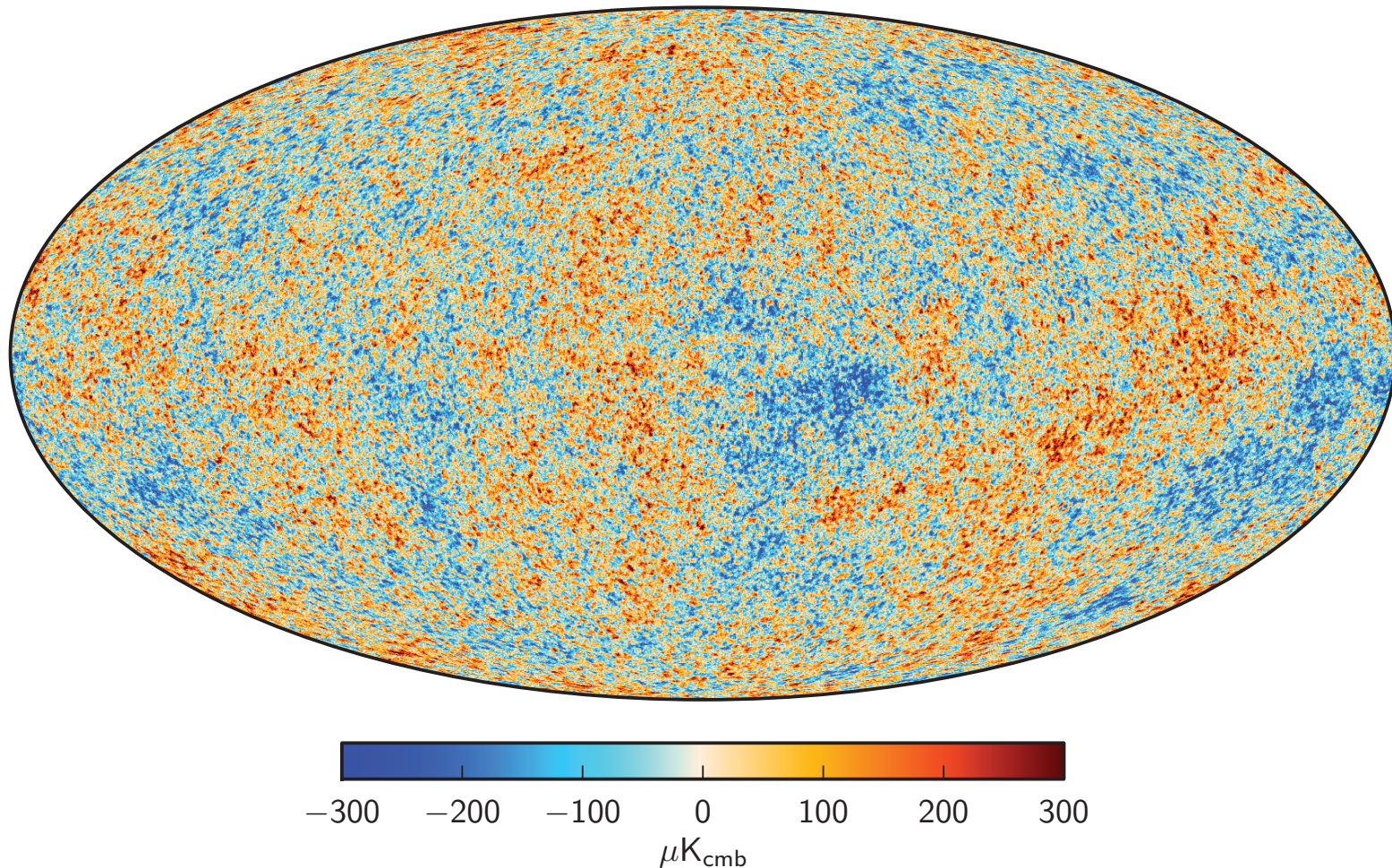