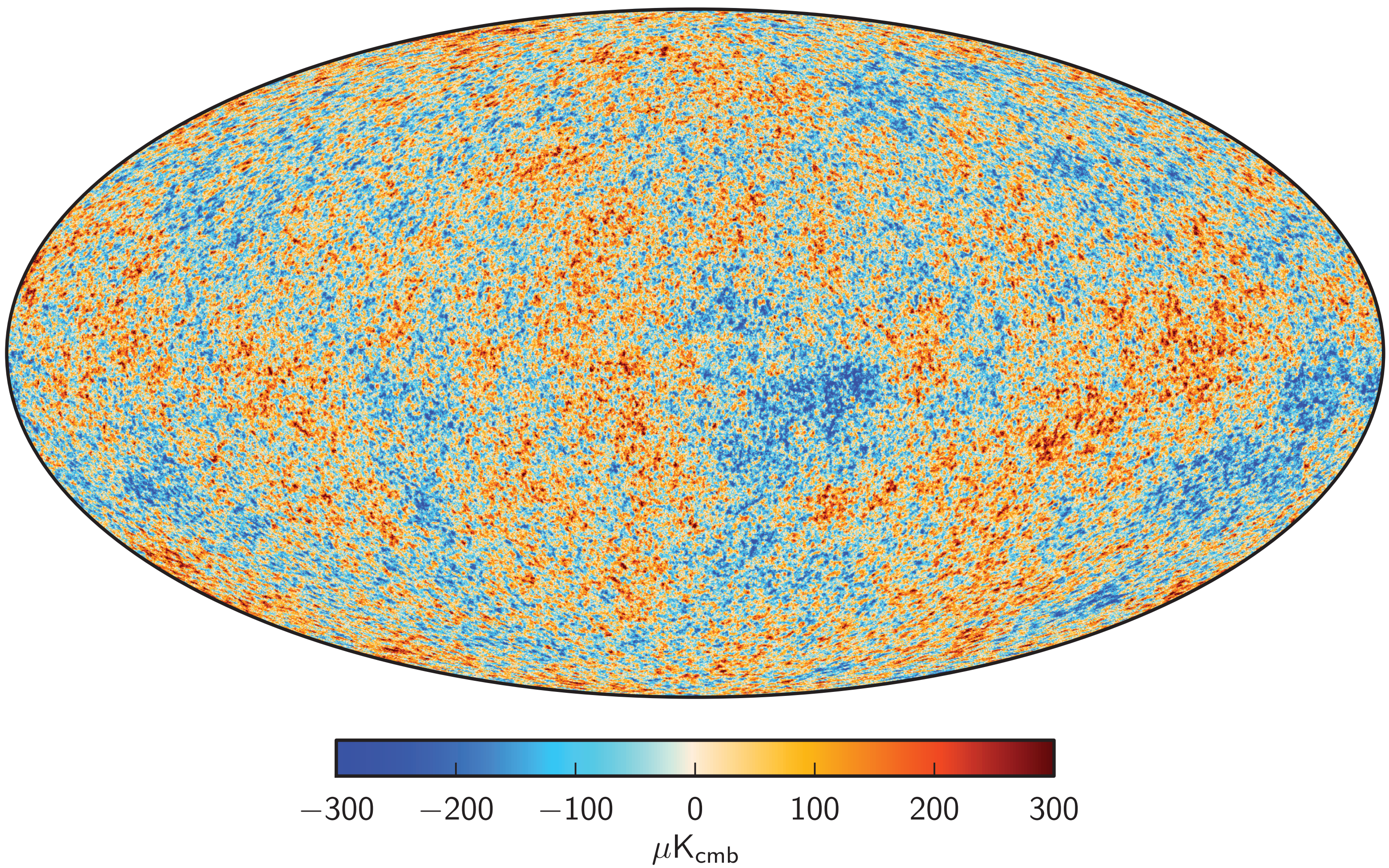
Set 7:

CMB and Large Scale Structure

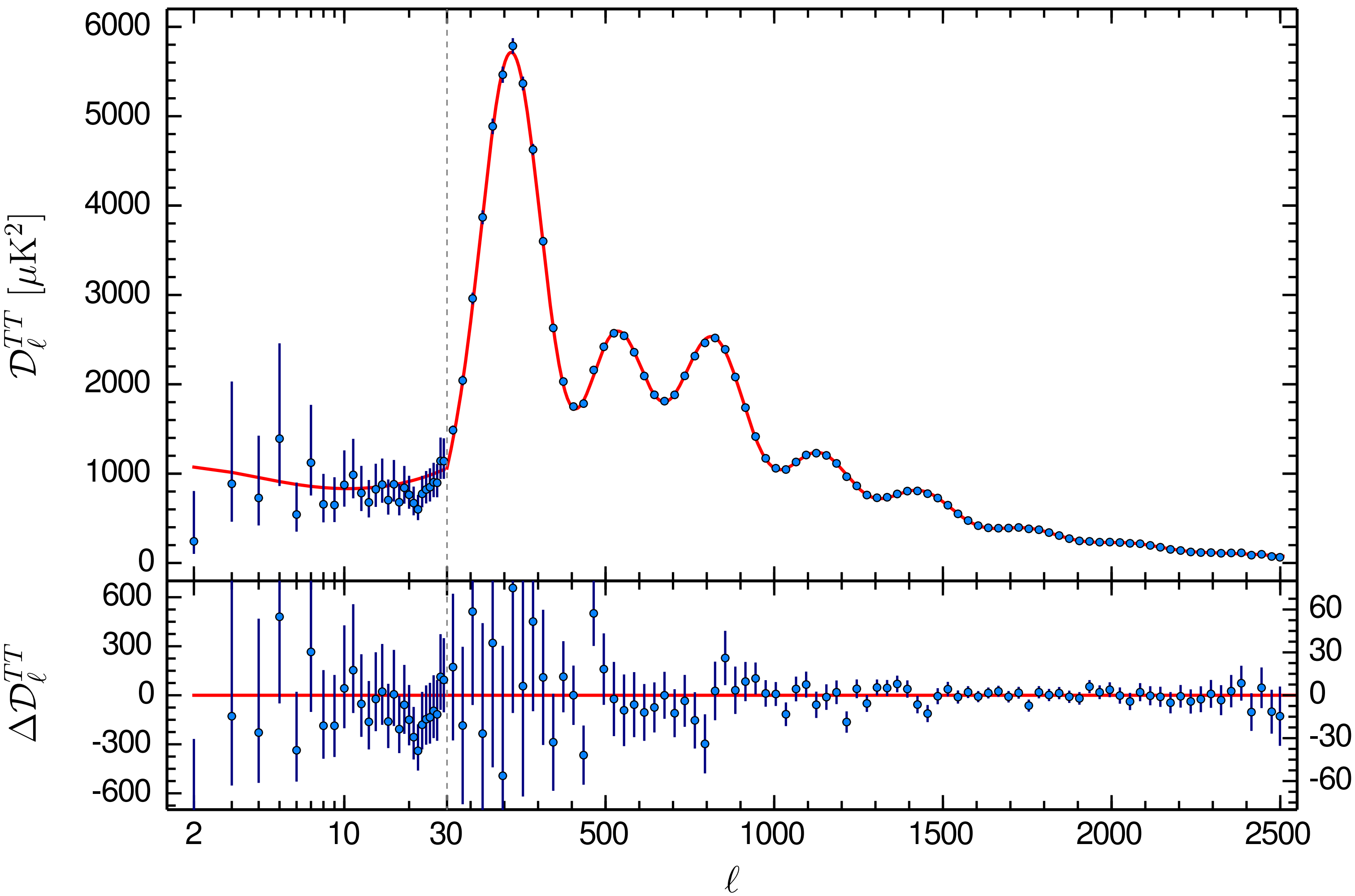
CMB Temperature Anisotropy

- Planck 2015 map of the temperature anisotropy (first discovered by COBE) from recombination:



CMB Temperature Anisotropy

- Power spectrum shows characteristic scales where the intensity of variations peak - reveals geometry and contents of the universe:



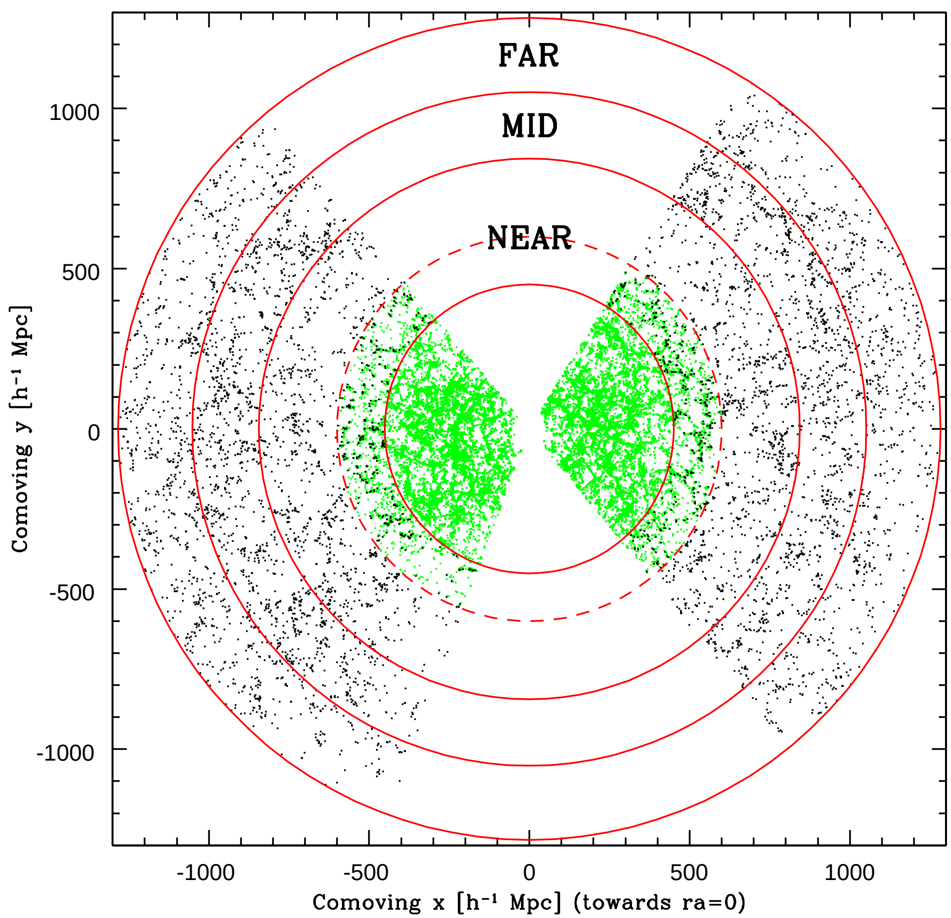
CMB Parameter Inferences

- Spectrum constrains the matter-energy contents of the universe

Parameter	[1] *Planck* TT+lowP	[2] *Planck* TE+lowP	[3] *Planck* EE+lowP	[4] *Planck* TT,TE,EE+lowP
$\Omega_b h^2$	0.02222 ± 0.00023	0.02228 ± 0.00025	0.0240 ± 0.0013	0.02225 ± 0.00016
$\Omega_c h^2$	0.1197 ± 0.0022	0.1187 ± 0.0021	$0.1150^{+0.0048}_{-0.0055}$	0.1198 ± 0.0015
$100\theta_{MC}$	1.04085 ± 0.00047	1.04094 ± 0.00051	1.03988 ± 0.00094	1.04077 ± 0.00032
τ	0.078 ± 0.019	0.053 ± 0.019	$0.059^{+0.022}_{-0.019}$	0.079 ± 0.017
$\ln(10^{10} A_s)$	3.089 ± 0.036	3.031 ± 0.041	$3.066^{+0.046}_{-0.041}$	3.094 ± 0.034
n_s	0.9655 ± 0.0062	0.965 ± 0.012	0.973 ± 0.016	0.9645 ± 0.0049
H_0	67.31 ± 0.96	67.73 ± 0.92	70.2 ± 3.0	67.27 ± 0.66
Ω_m	0.315 ± 0.013	0.300 ± 0.012	$0.286^{+0.027}_{-0.038}$	0.3156 ± 0.0091
σ_8	0.829 ± 0.014	0.802 ± 0.018	0.796 ± 0.024	0.831 ± 0.013
$10^9 A_s e^{-2\tau}$	1.880 ± 0.014	1.865 ± 0.019	1.907 ± 0.027	1.882 ± 0.012

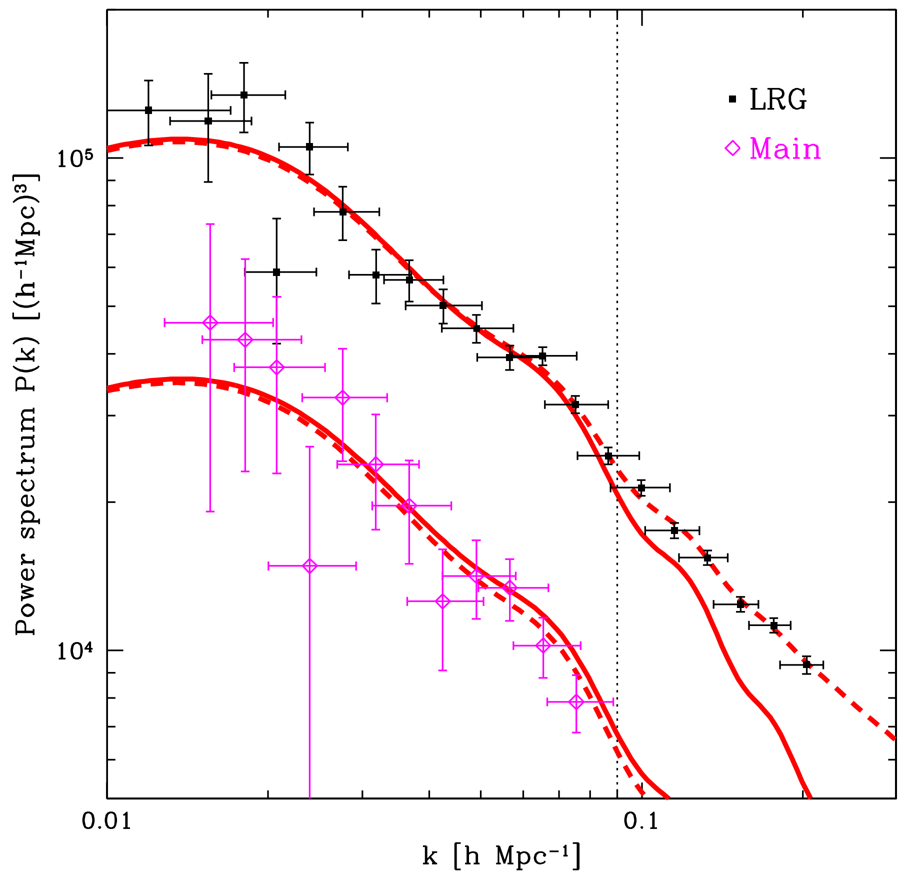
Galaxy Redshift Surveys

- Galaxy redshift surveys (e.g. 2dF and SDSS) measure the three dimensional distribution of galaxies today:



Galaxy Power Spectrum

- SDSS LRG and Main power spectrum:

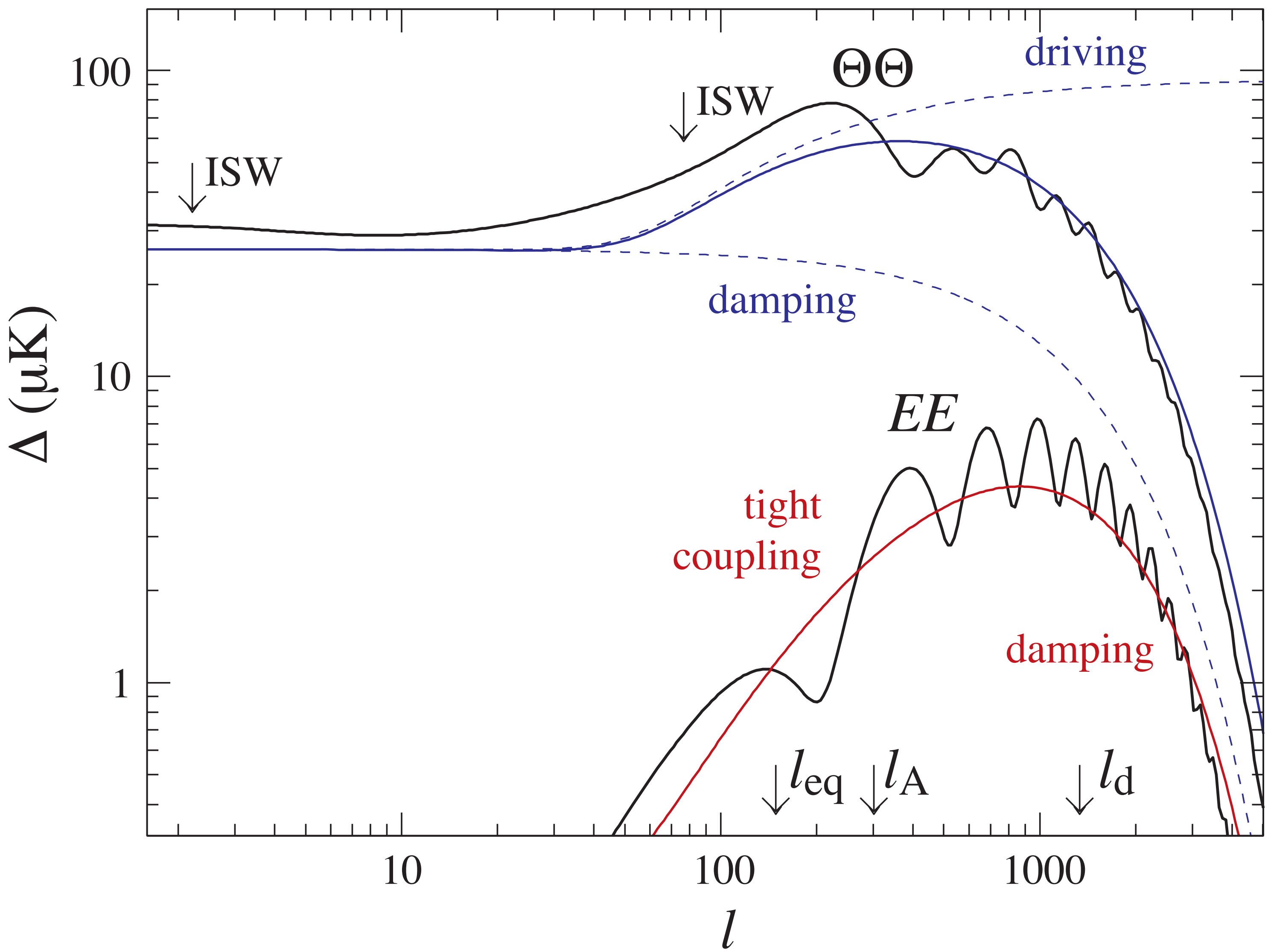


Structure Formation

- Small perturbations from inflation over the course of the 14Gyr life of the universe are gravitationally enhanced into all of the structure seen today
- Cosmic microwave background shows a snapshot at a few hundred thousand years old at recombination
- Discovery in 1992 of cosmic microwave background anisotropy provided the observational breakthrough - convincing support for adiabatic initial density fluctuations of amplitude 10^{-5}
- Combine with galaxy clustering - large scale structure seen in galaxy surveys - right amplitude given cold dark matter

Schematic CMB Spectrum

- Take apart features in the power spectrum



Fluid Approximation

- Thomson scattering of photons and free electrons before recombination is sufficiently rapid that the bayrons and photons are in equilibrium and hence move together
- Mean free path of the photons for $z \approx 10^3$ and $\Omega_b h^2 \approx 0.02$

$$\lambda_C \equiv \frac{1}{n_e \sigma_T a} \sim 2.5\mathrm{Mpc}$$

small by cosmological standards!

- On scales $\lambda \gg \lambda_C$ photons are tightly coupled to the electrons by Thomson scattering which in turn are tightly coupled to the baryons by Coulomb interactions
- Specifically, their bulk velocities are defined by a single fluid velocity $v_\gamma = v_b$ and the photons carry no anisotropy in the rest frame of the baryons

Zeroth Order Approximation

- Momentum density of a fluid is $(\rho + p)v$, where p is the pressure
- Neglect the momentum density of the baryons

$$R \equiv \frac{(\rho_b + p_b)v_b}{(\rho_\gamma + p_\gamma)v_\gamma} = \frac{\rho_b + p_b}{\rho_\gamma + p_\gamma} = \frac{3\rho_b}{4\rho_\gamma}$$
$$\approx 0.6 \left(\frac{\Omega_b h^2}{0.02}\right)\left(\frac{a}{10^{-3}}\right)$$

since $\rho_\gamma \propto T^4$ is fixed by the CMB temperature $T = 2.73(1+z)$K – OK substantially before recombination

- Neglect radiation in the expansion

$$\frac{\rho_m}{\rho_r} = 3.6 \left(\frac{\Omega_m h^2}{0.15}\right)\left(\frac{a}{10^{-3}}\right)$$

- Neglect gravity

Fluid Equations

- Density $\rho_\gamma \propto T^4$ so define temperature fluctuation Θ

$$\delta_\gamma = 4\frac{\delta T}{T} \equiv 4\Theta$$

- Real space continuity eqn.: the local number or energy density of photons changes if there is a divergence of the velocity field - a flow inwards or outwards from the volume

- Transformed to Fourier space $\nabla(e^{i\mathbf{k}\cdot\mathbf{x}}) \to i\mathbf{k}(e^{i\mathbf{k}\cdot\mathbf{x}})$ and $\nabla \cdot \mathbf{v} = -kv$

$$\dot{\delta}_\gamma = -(1 + w_\gamma)kv_\gamma$$

$$\dot{\Theta} = -\frac{1}{3}kv_\gamma$$

Fluid Equations

- Euler equation (neglecting gravity for now): momentum conservation says that pressure gradients generate changes in momentum density $k\delta p_\gamma = kc_s^2\delta\rho_\gamma$

$$\begin{aligned}\dot{v}_\gamma &= \frac{kc_s^2}{1+w_\gamma}\delta_\gamma \\ &= kc_s^2\frac{3}{4}\delta_\gamma = 3c_s^2 k\Theta\end{aligned}$$

  where the sound speed $c_s^2 = \delta p/\delta\rho$ is the pressure response to a density fluctuation

- So if you squeeze the photon gas to raise its density, its going to respond with a restoring force by raising the pressure and resisting compression $\rightarrow$ acoustic oscillations

