Set 5:

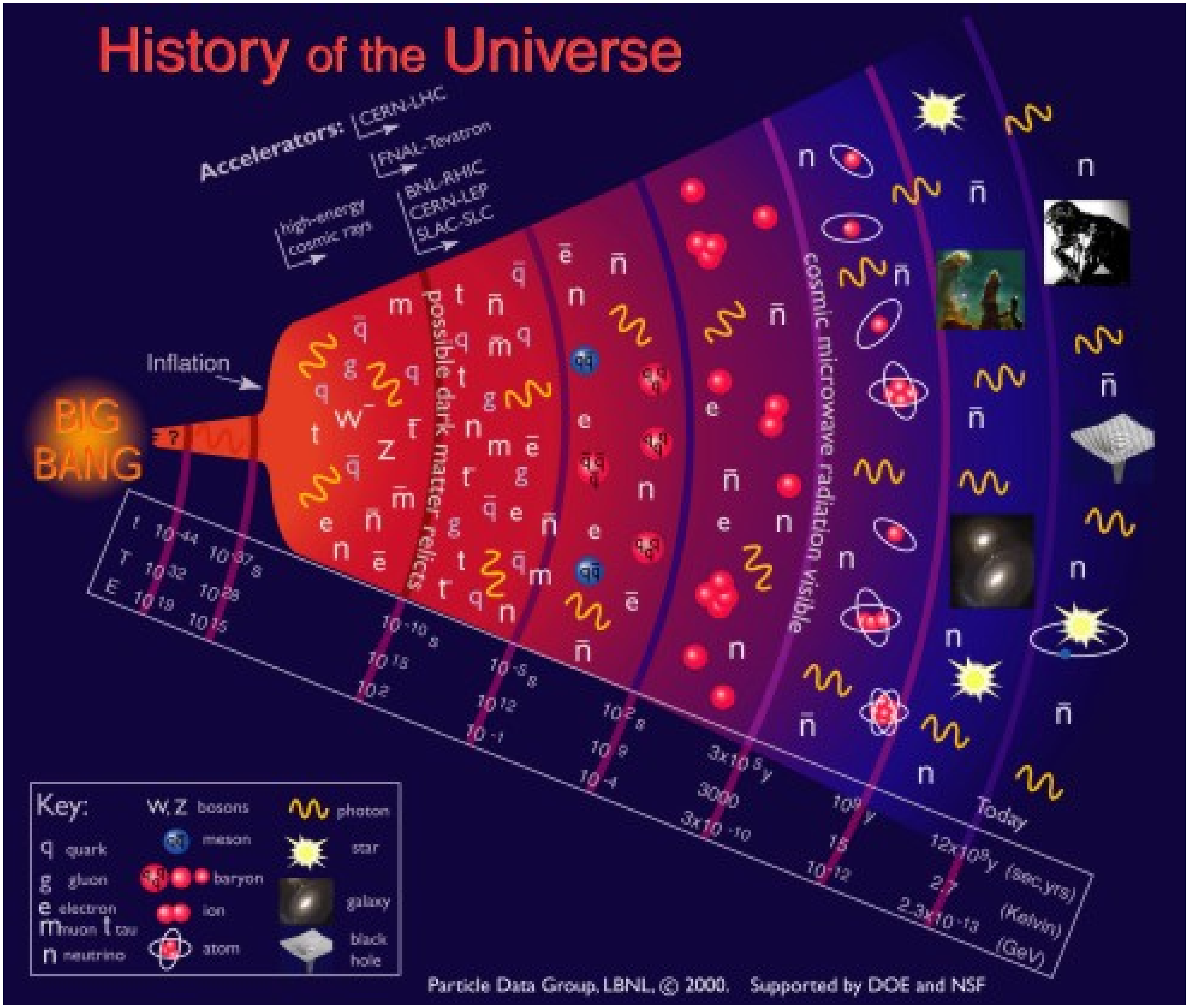
Hot Big Bang and Origin of Species

Hot Big Bang

- CMB blackbody at 2.7K and the redshifting of the energy and hence temperature, implies that the Universe began in a hot dense state
- Rapid interactions in the hot dense plasma kept particle species in equilibrium
- When reaction rates become too low compared with the expansion rate, various particle species freeze out
- "Origin of species" as relics of the hot big bang

- For those of you who haven't had statistical physics and want a crash course or those of you who have but want a review, see supplemental notes

Brief Thermal History



Origin Examples

- Neutrino background (weak freezeout)
- CDM freezeout (annihilation freezout)
- Light elements (nuclear statistical equilibrium freezeout)
- Baryogenesis (freezeout of baryon number changing processes)
- Blackbody freezeout (thermalization)
- Atomic hydrogen (recombination; free electron freezout)
- Next lecture set: origin of structure from inflation - freezeout of quantum fluctuations

Particle Distributions

- Phase space distribution of particles f gives the occupancy of quantum states of a given allowed momentum $\mathbf{q}$ and position $\mathbf{x}$
- Number density is the integral over momentum states

$$n = \int \frac{g}{(2\pi)^3} f d^3 q$$

where g is quantum degeneracy of state (e.g. spin)

- Energy density is the integral weighted by energy

$$\rho = \int \frac{g}{(2\pi)^3} E(q) f d^3 q, \qquad E(q) = (q^2 + m^2)^{1/2}$$

- Pressure from change in momentum reflection of a wall

$$p(\mathbf{x}, t) = g \int \frac{d^3 q}{(2\pi)^3} \frac{|q|^2}{3E(q)} f$$

Vacuum Energy

- Aside for advanced students: we've excluded the energy density associated with the state of no particles or "vacuum"
- In QFT, like the simple harmonic oscillator in ordinary quantum mechanics, there is a zero point energy to the ground state
- For bosons, $\hbar\omega/2 = E(q)/2$, so the most naive version of the cosmological constant problem is that $\rho \propto M^4$ where $M = M_{\rm Pl} = 1/\sqrt{8\pi G}$ if the theory applies out to the Planck scale
- The critical energy density $\rho_c = 3H_0^2/8\pi G \approx 8 \times 10^{-47} h^2 \text{GeV}^4$ is more than 10^{120} off $M_{\rm Pl}^4 \approx 2 \times 10^{76}$ GeV^4.
- This is the cosmological constant problem in its basic form - a more sophisticated QFT version is even given renormalization we expect $\rho_{\rm vac} \sim m^4$ for each particle of mass m