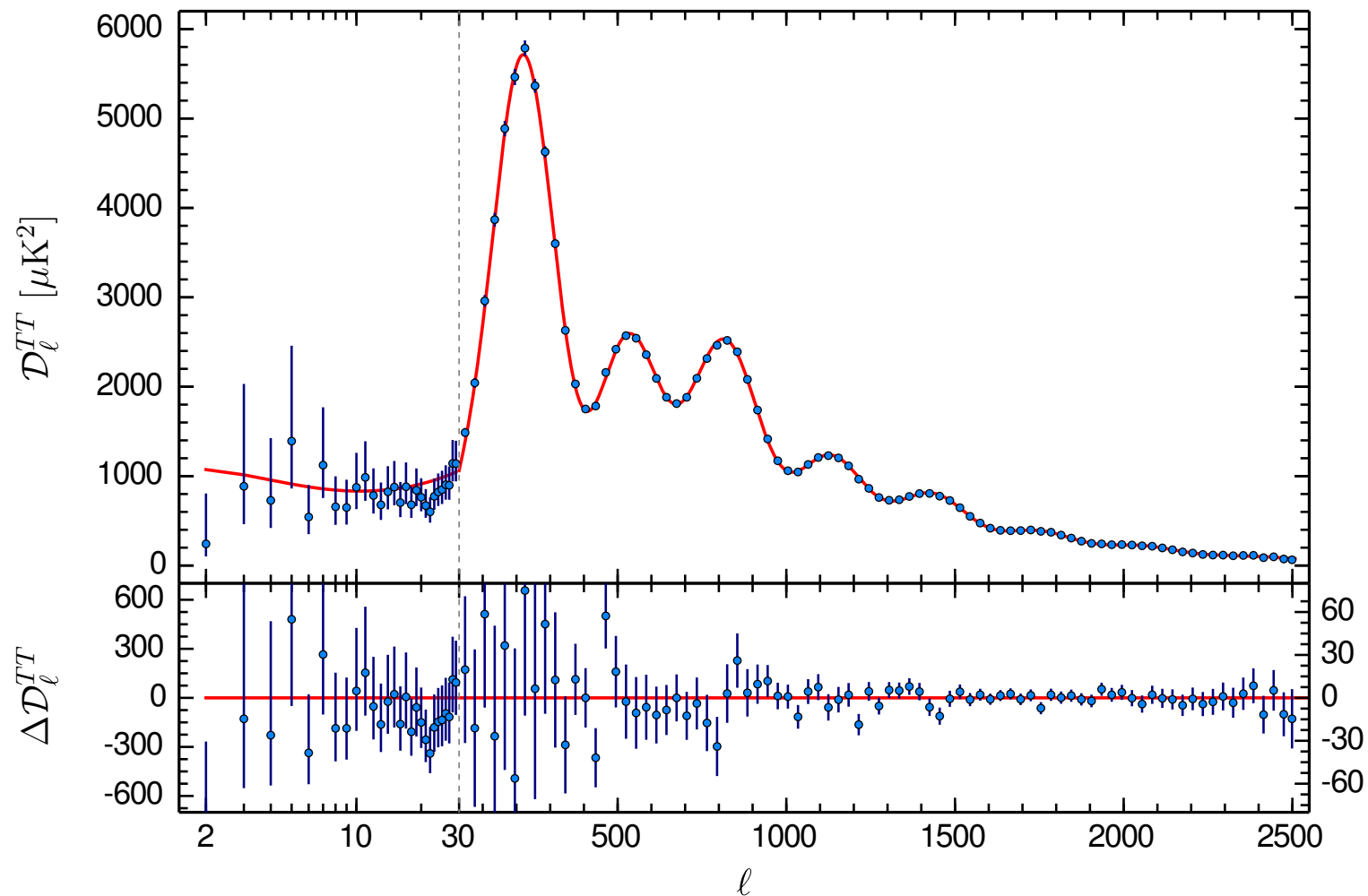# Set 7:

# CMB and Large Scale Structure

# CMB Temperature Anisotropy

- Planck 2015 map of the temperature anisotropy (first discovered by COBE) from recombination:

# CMB Temperature Anisotropy

- Power spectrum shows characteristic scales where the intensity of variations peak - reveals geometry and contents of the universe:

# CMB Parameter Inferences

- Spectrum constrains the matter-energy contents of the universe

- Planck 2018 results [arXiv:1807.06209]

| Parameter | TT+lowE 68% limits | TE+lowE 68% limits | EE+lowE 68% limits | TT,TE,EE+lowE 68% limits | TT,TE,EE+lowE+lensing 68% limits | TT,TE,EE+lowE+lensing+BAO 68% limits |
|---|---|---|---|---|---|---|
| $\Omega_b h^2$ . . . . . . . . . | $0.02212 \pm 0.00022$ | $0.02249 \pm 0.00025$ | $0.0240 \pm 0.0012$ | $0.02236 \pm 0.00015$ | $0.02237 \pm 0.00015$ | $0.02242 \pm 0.00014$ |
| $\Omega_c h^2$ . . . . . . . . . | $0.1206 \pm 0.0021$ | $0.1177 \pm 0.0020$ | $0.1158 \pm 0.0046$ | $0.1202 \pm 0.0014$ | $0.1200 \pm 0.0012$ | $0.11933 \pm 0.00091$ |
| $100\theta_{MC}$ . . . . . . . | $1.04077 \pm 0.00047$ | $1.04139 \pm 0.00049$ | $1.03999 \pm 0.00089$ | $1.04090 \pm 0.00031$ | $1.04092 \pm 0.00031$ | $1.04101 \pm 0.00029$ |
| $\tau$ . . . . . . . . . . . | $0.0522 \pm 0.0080$ | $0.0496 \pm 0.0085$ | $0.0527 \pm 0.0090$ | $0.0544^{+0.0070}_{-0.0081}$ | $0.0544 \pm 0.0073$ | $0.0561 \pm 0.0071$ |
| $\ln(10^{10} A_s)$ . . . . . . . | $3.040 \pm 0.016$ | $3.018^{+0.020}_{-0.018}$ | $3.052 \pm 0.022$ | $3.045 \pm 0.016$ | $3.044 \pm 0.014$ | $3.047 \pm 0.014$ |
| $n_s$ . . . . . . . . . . . | $0.9626 \pm 0.0057$ | $0.967 \pm 0.011$ | $0.980 \pm 0.015$ | $0.9649 \pm 0.0044$ | $0.9649 \pm 0.0042$ | $0.9665 \pm 0.0038$ |
| $H_0$ [km s$^{-1}$ Mpc$^{-1}$] . . | $66.88 \pm 0.92$ | $68.44 \pm 0.91$ | $69.9 \pm 2.7$ | $67.27 \pm 0.60$ | $67.36 \pm 0.54$ | $67.66 \pm 0.42$ |