Oscillator: Take One

- Combine these to form the simple harmonic oscillator equation

$$\ddot{\Theta} + c_s^2 k^2 \Theta = 0$$

where the sound speed is adiabatic

$$c_s^2 = \frac{\delta p}{\delta \rho} = \frac{\dot{p}_\gamma}{\dot{\rho}_\gamma}$$

here $c_s^2 = 1/3$ since we are photon-dominated

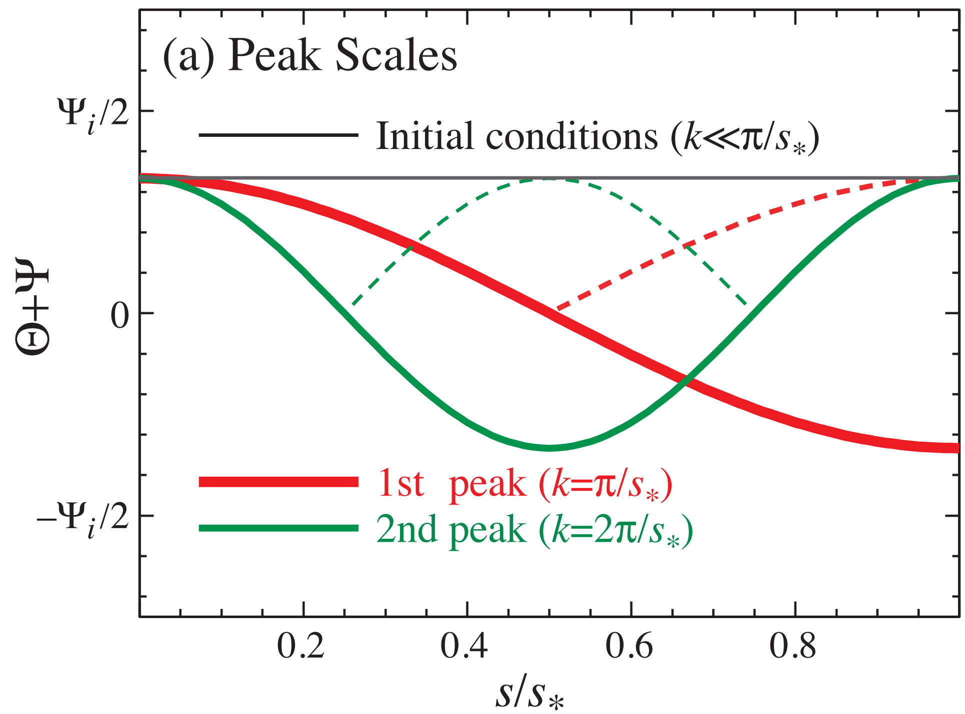
- General solution:

$$\Theta(\eta) = \Theta(0)\cos(ks) + \frac{\dot{\Theta}(0)}{kc_s}\sin(ks)$$

where the sound horizon is defined as $s \equiv \int c_s d\eta$

Harmonic Extrema

- All modes begin at end of inflation and are frozen in at recombination (denoted with a subscript $*$)
- Temperature perturbations of different amplitude for different modes.



- For the adiabatic (curvature mode) initial conditions

$$\dot{\Theta}(0) = 0$$

- So solution

$$\Theta(\eta_*) = \Theta(0)\cos(ks_*)$$

Harmonic Extrema

- Modes caught in the extrema of their oscillation will have enhanced fluctuations

$$k_n s_* = n\pi$$

yielding a fundamental scale or frequency, related to the inverse sound horizon

$$k_A = \pi / s_*$$

and a harmonic relationship to the other extrema as $1 : 2 : 3 \ldots$

Peak Location

- The fundmental physical scale is translated into a fundamental angular scale by simple projection according to the angular diameter distance D_A

$$\begin{aligned} \theta_A &= \lambda_A / D_A \\ \ell_A &= k_A D_A \end{aligned}$$

- In a flat universe, the distance is simply $D_A = D \equiv \eta_0 - \eta_* \approx \eta_0$, the horizon distance, and $k_A = \pi / s_* = \sqrt{3}\pi/\eta_*$ so

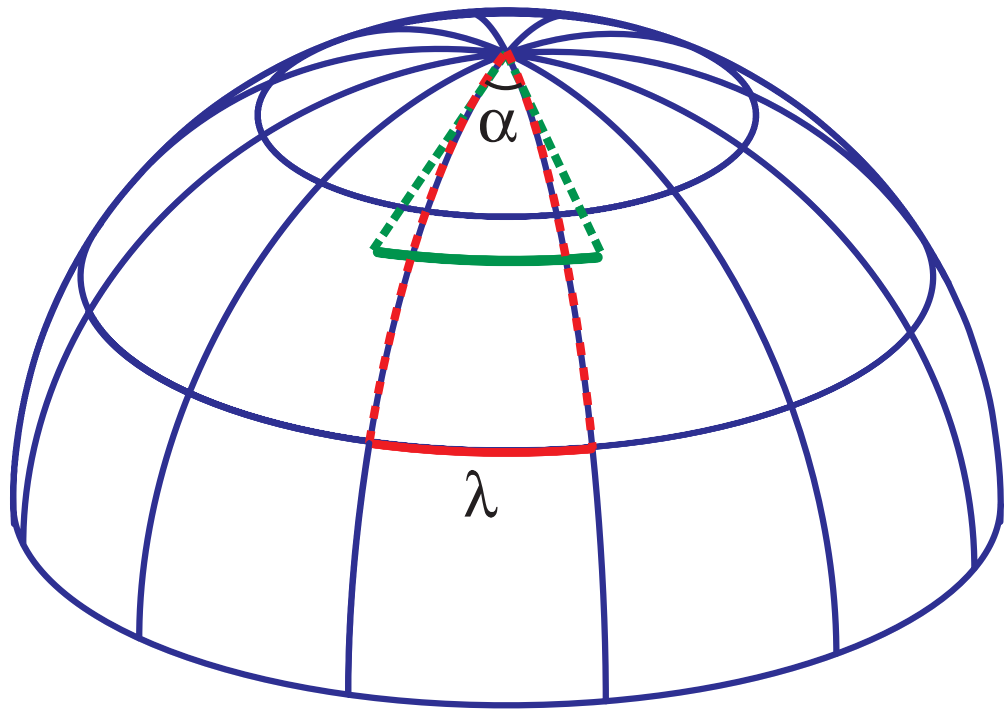
$$\theta_A \approx \frac{\eta_*}{\eta_0}$$

- In a matter-dominated universe $\eta \propto a^{1/2}$ so $\theta_A \approx 1/30 \approx 2^\circ$ or

$$\ell_A \approx 200$$

Curvature

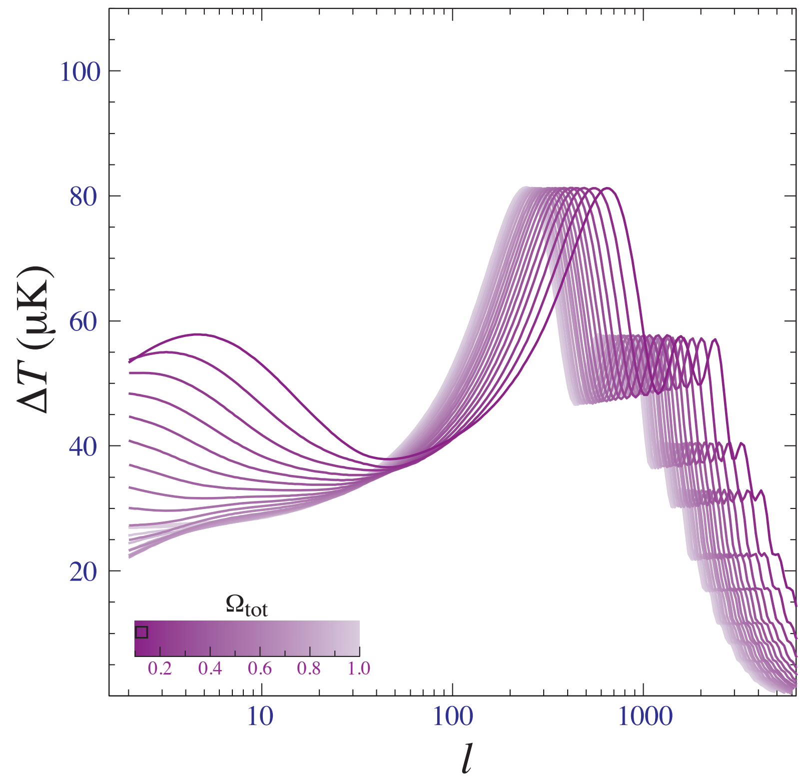
- In a curved universe, the apparent or angular diameter distance is no longer the conformal distance $D_A = R\sin(D/R) \neq D$



- Objects in a closed universe are further than they appear! gravitational lensing of the background...
- Curvature scale of the universe must be substantially larger than current horizon

Curvature

- Flat universe indicates critical density and implies missing energy given local measures of the matter density "dark energy"
- D also depends on dark energy density $\Omega_{\rm DE}$ and equation of state $w = p_{\rm DE}/\rho_{\rm DE}$.
- Expansion rate at recombination or matter-radiation ratio enters into calculation of k_A.