Freezeout Rule of Thumb

- Non expanding medium - given Γ, rate of thermalizing interactions

$$\frac{\partial f}{\partial t} = \Gamma\,(f - f_{\rm eq})$$

- Add in expansion in a homogeneous medium - de Broglie wavelength $\lambda \propto q^{-1} \propto a$ stretches with the expansion

$$\frac{\partial f}{\partial t} + \frac{dq}{dt}\frac{\partial f}{\partial q} = \Gamma\,(f - f_{\rm eq})$$

$$(q \propto a^{-1} \rightarrow \frac{1}{q}\frac{dq}{dt} = -\frac{1}{a}\frac{da}{dt} = H)$$

$$\frac{\partial f}{\partial t} - H\frac{\partial f}{\partial \ln q} = \Gamma\,(f - f_{\rm eq})$$

- So equilibrium will be maintained if collision rate exceeds expansion rate $\Gamma = n\langle\sigma v\rangle > H$

Thermal Physics

- In thermal physics there are two quantities of interest that become equal on the two sides of an interaction
  - Temperature T - from energy exchange
  - Chemical potential μ - from particle exchange
- These quantities maximize the entropy or number of accessible states between systems that can exchange energy and particles
- The latter is associated with the law of mass action - for a change in number of species i, $\sum_i \mu_i dN_i = 0$ – e.g. $e^- + p \leftrightarrow H + \gamma$ sets $\mu_e + \mu_p = \mu_H + \mu_\gamma$
- If a particle can be created freely then its chemical potential is driven to zero, e.g. bremsstrahlung $e^- + p \leftrightarrow e^- + p + \gamma$ implies $\mu_e + \mu_p = \mu_e + \mu_p + \mu_\gamma$ or $\mu_\gamma = 0$

Statistical Mechanics

- All allowed quantum states are equally likely to be occupied - so the average number of particles in thermal equilibrium can just be found by maximizing the total number of allowed states between a system and a larger reservoir
- In the supplement we derive the probability of system being in state of energy E_i and number N_i (Gibbs Factor)

$$P(E_i, N_i) \propto \exp[-(E_i - \mu N_i)/T]$$

- Mean occupation of the state in thermal equilibrium

$$f \equiv \frac{\sum N_i P(E_i, N_i)}{\sum P(E_i, N_i)}$$

where the total energy is related to the particle energy as $E_i = N_i E$ (ignoring zero pt)

Fermi-Dirac Distribution

- For fermions, the occupancy can only be $N_i = 0, 1$

$$
\begin{aligned}
f &= \frac{P(E,1)}{P(0,0) + P(E,1)} \\
&= \frac{e^{-(E-\mu)/T}}{1 + e^{-(E-\mu)/T}} \\
&= \frac{1}{e^{(E-\mu)/T} + 1}
\end{aligned}
$$

- In the non-relativistic, non-degenerate limit

$$
E = (q^2 + m^2)^{1/2} \approx m + \frac{1}{2}\frac{q^2}{m}
$$

and $m \gg T$ so the distribution is Maxwell-Boltzmann

$$
f = e^{-(m-\mu)/T} e^{-q^2/2mT} = e^{-(m-\mu)/T} e^{-mv^2/2T}
$$

Bose-Einstein Distribution

- For bosons each state can have multiple occupation,

$$f = \frac{\frac{d}{d\mu/T}\sum_{N=0}^{\infty}(e^{-(E-\mu)/T})^N}{\sum_{N=0}^{\infty}(e^{-(E-\mu)/T})^N} \qquad \text{with} \sum_{N=0}^{\infty} x^N = \frac{1}{1-x}$$

$$= \frac{1}{e^{(E-\mu)/T}-1}$$

- Again, non relativistic distribution is Maxwell-Boltzmann

$$f = e^{-(m-\mu)/T}e^{-q^2/2mT} = e^{-(m-\mu)/T}e^{-mv^2/2T}$$

with a spatial number density

$$n = ge^{-(m-\mu)/T}\int \frac{d^3q}{(2\pi)^3}e^{-q^2/2mT}$$

$$= ge^{-(m-\mu)/T}\left(\frac{mT}{2\pi}\right)^{3/2}$$

Ultra-Relativistic Bulk Properties

- Chemical potential $\mu = 0$, $\zeta(3) \approx 1.202$
- Number density

$$n_{\text{boson}} = gT^3 \frac{\zeta(3)}{\pi^2} \qquad \zeta(n+1) \equiv \frac{1}{n!} \int_0^\infty dx \frac{x^n}{e^x - 1}$$

$$n_{\text{fermion}} = \frac{3}{4} gT^3 \frac{\zeta(3)}{\pi^2}$$

- Energy density

$$\rho_{\text{boson}} = gT^4 \frac{3}{\pi^2} \zeta(4) = gT^4 \frac{\pi^2}{30}$$

$$\rho_{\text{fermion}} = \frac{7}{8} gT^4 \frac{3}{\pi^2} \zeta(4) = \frac{7}{8} gT^4 \frac{\pi^2}{30}$$