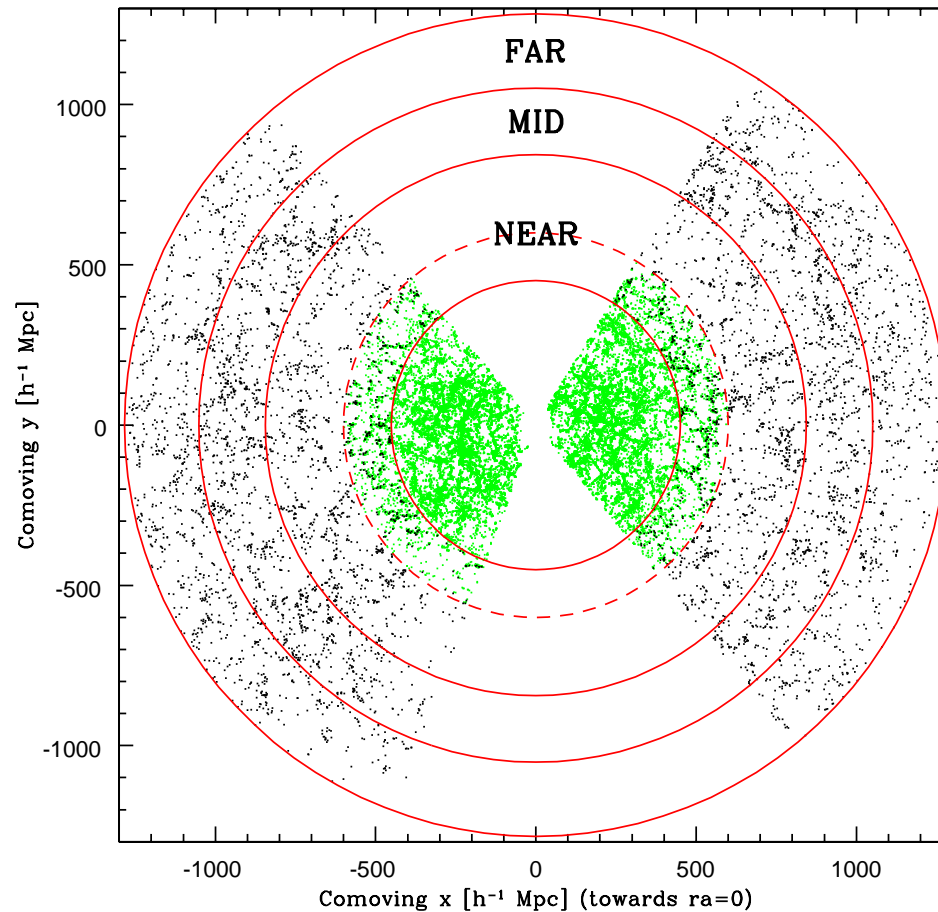# CMB Power Spectra

- **Power spectra of CMB**
  - temperature
  - polarization
  - lensing