Restoring Gravity

- Take a simple photon dominated system with gravity
- Continuity altered since a gravitational potential represents a stretching of the spatial fabric that dilutes number densities – formally a spatial curvature perturbation
- Think of this as a perturbation to the scale factor $a \to a(1 + \Phi)$ so that the cosmogical redshift is generalized to

$$\frac{\dot{a}}{a} \to \frac{\dot{a}}{a} + \dot{\Phi}$$

so that the continuity equation becomes

$$\dot{\Theta} = -\frac{1}{3}kv_\gamma - \dot{\Phi}$$

Restoring Gravity

- Gravitational force in momentum conservation $\mathbf{F} = -m\nabla\Psi$ generalized to momentum density modifies the Euler equation to

$$\dot{v}_\gamma = k(\Theta + \Psi)$$

- General relativity says that Φ and Ψ are the relativistic analogues of the Newtonian potential and that $\Phi \approx -\Psi$.
- In our matter-dominated approximation, Φ represents matter density fluctuations through the cosmological Poisson equation

$$k^2\Phi = 4\pi G a^2 \rho_m \Delta_m$$

where the difference comes from the use of comoving coordinates for k (a^2 factor), the removal of the background density into the background expansion $(\rho\Delta_m)$ and finally a coordinate subtlety that enters into the definition of Δ_m

Constant Potentials

- In the matter dominated epoch potentials are constant because infall generates velocities as $v_m \sim k\eta\Psi$
- Velocity divergence generates density perturbations as $\Delta_m \sim -k\eta v_m \sim -(k\eta)^2\Psi$
- And density perturbations generate potential fluctuations

$$\Phi = \frac{4\pi G a^2 \rho \Delta}{k^2} \approx \frac{3}{2}\frac{H^2 a^2}{k^2}\Delta \sim \frac{\Delta}{(k\eta)^2} \sim -\Psi$$

keeping them constant. Note that because of the expansion, density perturbations must grow to keep potentials constant.

Constant Potentials

- More generally, if stress perturbations are negligible compared with density perturbations ($\delta p \ll \delta\rho$) then potential will remain roughly constant
- More specifically a variant called the Bardeen or comoving curvature is strictly constant

$$\mathcal{R} = \mathrm{const} \approx \frac{5+3w}{3+3w}\Phi$$

where the approximation holds when $w \approx$const.

Oscillator: Take Two

- Combine these to form the simple harmonic oscillator equation

$$\ddot{\Theta} + c_s^2 k^2 \Theta = -\frac{k^2}{3}\Psi - \ddot{\Phi}$$

- In a CDM dominated expansion $\dot{\Phi} = \dot{\Psi} = 0$. Also for photon domination $c_s^2 = 1/3$ so the oscillator equation becomes

$$\ddot{\Theta} + \ddot{\Psi} + c_s^2 k^2 (\Theta + \Psi) = 0$$

- Solution is just an offset version of the original

$$[\Theta + \Psi](\eta) = [\Theta + \Psi](0)\, \cos(ks)$$

- $\Theta + \Psi$ is also the observed temperature fluctuation since photons lose energy climbing out of gravitational potentials at recombination

Effective Temperature

- Photons climb out of potential wells at last scattering
- Lose energy to gravitational redshifts
- Observed or effective temperature

$$\Theta + \Psi$$

- Effective temperature oscillates around zero with amplitude given by the initial conditions
- Note: initial conditions are set when the perturbation is outside of horizon, need inflation or other modification to matter-radiation FRW universe.
- GR says that initial temperature is given by initial potential

Sachs-Wolfe Effect and the Magic 1/3

- A gravitational potential is a perturbation to the temporal coordinate [formally a gauge transformation]

$$\frac{\delta t}{t} = \Psi$$

- Convert this to a perturbation in the scale factor, in a matter dominated expansion $a \propto t^{2/3}$ so

$$\frac{\delta a}{a} = \frac{2}{3}\frac{\delta t}{t}$$

- CMB temperature is cooling as $T \propto a^{-1}$ so

$$\Theta + \Psi \equiv \frac{\delta T}{T} + \Psi = -\frac{\delta a}{a} + \Psi = \frac{1}{3}\Psi$$

Sachs-Wolfe Normalization

- Use measurements of $\Delta T/T \approx 10^{-5}$ in the Sachs-Wolfe effect to infer $\Delta^2_{\mathcal{R}}$
- Recall in matter domination $\Psi = -3\mathcal{R}/5$ and so $\Delta T/T = -\mathcal{R}/5$
- So that the amplitude of initial curvature fluctuations is $\Delta_R \approx 5 \times 10^{-5}$
- This then determines the amplitude of the inflationary power spectrum $A_S = \Delta^2_{\mathcal{R}}$ in the previous lecture set

Baryon Loading

- Baryons add extra mass to the photon-baryon fluid
- Controlling parameter is the momentum density ratio:

$$R \equiv \frac{p_b + \rho_b}{p_\gamma + \rho_\gamma} \approx 30\Omega_b h^2 \left(\frac{a}{10^{-3}}\right)$$

of order unity at recombination

- Momentum density of the joint system is conserved

$$\begin{aligned}(\rho_\gamma + p_\gamma)v_\gamma + (\rho_b + p_b)v_b &\approx (p_\gamma + p_\gamma + \rho_b + \rho_\gamma)v_\gamma \\ &= (1+R)(\rho_\gamma + p_\gamma)v_{\gamma b}\end{aligned}$$

- Momentum density ratio enters as

$$[(1+R)v_{\gamma b}]^{\cdot} = k\Theta + (1+R)k\Psi$$

New Euler Equation

- Photon continuity remains the same

$$\dot{\Theta} = -\frac{k}{3} v_{\gamma b} - \dot{\Phi}$$

- Modification of oscillator equation

$$\frac{d}{d\eta}[(1+R)\dot{\Theta}] + \frac{1}{3}k^2\Theta = -\frac{1}{3}k^2(1+R)\Psi - \frac{d}{d\eta}[(1+R)\dot{\Phi}]$$

- In a CDM dominated expansion $\dot{\Phi} = \dot{\Psi} = 0$ and the adiabatic approximation where the sound speed evolves slowly

$$c_s = \sqrt{\frac{1}{3}\frac{1}{1+R}}$$

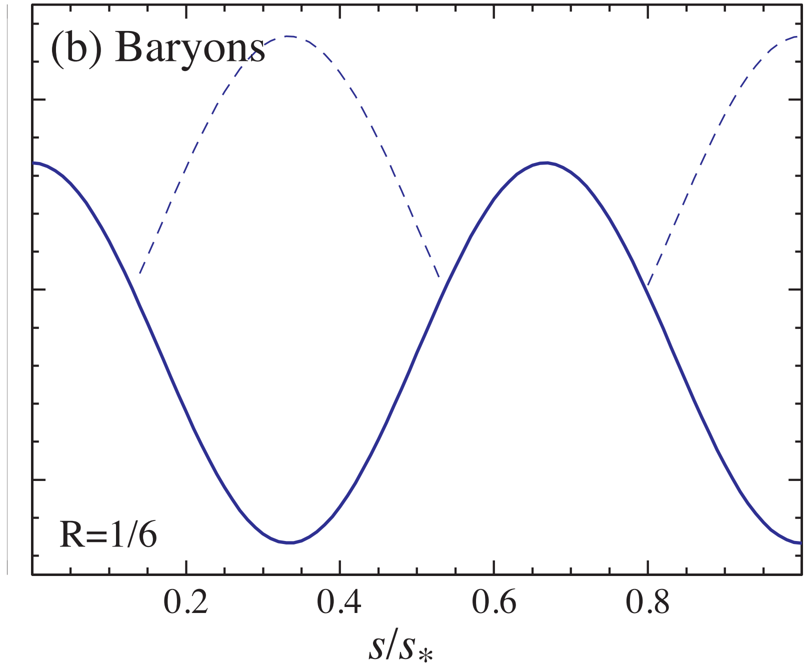
$$[\Theta + (1+R)\Psi](\eta) = [\Theta + (1+R)\Psi](0)\cos(ks)$$

