Astro 243: Problem Set 1

1 Problem 1: Natural Units and Cosmological Scales

Convert the following quantities by inserting the appropriate factors of c, $\hbar$, k_B and unit conversions. The easiest way to think about this is from the natural unit standpoint where $c = \hbar = k_B = 1$ so that time, space, (inverse) mass, (inverse) energy, (inverse) temperature all have the same dimensions. For example $c = 2.9979 \times 10^5\,\mathrm{km\,s^{-1}} = 1$ means $2.9979 \times 10^5\,\mathrm{km} = 1\mathrm{s}$, etc. so we just choose to measure space in seconds or time in km and convert hybrid units appropriately. In this sense the conversions below are just like changing from cm to km.

Note that h in the formulae below is the reduced Hubble parameter not the Planck constant and should just be carried through in your conversion formulae (e.g. numbers in Mpc will carry an h^{-1} factor and become units of h^{-1} Mpc denoting distances that were really converted from redshift measurements using the expansion rate).

- Expansion rate $H_0 = 100h$km s^{-1} Mpc^{-1} into (a) eV, (b) Mpc^{-1}, (c) Gyr^{-1}. [We will see later that these corresponds to upper limit on the mass of a dark energy particle, the inverse Hubble length, inverse approximate age of Universe.]

- Critical density $\rho_{\rm crit} = 3H_0^2/8\pi G$ into (a) g cm^{-3}, (b) GeV^4, (c) eV cm^{-3}, (d) proton (mass) cm^{-3}, (e) $M_\odot$ Mpc^{-3}. If a (cosmological) constant energy density has $\rho_\Lambda = 2\rho_{\rm crit}/3$, what is its energy scale in eV (i.e. $\rho_\Lambda^{1/4}$). Compare that to the Planck mass; that these numbers are so different is the cosmological constant problem.

- The properties of the CMB with temperature $T_{\rm CMB} = 2.7255$K can be thought of as a special type of unit conversion that assumes photons are distributed as a blackbody, which means its phase space is occupied by on average around one photon at peak (see Ryden §2.4 if you have not had stat mech): number and energy densities
$$n_\gamma = \frac{2\zeta(3)}{\pi^2}T^3, \qquad \rho_\gamma = \frac{\pi^2}{15}T^4 \tag{1}$$
where $\zeta(3) \approx 1.202$. Use the conversion factors to bring these into (a) photons cm^{-3} (b) eV cm^{-3} (c) average photon energy in eV (d) $\Omega_\gamma = \rho_\gamma/\rho_{\rm crit}$.

2 Problem 2: Ryden 2.1, 2.2

For the purposes of this problem, assume you are spherical with a mass of 70kg and density of 1000 kg m^{-3} for definiteness.

1) Assume that you emit as a perfect blackbody at a temperature of $T = 310$K. What is the rate, in watts, at which you radiate energy?

2) Since you are made mostly of water, you are very efficient at absorbing microwave photons. What is the rate (in watts) at which you absorb radiative energy from the CMB? (Hint: what rate would you be emitting energy if you were in equilibrium with the CMB). Estimate the number of photons you absorb per second.