Planck (2018) I
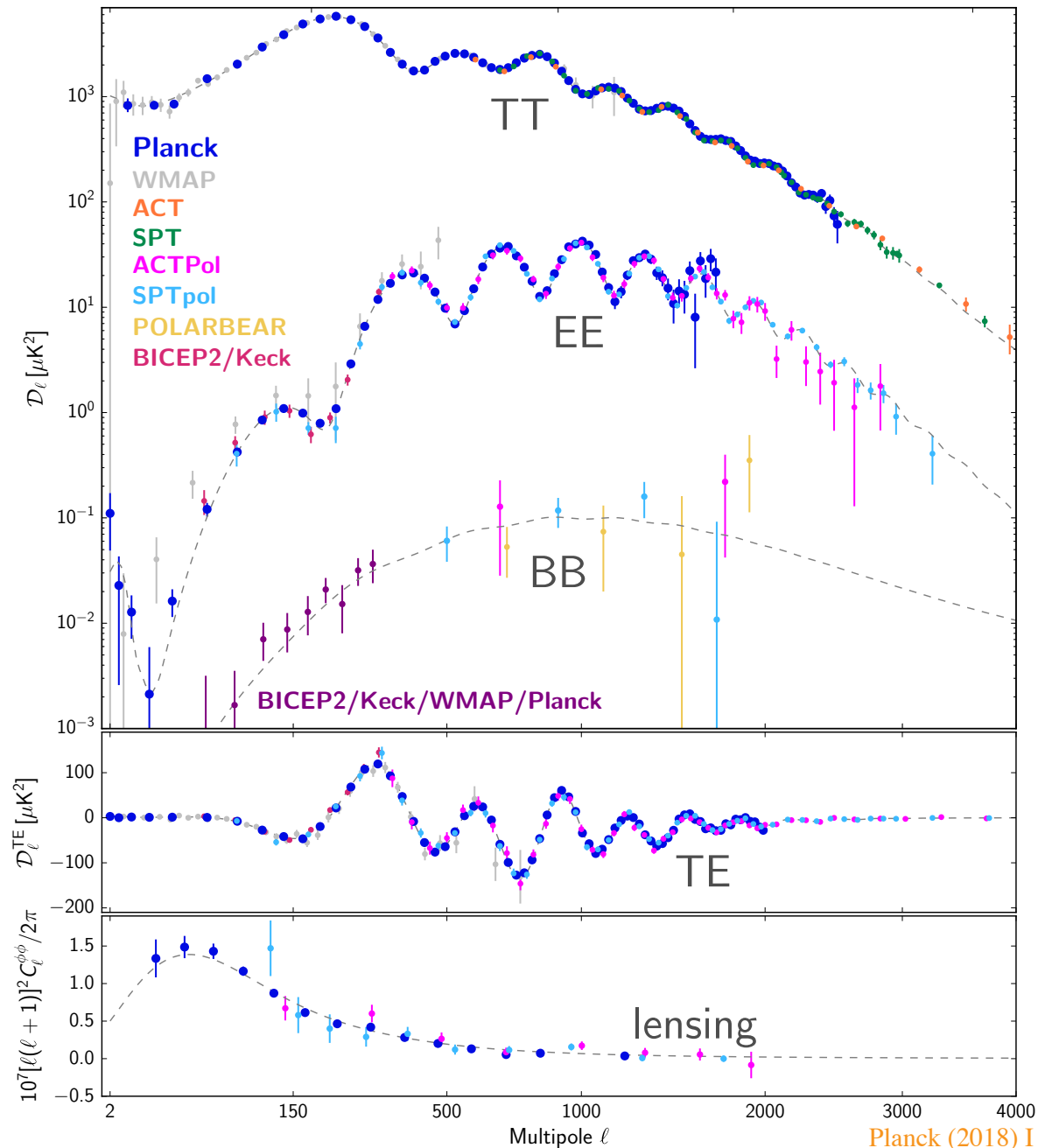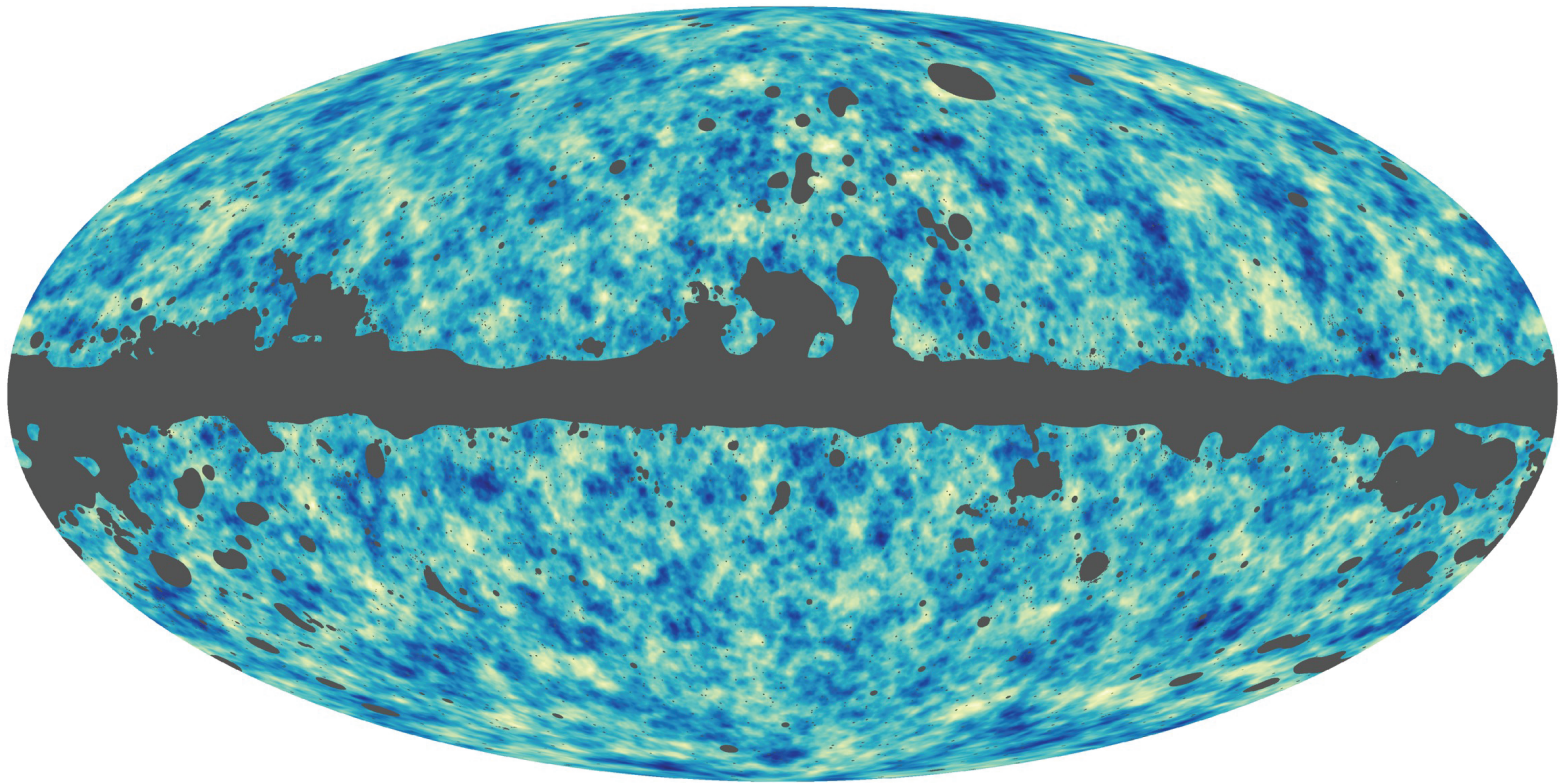
# Galaxy Redshift Surveys

