Set 1:
Expansion of the Universe
Course text book: Ryden, Introduction to Cosmology, 2nd edition

- Olber’s paradox, expansion of the universe: Ch 2
- Cosmic geometry, expansion rate, acceleration: Ch 3, 6
- Cosmic dynamics and composition: Ch 4, 5
- Dark matter and dark energy: Ch 5, 7

Midterm (in class)

- Hot big bang and origin of species: Ch 9
- Inflation: Ch 10
- Cosmic microwave background: Ch 8
- Gravitational instability and structure formation: Ch 11, 12

Final
Observables

- Most cosmological inferences are based on interpreting the radiation we receive on Earth from astrophysical objects, stars, galaxies, clusters of galaxies, cosmic microwave background - mostly electromagnetic radiation but now also neutrinos and most recently, gravitational waves.

- At the most basic level we can make a 2D map of the sky as a function of radiation frequency, polarization and time.

- For light coming from a single object, e.g. a star or galaxy, we can quantify the flux or energy per unit time per unit area of a detector.
Starlight: Optical Image

- Color overlay: microwave background
• Energy
Flux given a luminosity of the source $L = \frac{dE}{dt}$ (energy/time)

$$F = \frac{dE}{dtdA}$$

• Astro units: often
in cgs erg s$^{-1}$ cm$^{-2}$ (mks W m$^{-2}$)

• Energy conservation says flux decreases as $1/r^2$ from the source

$$F(r_1)4\pi r_1^2 = F(r_2)4\pi r_2^2$$

$$F \propto r^{-2}$$
From 2D to 3D

- So even without knowing the luminosity of a set of standard objects, we can judge relative distance from flux:
  \[ \frac{r_2}{r_1} = \left( \frac{F_1}{F_2} \right)^{1/2} \]

- This is the idea of a standard candle, star of the same type, supernovae etc.

- 2D map becomes a 3D map and we can start to talk about the physical structure of the universe.

- Aside: certain variable stars called Cepheids are good standard candles, the pulsation period is linked to the luminosity so we can pick out objects of the same luminosity and use the measured flux to put place their relative position on a map.

- We can calibrate the absolute distance to the nearby ones by other methods, ultimately parallax - change in angle on the sky as earth orbits the sun at 1AU indicating distance as \( d\alpha = \pm (1\text{AU}/d) \).
Cosmological Units

- Astro and cosmo units often look bizarre at first sight - however they are useful in that they tell you something about the observation behind the inference

- Cosmologists favorite unit is the megaparsec

\[1 \text{Mpc} = 3.0856 \times 10^{24}\text{cm}\]

because it is the typical separation between galaxies

- 1 AU subtends 1 arcsec at 1 pc – parallax to close objects allows us to convert relative distance to absolute distance

- 1 pc: nearest stars, 1 kpc distances in the galaxy, 1 Mpc distance between galaxies, 1 Gpc distances across observable universe

- Because of the finite light travel time, light from distant objects is emitted at earlier times so sky maps become snapshots of the history of the universe
Observables: Surface Brightness

- If the 2D map resolves the object in question we can do better and measure the surface brightness.

- Direction: colunate (e.g. pinhole) in an acceptance angle $d\Omega$ normal to $dA \rightarrow$ surface brightness

$$S(\Omega) = \frac{dE}{dt dA d\Omega}$$

- Units: for example in cgs, erg s$^{-1}$ cm$^{-2}$ sr$^{-1}$ or mks, W m$^{-2}$ sr$^{-1}$
Olber’s Paradox

- Surface brightness $\Delta \Omega = (\lambda/d)^2$ for a region with fixed size $\lambda$

$$S = \frac{F}{\Delta \Omega} = \frac{L}{4\pi d^2} \frac{d^2}{\lambda^2}$$

- In a non-expanding geometry, these two distances cancel

$$S = \text{const.}$$

- Aside: because of this conservation, astro/cosmo units for galactic scale objects are also often quoted in $L_\odot/\text{pc}^2$

- So since each site line in universe full of stars will eventually end on surface of star, night sky should be as bright as sun (not infinite)

- We shall see that the resolution lies in the expansion of the universe and that the two distance factors don’t cancel
Observables: Redshift