Baryon Peak Phenomenology

- Photon-baryon ratio enters in three ways
- Overall larger amplitude:

$$[\Theta + (1+R)\Psi](0) = \frac{1}{3}(1+3R)\Psi(0)$$

- Even-odd peak modulation of effective temperature



$$[\Theta + \Psi]_{\rm peaks} = [\pm(1+3R) - 3R]\,\frac{1}{3}\Psi(0)$$

$$[\Theta + \Psi]_1 - [\Theta + \Psi]_2 = [-6R]\frac{1}{3}\Psi(0)$$

- Shifting of the sound horizon down or ℓ_A up

$$\ell_A \propto \sqrt{1+R}$$

Photon Baryon Ratio Evolution

- Actual effects smaller since R evolves
- Oscillator equation has time evolving mass

$$c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Theta}) + c_s^2 k^2 \Theta = 0$$

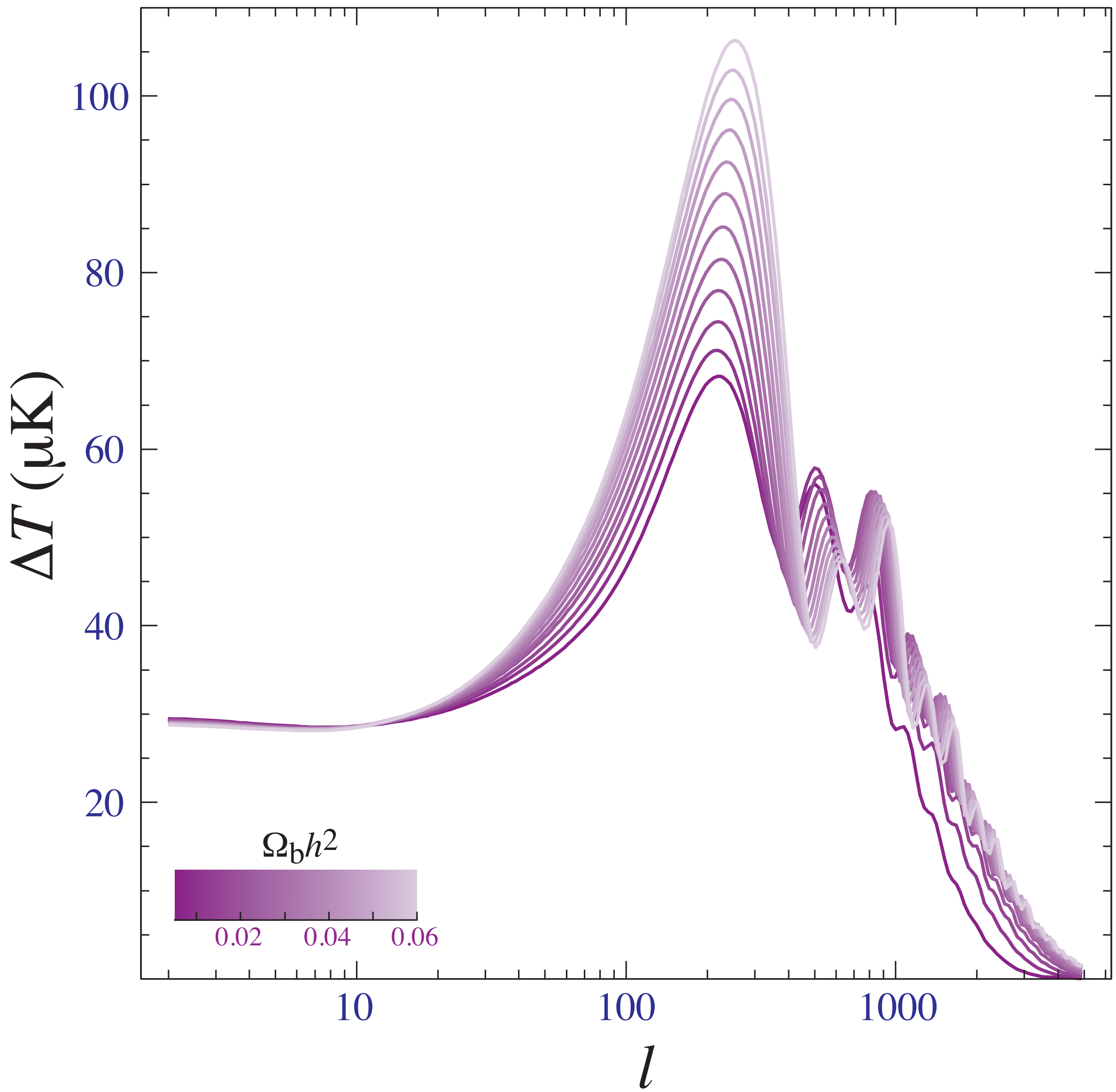
- Effective mass is is $m_{\rm eff} = 3c_s^{-2} = (1+R)$
- Adiabatic invariant

$$\frac{E}{\omega} = \frac{1}{2} m_{\rm eff} \omega A^2 = \frac{1}{2} 3 c_s^{-2} k c_s A^2 \propto A^2 (1+R)^{1/2} = const.$$

- Amplitude of oscillation $A \propto (1+R)^{-1/4}$ decays adiabatically as the photon-baryon ratio changes

Baryons in the Power Spectrum

- Relative heights of peaks



Oscillator: Take Three and a Half

- The not-quite-so simple harmonic oscillator equation is a forced harmonic oscillator

$$c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Theta}) + c_s^2 k^2 \Theta = -\frac{k^2}{3}\Psi - c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Phi})$$

  changes in the gravitational potentials alter the form of the acoustic oscillations

- If the forcing term has a temporal structure that is related to the frequency of the oscillation, this becomes a driven harmonic oscillator
- Term involving Ψ is the ordinary gravitational force
- Term involving Φ involves the $\dot{\Phi}$ term in the continuity equation as a (curvature) perturbation to the scale factor