- Pressure $q^2/3E = E/3 \to p = \rho/3$, $w_r = 1/3$

Entropy Density

- First law of thermodynamics

$$dS = \frac{1}{T}(d\rho(T)V + p(T)dV)$$

so that

$$\left.\frac{\partial S}{\partial V}\right|_T = \frac{1}{T}[\rho(T) + p(T)], \qquad \left.\frac{\partial S}{\partial T}\right|_V = \frac{V}{T}\frac{d\rho}{dT}$$

- Since $S(V,T) \propto V$ is extensive

$$S = \frac{V}{T}[\rho(T) + p(T)] \quad \sigma = \frac{S}{V} = \frac{1}{T}[\rho(T) + p(T)]$$

So

$$\frac{\partial S}{\partial V} = \sigma, \qquad \frac{\partial}{\partial V}\left(\frac{\partial S}{\partial T}\right) = \frac{1}{T}\frac{d\rho}{dT}$$

Entropy Density

- Integrability condition $\partial^2 S/\partial V \partial T = \partial^2 S/\partial T \partial V$ relates the evolution of entropy density

$$\frac{d\sigma}{dT} = \frac{1}{T}\frac{d\rho}{dT}$$
$$\frac{d\sigma}{dt} = \frac{1}{T}\frac{d\rho}{dt} = \frac{1}{T}[-3(\rho+p)]\frac{d\ln a}{dt} = -3\sigma\frac{d\ln a}{dt}$$
$$\rightarrow \frac{d\ln\sigma}{dt} = -3\frac{d\ln a}{dt} \rightarrow \sigma \propto a^{-3}$$

comoving entropy density is conserved in thermal equilibrium

- Ultra relativisitic bosons $\sigma_{\rm boson} = 3.602 n_{\rm boson}$; for fermions $\times 7/8$ given scaling of ρ

$$g_* = \sum_{\rm bosons} g_b + \frac{7}{8}\sum g_f$$

Evolution of Temperature

- We will use this to derive the evolution of T as different particle species annihilate in thermal equilibrium
- Setting the entropy density before and after is equivalent to setting

$$g_* T^3 \Big|_{\rm initial} = g_* T^3 \Big|_{\rm final}$$

- When particle species disappear through annihilation, they dump their entropy into the remaining species and hence raise the temperature

Neutrino Freezeout

- Neutrino equilibrium maintained by weak interactions, e.g. $e^+ + e^- \leftrightarrow \nu + \bar{\nu}$
- Weak interaction cross section $T_{10} = T/10^{10}K \sim T/1\mathrm{MeV}$

$$\sigma_w \sim G_F^2 E_\nu^2 \approx 4 \times 10^{-44}\, T_{10}^2 \mathrm{cm}^2$$

- Rate $\Gamma = n_\nu \sigma_w = H$ at $T_{10} \sim 3$ or $t \sim 0.2$s
- After neutrino freezeout, electrons and positrons annihilate dumping their entropy into the photons
- Before g_*: $\gamma, e^+, e^- = 2 + \frac{7}{8}(2+2) = \frac{11}{2}$
- After g_*: γ = 2; so conservation of entropy gives

$$g_* T^3 \Big|_{\mathrm{initial}} = g_* T^3 \Big|_{\mathrm{final}} \qquad T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$$

Relic Neutrinos

- Relic number density (zero chemical potential; now required by oscillations & BBN)

$$n_\nu = n_\gamma \frac{3}{4}\frac{4}{11} = 112\mathrm{cm}^{-3}$$

- Relic energy density assuming one species with finite m_ν:

$\rho_\nu = m_\nu n_\nu$

$$\rho_\nu = 112\frac{m_\nu}{\mathrm{eV}}\,\mathrm{eV\,cm}^{-3} \qquad \rho_c = 1.05 \times 10^4 h^2\,\mathrm{eV cm}^{-3}$$

$$\Omega_\nu h^2 = \frac{m_\nu}{93.7\mathrm{eV}}$$

- Candidate for dark matter? an eV mass neutrino goes non relativistic around $z \sim 1000$ and retains a substantial velocity dispersion σ_ν.

Hot Dark Matter

- Momenta for a nonrelativistic species redshifts like temperature for a relativistic one, so average momentum is still given by

$$\langle q \rangle = 3T_\nu = m_\nu \sigma_\nu$$

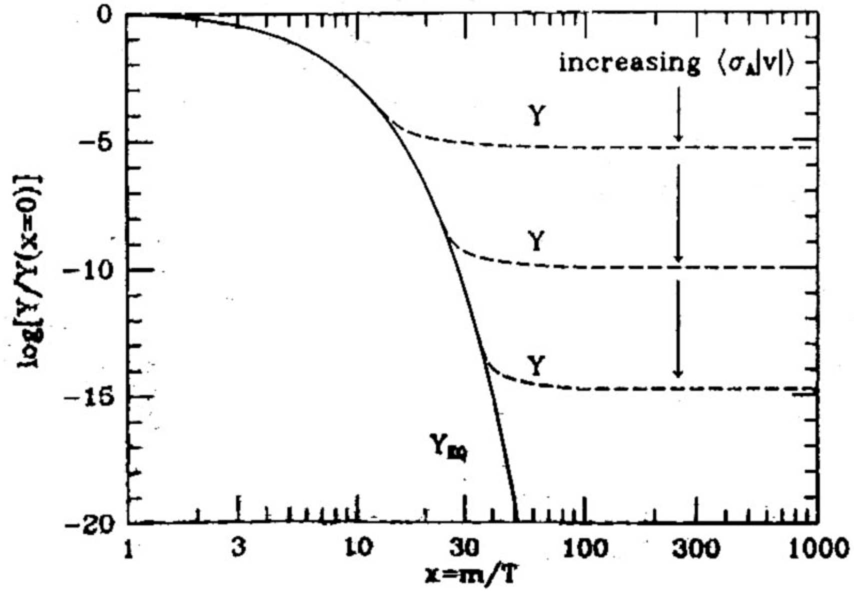
$$\sigma_\nu = 3\left(\frac{m_\nu}{1\mathrm{eV}}\right)^{-1}\left(\frac{T_\nu}{1\mathrm{eV}}\right) = 3\left(\frac{m_\nu}{1\mathrm{eV}}\right)^{-1}\left(\frac{T_\nu}{10^4\mathrm{K}}\right)$$

$$= 6 \times 10^{-4}\left(\frac{m_\nu}{1\mathrm{eV}}\right)^{-1} = 200\mathrm{km/s}\left(\frac{m_\nu}{1\mathrm{eV}}\right)^{-1}$$