• We can also measure the frequency of radiation from objects

• If emission contains atomic lines with a natural rest frequency $\nu_{\text{rest}}$ we can measure the redshift or velocity by the ratio of observed to rest frequency

$$1 + z = \frac{\nu_{\text{rest}}}{\nu_{\text{obs}}} = \frac{1 + v/c}{\sqrt{1 - v^2/c^2}}$$

and for $v \ll c$, $z = v/c$

• Here $v$ is the recession velocity - i.e. light is shifted to the red if the object is receding from us

• In this class we’ll often use units where $c = \hbar = 1$ which means time and length are energy are measured in the same units

• If units don’t make sense add $c$, $\hbar$ until they do (see problem set)!
Expansion of the Universe

• Now let’s put together these observational tools

• Given a standard candle with known luminosity $L$, we measure its distance $d$ away from us from the measured flux $F = \frac{L}{4\pi d^2}$

• Given atomic line transitions, we measure the redshift $z$ or equivalently the recession velocity $v$

• Hubble then plotted out recession velocity as a function of distance....
Hubble Law

- Hubble in 1929 used the Cepheid period luminosity relation to infer distances to nearby galaxies thereby discovering the expansion of the universe.

- Hubble actually inferred too large a Hubble constant of $H_0 \sim 500 \text{km/s/Mpc}$ due to a miscalibration of the Cepheid distance scale.

- $H_0$ now measured as $74.03 \pm 1.42 \text{km/s/Mpc}$ by combining a suite of distance measurements [arXiv:1903.07603]
Fundamental Properties

- Hubble law: objects are receding from us at a velocity that is proportional to their distance
- Universe is highly isotropic at sufficiently large distances
- Universe is homogeneous on large scales
- Let’s see why the first property, along with the implications of the second two that we are not in a special position, imply the universe is expanding

- The Hubble law sounds much like we are at the center of an explosion outwards but that would violate homogeneity and put us in a special place
- To be consistent with both, we posit space itself is expanding...
Expansion of the Universe

- Consider a 1 dimensional expansion traced out by galaxies

\[ \Delta x = 2a \quad a \quad \Delta x/\Delta t = H_0d \]

- From the perspective of the central galaxy the others are receding with a velocity proportional to distance

- Proportionality constant is called the *Hubble Constant* \( H_0 \)

- Each observer in the expansion will see the same relative recession of galaxies
Expansion of the Universe

- Generalizes to a three dimensional expansion. Consider the observer at the origin and two galaxies at position $d_A$ and $d_B$
- Recession velocities according to the observer
  \[ v_A = H_0 d_A, \quad v_B = H_0 d_B \]
- According to galaxy $B$, the recession velocity of galaxy $A$ is
  \[ v_B - v_A = H_0 d_B - H_0 d_A = H_0 d_{AB} \]
  so that $B$ will see the same expansion rate as the observer at the origin given the linearity of Hubble’s law
- Hubble’s law is best thought of as an expansion of space itself, with galaxies carried along the “Hubble flow”
Olber’s Paradox Redux

- In an expanding universe Olber’s paradox is resolved
- First piece: age finite so even if stars exist in the early universe, not all site lines end on stars
- But even as age goes to infinity and the number of site lines goes to 100%, surface brightness of distant objects (of fixed physical size) goes to zero
  - Angular size increases
  - Redshift of energy and arrival time

we’ll see in the next set of lectures

\[ S \propto (1 + z)^{-4} \]
Supernovae as Standard Candles

- Type 1A supernovae are white dwarfs that reach Chandrashekar mass where electron degeneracy pressure can no longer support the star, hence a very regular explosion.

- Moreover, the scatter in luminosity (absolute magnitude) is correlated with the shape of the light curve - the rate of decline from peak light, empirical “Phillips relation”.

- Higher $^{56}\text{Ni}$, brighter SN, higher opacity, longer light curve duration.
Beyond Hubble’s Law

- Type 1A are therefore “standardizable” candles leading to a very low scatter $\delta m \sim 0.15$ and visible out to high redshift $z \sim 1$
- Two groups in 1999 found that SN more distant at a given redshift than expected
- Cosmic acceleration discovery won the 2011 Nobel Prize in Physics
- Requires more on cosmic geometry to understand...