Potential Decay

- Matter-to-radiation ratio

$$\frac{\rho_m}{\rho_r} \approx 24\Omega_m h^2 \left(\frac{a}{10^{-3}}\right)$$

of order unity at recombination in a low Ω_m universe

- Radiation is not stress free and so impedes the growth of structure

$$k^2\Phi = 4\pi G a^2 \rho_r \Delta_r$$

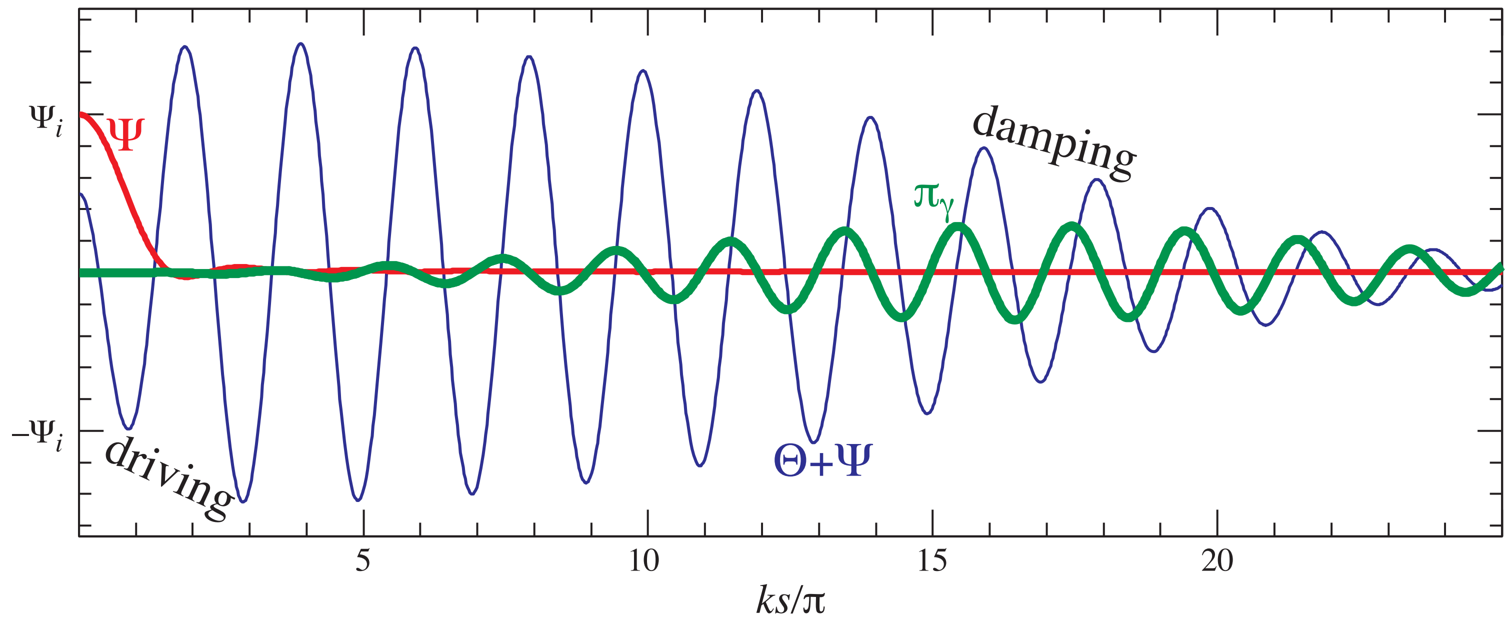
$\Delta_r \sim 4\Theta$ oscillates around a constant value, $\rho_r \propto a^{-4}$ so the Netwonian curvature decays.

- General rule: potential decays if the dominant energy component has substantial stress fluctuations, i.e. below the generalized sound horizon or Jeans scale

Radiation Driving

- Decay is timed precisely to drive the oscillator - close to fully coherent

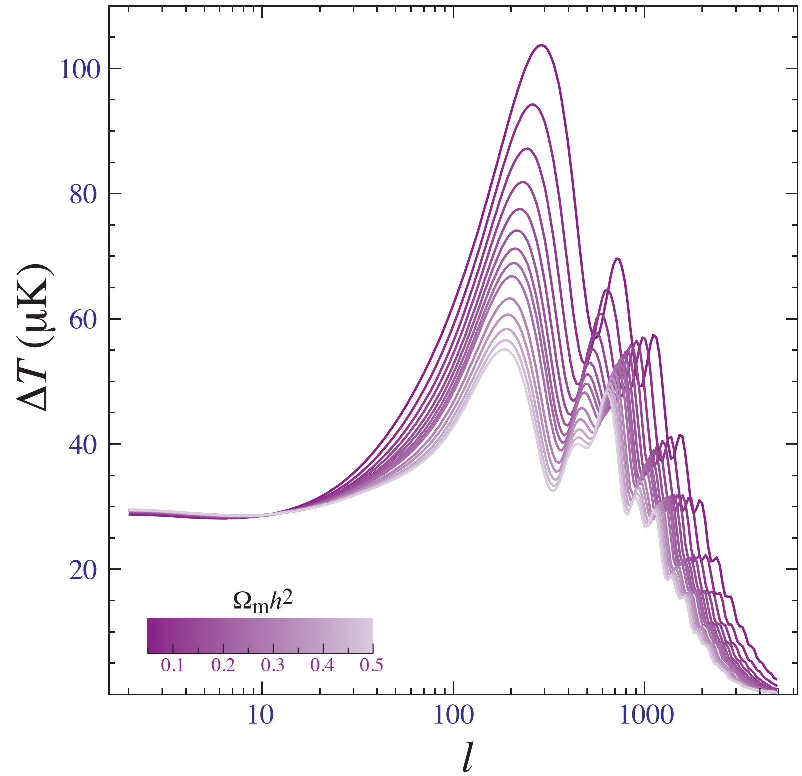
$$
\begin{aligned}
|[\Theta + \Psi](\eta)| &= |[\Theta + \Psi](0) + \Delta\Psi - \Delta\Phi| \\
&= |\frac{1}{3}\Psi(0) - 2\Psi(0)| = |\frac{5}{3}\Psi(0)|
\end{aligned}
$$



- $5\times$ the amplitude of the Sachs-Wolfe effect!

Matter-Radiation in the Power Spectrum

- Coherent approximation is exact for a photon-baryon fluid but reality is reduced to $\sim 4\times$ because neutrino contribution is free streaming not fluid like
- Neutrinos drive the oscillator less efficiently and also slightly change the phase of the oscillation



- Actual initial conditions are $\Theta + \Psi = \Psi/2$ for radiation domination but comparison to matter dominated SW correct
- With 3 peaks, it is possible to solve for both the baryons and dark matter densities, providing a calibration for the sound horizon
- Higher peaks check consistency with assumptions: e.g. extra relativistic d.o.f.s

Damping

- Tight coupling equations assume a perfect fluid: no viscosity, no heat conduction
- Fluid imperfections are related to the mean free path of the photons in the baryons

$$\lambda_C = \dot{\tau}^{-1} \quad \text{where} \quad \dot{\tau} = n_e \sigma_T a$$

  is the conformal opacity to Thomson scattering
- Dissipation is related to the diffusion length: random walk approximation

$$\lambda_D = \sqrt{N}\lambda_C = \sqrt{\eta/\lambda_C}\,\lambda_C = \sqrt{\eta\lambda_C}$$

  the geometric mean between the horizon and mean free path
- $\lambda_D/\eta_* \sim$ few %, so expect the peaks :> 3 to be affected by dissipation

Transfer Function

- Transfer function transfers the initial Newtonian curvature to its value today (linear response theory)

$$T(k) = \frac{\Phi(k, a=1)}{\Phi(k, a_{\rm init})} \frac{\Phi(k_{\rm norm}, a_{\rm init})}{\Phi(k_{\rm norm}, a=1)}$$

- Conservation of Bardeen curvature: Newtonian curvature is a constant when stress perturbations are negligible: above the horizon during radiation and dark energy domination, on all scales during matter domination
- When stress fluctuations dominate, perturbations are stabilized by the Jeans mechanism
- Hybrid Poisson equation: Newtonian curvature, comoving density perturbation $\Delta \equiv (\delta\rho/\rho)_{\rm com}$ implies Φ decays

$$(k^2 - 3K)\Phi = 4\pi G a^2 \rho \Delta \sim \eta^{-2} \Delta$$

Transfer Function

- Freezing of Δ stops at $\eta_{\rm eq}$

$$\Phi \sim (k\eta_{\rm eq})^{-2}\Delta_H \sim (k\eta_{\rm eq})^{-2}\Phi_{\rm init}$$

- Transfer function has a k^{-2} fall-off beyond $k_{\rm eq} \sim \eta_{\rm eq}^{-1}$

$$\eta_{\rm eq} = 15.7(\Omega_m h^2)^{-1}\left(\frac{T}{2.7K}\right)^2 \mathrm{Mpc}$$

- Small correction since growth with a smooth radiation component is logarithmic not frozen
- Transfer function is a direct output of an Einstein-Boltzmann code