- Of order the rotation velocity of galactic halos and higher at higher redshift - small objects can't form: top down structure formation – not observed – must not constitute the bulk of the dark matter

Cold Dark Matter

- Problem with neutrinos is they decouple while relativistic and hence have a comparable number density to photons - for a reasonable energy density, the mass must be small



- The equilibrium distribution for a non-relativistic species declines exponentially beyond the mass threshold

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$$

- Exponential will eventually win soon after $T < m$, suppressing annihilation rates

WIMP Miracle

- Freezeout when annihilation rate equal expansion rate $\Gamma \propto \sigma_A$, increasing annihilation cross section decreases abundance

$$\Gamma = n\langle\sigma_A v\rangle = H$$

$$H \propto T^2 \sim m^2$$

$$\rho_{\rm freeze} = mn \propto \frac{m^3}{\langle\sigma_A v\rangle}$$

$$\rho_c = \rho_{\rm freeze}(T/T_0)^{-3} \propto \frac{1}{\langle\sigma_A v\rangle}$$

independently of the mass of the CDM particle

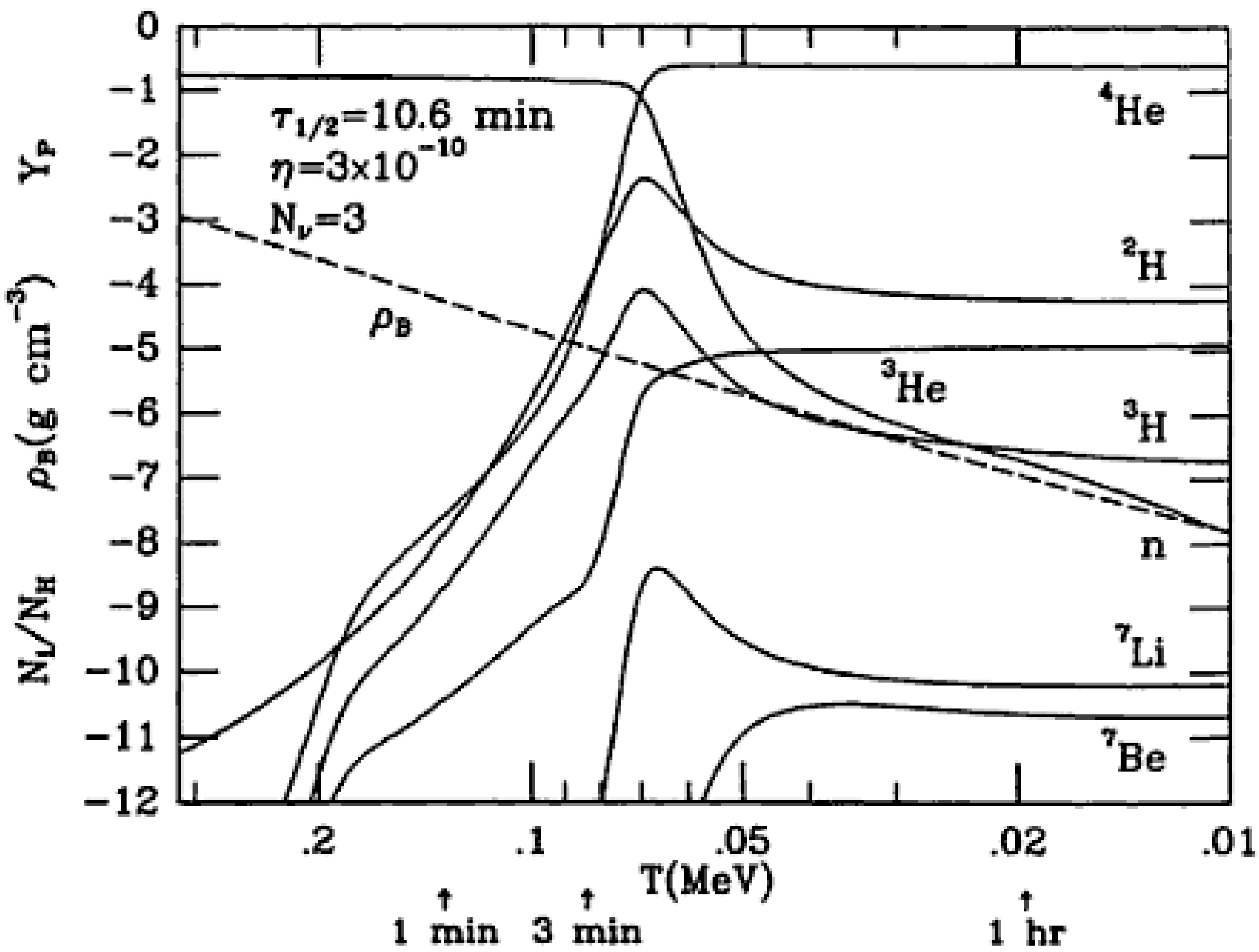
- Plug in some typical numbers for a particle with weak interaction scale cross sections or WIMPs (weakly interacting massive particles) of $\langle\sigma_A v\rangle \approx 10^{-36}$ cm^2 and restore the proportionality constant $\Omega_c h^2$ is of the right order of magnitude (~ 0.1)!

Axions

- Alternate solution: keep light particle but not created in thermal equilibrium
- Example: axion dark matter - particle that solves the strong CP problem
- Inflation sets initial conditions, fluctuation from potential minimum
- Once Hubble scale smaller than the mass scale, field unfreezes
- Coherent oscillations of the axion field - condensate state. Can be very light $m \ll 1\text{eV}$ and yet remain cold.
- Same reason a quintessence dark energy candidate must be lighter than the Hubble scale today

Big Bang Nucleosynthesis

- Integrating the Boltzmann equation for nuclear processes during first few minutes leads to synthesis and freezeout of light elements



Big Bang Nucleosynthesis

- Most of light element synthesis can be understood through nuclear statistical equilibrium and reaction rates
- Equilibrium abundance of species with mass number A and charge Z (Z protons and $A - Z$ neutrons)

$$n_A = g_A(\frac{m_A T}{2\pi})^{3/2} e^{(\mu_A - m_A)/T}$$

- In chemical equilibrium with protons and neutrons

$$\mu_A = Z\mu_p + (A - Z)\mu_n$$

$$n_A = g_A(\frac{m_A T}{2\pi})^{3/2} e^{-m_A/T} e^{(Z\mu_p + (A-Z)\mu_n)/T}$$

Big Bang Nucleosynthesis

- Eliminate chemical potentials with n_p, n_n

$$e^{\mu_p/T} = \frac{n_p}{g_p}\left(\frac{2\pi}{m_p T}\right)^{3/2} e^{m_p/T}$$

$$e^{\mu_n/T} = \frac{n_n}{g_n}\left(\frac{2\pi}{m_n T}\right)^{3/2} e^{m_n/T}$$

$$n_A = g_A g_p^{-Z} g_n^{Z-A} (\frac{m_A T}{2\pi})^{3/2}\left(\frac{2\pi}{m_p T}\right)^{3Z/2}\left(\frac{2\pi}{m_n T}\right)^{3(A-Z)/2}$$
$$\times\, e^{-m_A/T} e^{(Z\mu_p + (A-Z)\mu_n)/T} n_p^Z n_n^{A-Z}$$

$$(g_p = g_n = 2; m_p \approx m_n = m_b = m_A/A)$$

$$(B_A = Zm_p + (A-Z)m_n - m_A)$$

$$= g_A 2^{-A}\left(\frac{2\pi}{m_b T}\right)^{3(A-1)/2} A^{3/2} n_p^Z n_n^{A-Z} e^{B_A/T}$$

Big Bang Nucleosynthesis

- Convenient to define abundance fraction

$$X_A \equiv A\frac{n_A}{n_b} = Ag_A 2^{-A}\left(\frac{2\pi}{m_b T}\right)^{3(A-1)/2} A^{3/2} n_p^Z n_n^{A-Z} n_b^{-1} e^{B_A/T}$$

$$= Ag_A 2^{-A}\left(\frac{2\pi n_b^{2/3}}{m_b T}\right)^{3(A-1)/2} A^{3/2} e^{B_A/T} X_p^Z X_n^{A-Z}$$

$$(n_\gamma = \frac{2}{\pi^2}T^3\zeta(3) \qquad \eta_{b\gamma} \equiv n_b/n_\gamma)$$

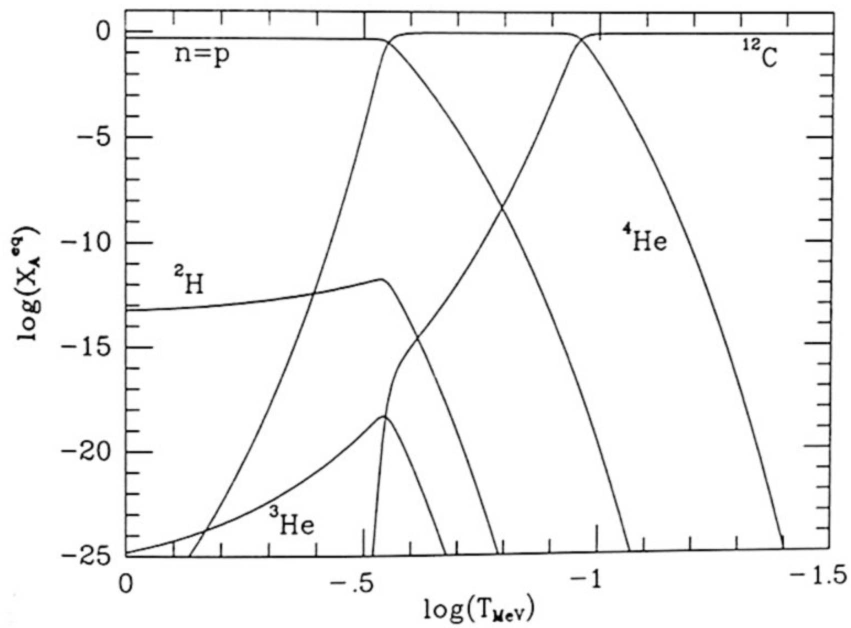
$$= A^{5/2} g_A 2^{-A}\left[\left(\frac{2\pi T}{m_b}\right)^{3/2}\frac{2\zeta(3)\eta_{b\gamma}}{\pi^2}\right]^{A-1} e^{B_A/T} X_p^Z X_n^{A-Z}$$

Deuterium

- Deuterium $A = 2$, $Z = 1$, $g_2 = 3$, $B_2 = 2.225$ MeV

$$X_2 = \frac{3}{\pi^2}\left(\frac{4\pi T}{m_b}\right)^{3/2} \eta_{b\gamma}\zeta(3)e^{B_2/T}X_pX_n$$

- Deuterium "bottleneck" is mainly due to the low baryon-photon number of the universe $\eta_{b\gamma} \sim 10^{-9}$, secondarily due to the low binding energy B_2