Fitting Function

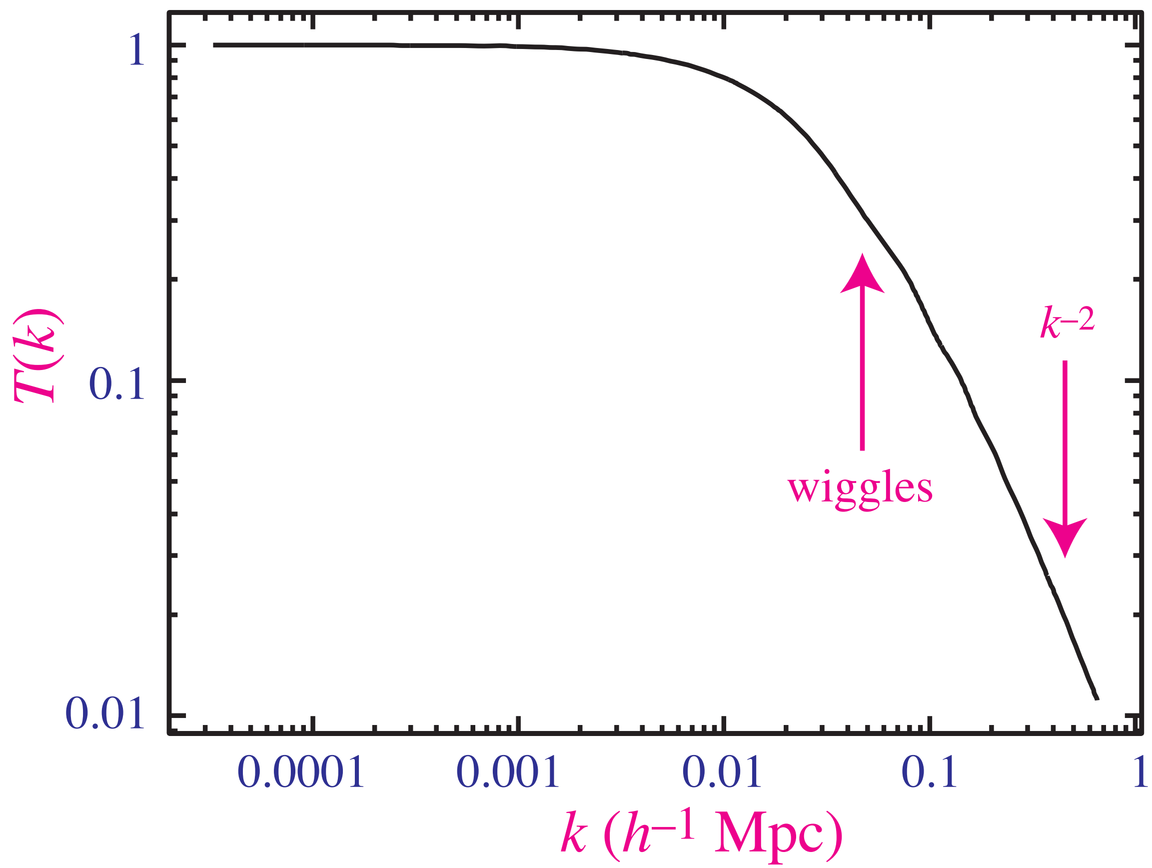
- Alternately accurate fitting formula exist, e.g. pure CDM form:

$$T(k(q)) = \frac{L(q)}{L(q) + C(q)q^2}$$
$$L(q) = \ln(e + 1.84q)$$
$$C(q) = 14.4 + \frac{325}{1 + 60.5q^{1.11}}$$
$$q = k/\Omega_m h^2 \mathrm{Mpc}^{-1} (T_{\mathrm{CMB}}/2.7K)^2$$

- In h Mpc^{-1}, the critical scale depends on $\Gamma \equiv \Omega_m h$ also known as the shape parameter

Transfer Function

- Numerical calculation

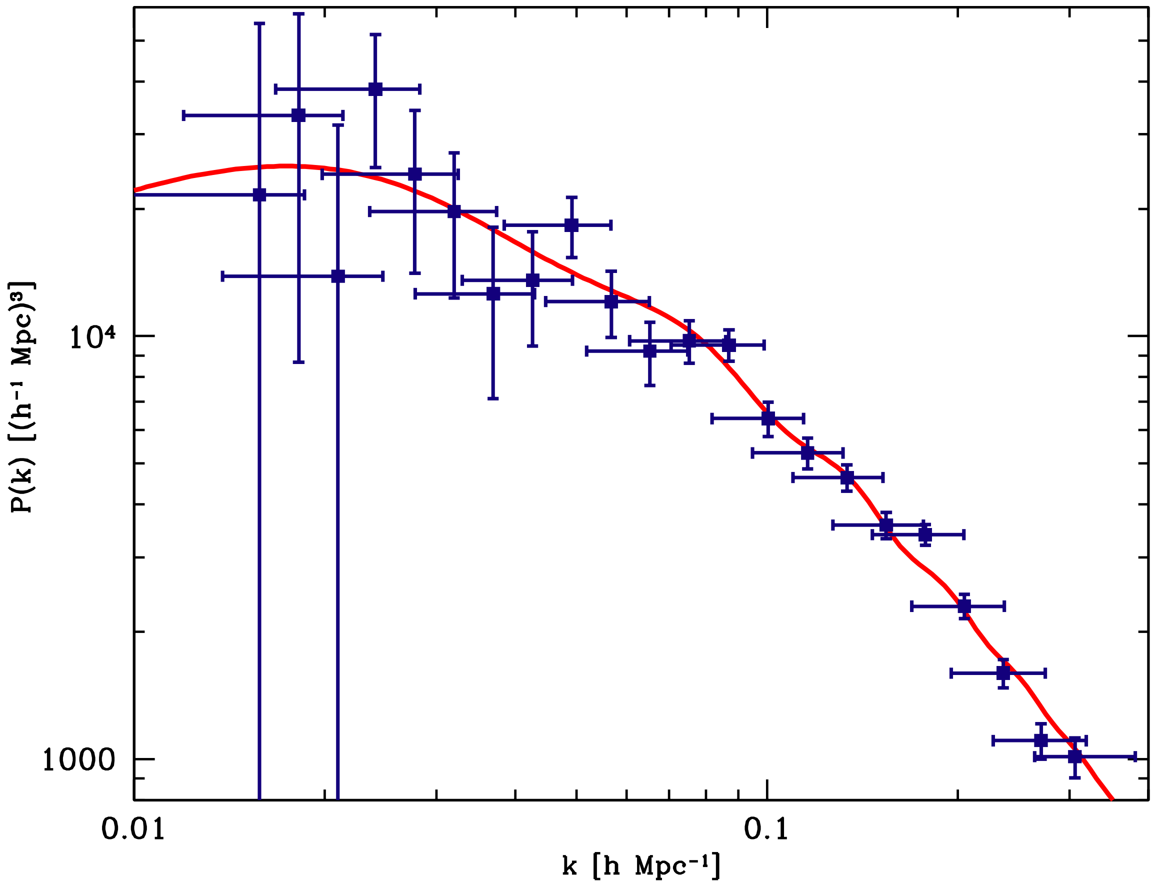


Baryon Acoustic Oscillations

- Baryons caught up in the acoustic oscillations of the CMB and impart acoustic oscillations to the transfer function. Density enhancements are produced kinematically through the continuity equation $\delta_b \sim (k\eta) v_b$ and hence are out of phase with CMB temperature peaks
- Dissipation of the acoustic oscillations eliminates both the CMB and baryon perturbations – known as Silk damping for the baryons. This suppression and the general fact that baryons are caught up with photons was one of the main arguments for CDM
- Detected first in the SDSS LRG survey.
- An excellent standard ruler for angular diameter distance $D_A(z)$ since it does not evolve in redshift in linear theory
- Radial extent of BAO gives $H(z)$

Power Spectrum

- SDSS data



- Power spectrum defines large scale structure observables: galaxy clustering, velocity field, Lyα forest clustering, cosmic shear