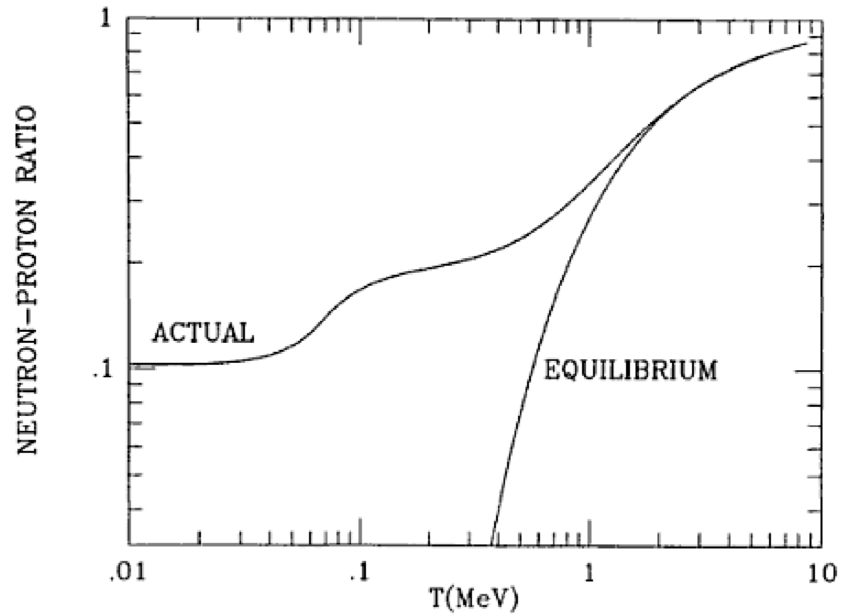


Deuterium

- $X_2/X_pX_n \approx \mathcal{O}(1)$ at $T \approx 100\text{keV}$ or 10^9 K, much lower than the binding energy B_2
- Most of the deuterium formed then goes through to helium via $\mathrm{D} + \mathrm{D} \to {}^3\mathrm{He} + p$, ${}^3\mathrm{He} + \mathrm{D} \to {}^4\mathrm{He} + n$
- Deuterium freezes out as number abundance becomes too small to maintain reactions $n_D =$ const. independent of n_b
- The deuterium freezeout fraction $n_D/n_b \propto \eta_{b\gamma}^{-1} \propto (\Omega_b h^2)^{-1}$ and so is fairly sensitive to the baryon density.
- Observations of the ratio in quasar absorption systems give $\Omega_b h^2 \approx 0.02$

Helium

- Essentially all neutrons around during nucleosynthesis end up in Helium
- In equilibrium, the neutron-to-proton ratio is determined by the mass difference $Q = m_n - m_p = 1.293$ MeV



$$\frac{n_n}{n_p} = \exp[-Q/T]$$

Helium

- Equilibrium is maintained through weak interactions, e.g. $n \leftrightarrow p + e^- + \bar{\nu}$, $\nu + n \leftrightarrow p + e^-$, $e^+ + n \leftrightarrow p + \bar{\nu}$ with rate

$$\frac{\Gamma}{H} \approx \frac{T}{0.8\mathrm{MeV}}$$

- Freezeout fraction

$$\frac{n_n}{n_p} = \exp[-1.293/0.8] \approx 0.2$$

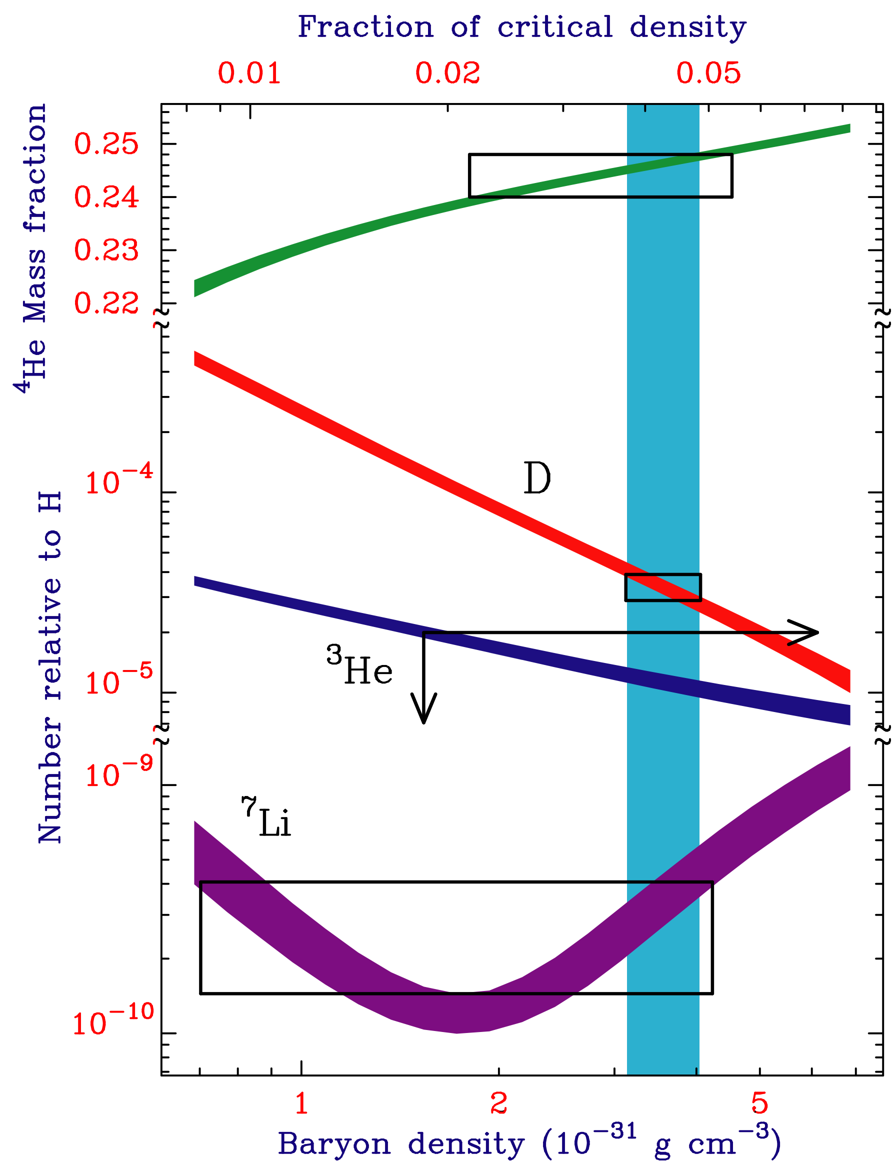
- Finite lifetime of neutrons brings this to $\sim 1/7$ by 10^9K
- Helium mass fraction

$$\begin{aligned} Y_{\mathrm{He}} &= \frac{4n_{He}}{n_b} = \frac{4(n_n/2)}{n_n + n_p} \\ &= \frac{2n_n/n_p}{1 + n_n/n_p} \approx \frac{2/7}{8/7} \approx \frac{1}{4} \end{aligned}$$

Helium

- Depends mainly on the expansion rate during BBN - measure number of relativistic species
- Traces of ^{7}Li as well. Measured abundances in reasonable agreement with deuterium measure $\Omega_b h^2 = 0.02$ but the detailed interpretation is still up for debate

Light Elements



Burles, Nollett, Turner (1999)

Baryogenesis

- What explains the small, but non-zero, baryon-to-photon ratio?

$$\eta_{b\gamma} = n_b/n_\gamma \approx 3 \times 10^{-8}\Omega_b h^2 \approx 6 \times 10^{-10}$$

- Must be a slight excess of baryons b to anti-baryons $\bar{b}$ that remains after annihilation
- Sakharov conditions
  - Baryon number violation: some process must change the net baryon number
  - CP violation: process which produces b and $\bar{b}$ must differ in rate
  - Out of equilibrium: else equilibrium distribution with vanishing chemical potential (processes exist which change baryon number) gives equal numbers for b and $\bar{b}$
- Expanding universe provides 3; physics must provide 1,2

Baryogenesis

- Example: out of equilibrium decay of some heavy boson $X, \bar{X}$
- Suppose X decays through 2 channels with baryon number b_1 and b_2 with branching ratio r and $1-r$ leading to a change in the baryon number per decay of

$$rb_1 + (1-r)b_2$$

- And $\bar{X}$ to $-b_1$ and $-b_2$ with ratio $\bar{r}$ and $1-\bar{r}$

$$-\bar{r}b_1 - (1-\bar{r})b_2$$

- Net production

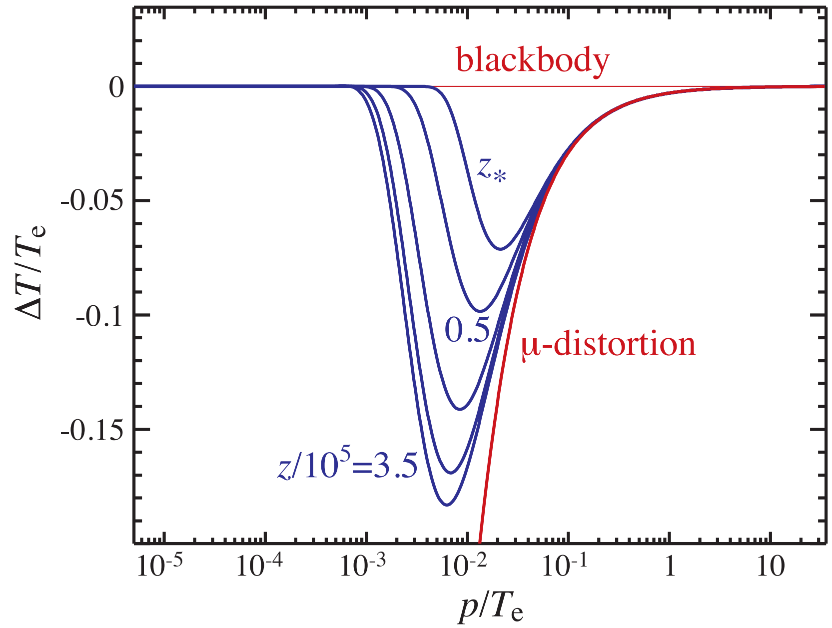
$$\Delta b = (r-\bar{r})(b_1 - b_2)$$

Baryogenesis

- Condition 1: $b_1 \neq 0, b_2 \neq 0$
- Condition 2: $\bar{r} \neq r$
- Condition 3: out of equilibrium decay
- GUT and electroweak (instanton) baryogenesis mechanisms exist
- Active subject of research

Black Body Formation

- After $z \sim 10^6$, photon creating processes $\gamma + e^- \leftrightarrow 2\gamma + e^-$ and bremmstrahlung $e^- + p \leftrightarrow e^- + p + \gamma$ drop out of equilibrium for photon energies $E \sim T$.



- Compton scattering remains effective in redistributing energy via exchange with electrons
- Out of equilibrium processes like decays leave residual photon chemical potential imprint
- Observed black body spectrum places tight constraints on any that might dump energy into the CMB

Recombination

- Maxwell-Boltzmann distribution determines the equilibrium distribution for reactions, e.g. big-bang nucleosynthesis, recombination:

$$p + e^- \leftrightarrow H + \gamma$$

$$\frac{n_p n_e}{n_H} \approx e^{-B/T} \left(\frac{m_e T}{2\pi} \right)^{3/2} e^{(\mu_p + \mu_e - \mu_H)/T}$$

where $B = m_p + m_e - m_H = 13.6\text{eV}$ is the binding energy, $g_p = g_e = \frac{1}{2} g_H = 2$, and $\mu_p + \mu_e = \mu_H$ in equilibrium

- Define ionization fraction

$$n_p = n_e = x_e n_b$$

$$n_H = n_{\rm b} - n_e = (1 - x_e) n_b$$

Recombination

- Saha Equation

$$\frac{n_e n_p}{n_H n_b} = \frac{x_e^2}{1 - x_e} = \frac{1}{n_b}\left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-B/T}$$

- Naive guess of $T_* = B$ wrong due to the low baryon-photon ratio – $T_* \approx 0.3$eV so recombination at $z_* \approx 1000$
- But the photon-baryon ratio is very low

$$\eta_{b\gamma} \equiv n_b/n_\gamma \approx 3 \times 10^{-8} \Omega_b h^2$$

Recombination

- Eliminate in favor of $\eta_{b\gamma}$ and B/T through

$$n_\gamma = 0.244T^3\,, \quad \frac{m_e}{B} = 3.76 \times 10^4$$

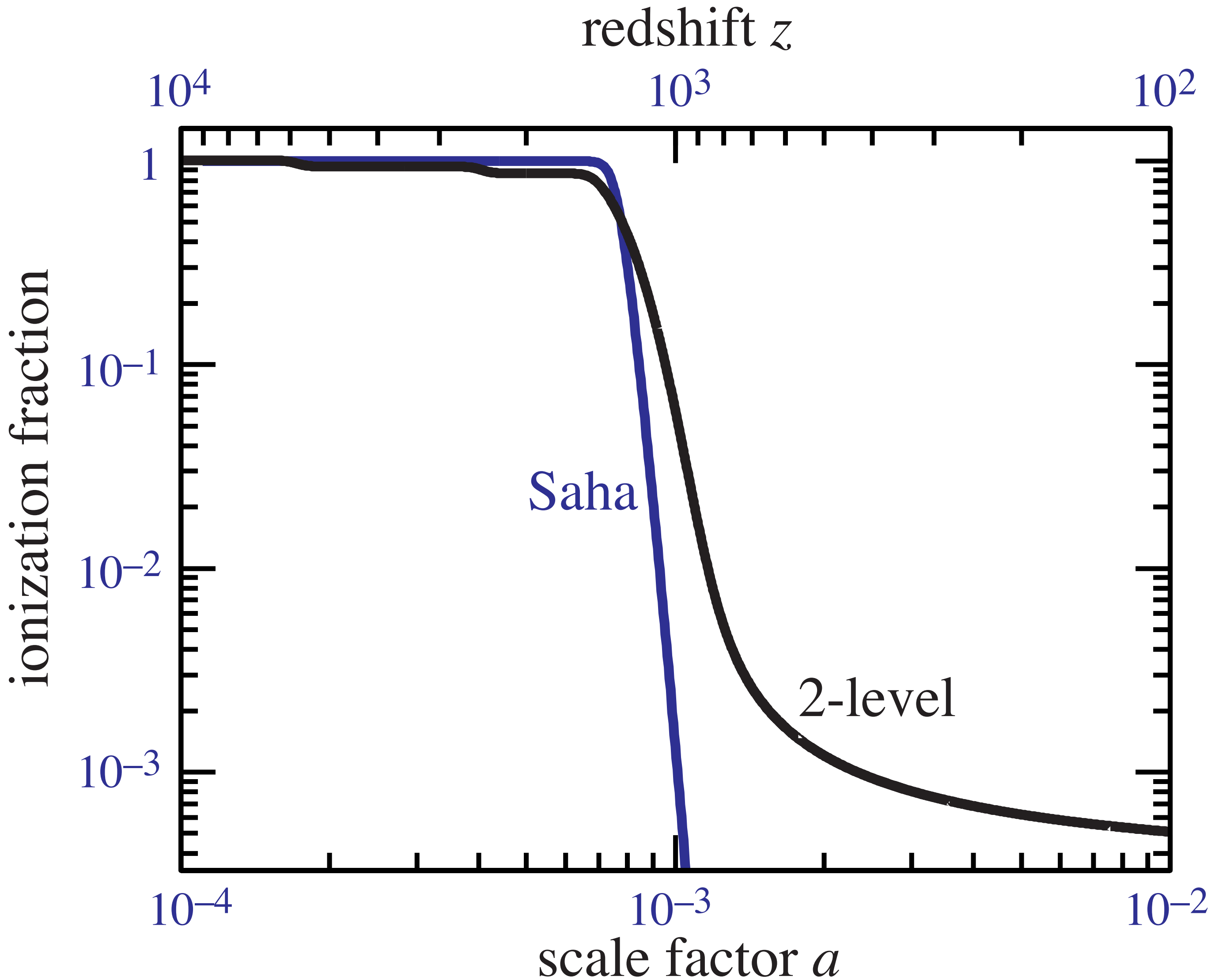
- Big coefficient

$$\frac{{x_e}^2}{1 - x_e} = 3.16 \times 10^{15} \left(\frac{B}{T}\right)^{3/2} e^{-B/T}$$

$T = 1/3\text{eV} \rightarrow x_e = 0.7$, $T = 0.3\text{eV} \rightarrow x_e = 0.2$

- Further delayed by inability to maintain equilibrium since net is through 2γ process and redshifting out of line

Recombination



CMB Anisotropy

- Recombination can be viewed as the epoch where (most) of the CMB fluctuations freeze out
- Once neutral hydrogen forms, photons largely propagate unimpeded to the observer today
- CMB fluctuations thus provide an image of the universe at $z \sim 1000$
- This leads to the famous horizon problem whose resolution is the subject of the next set of lectures…