- Galaxy redshift surveys (e.g. 2dF and SDSS) measure the three dimensional distribution of galaxies today:
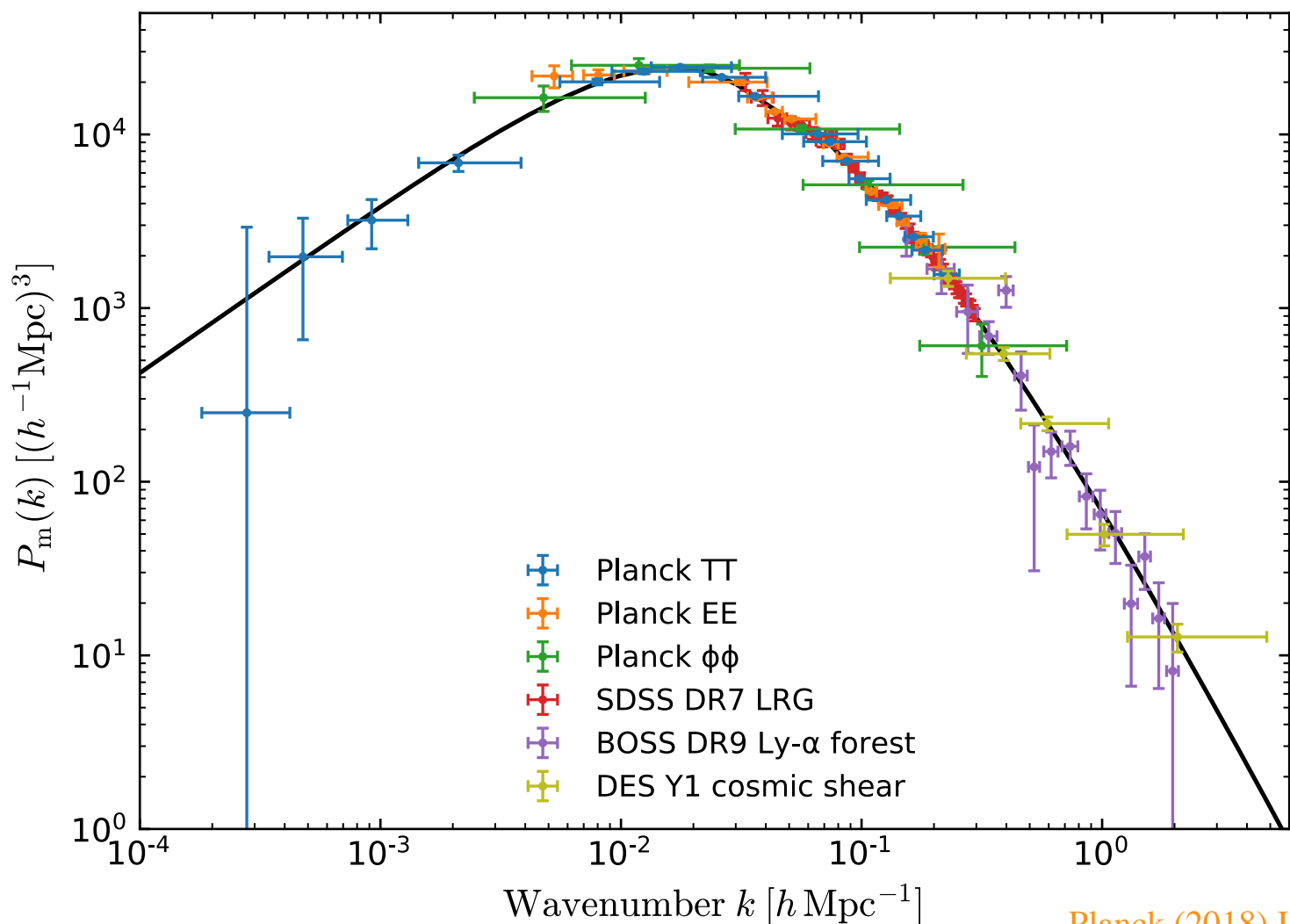
# Gravitational Lensing

- Gravitational Lensing measures projected mass

- Planck CMB lensing map

# Matter Power Spectrum

- Compilation of Redshift Surveys, Lensing, CMB



Planck (2018) I

# Structure Formation

- Small perturbations from inflation over the course of the 14Gyr life of the universe are gravitationally enhanced into all of the structure seen today

- Cosmic microwave background shows a snapshot at a few hundred thousand years old at recombination

- Discovery in 1992 of cosmic microwave background anisotropy provided the observational breakthrough - convincing support for adiabatic initial density fluctuations of amplitude $10^{-5}$

- Combine with galaxy clustering - large scale structure seen in galaxy surveys - right amplitude given cold dark matter

# Schematic CMB Spectrum

- Take apart features in the power spectrum
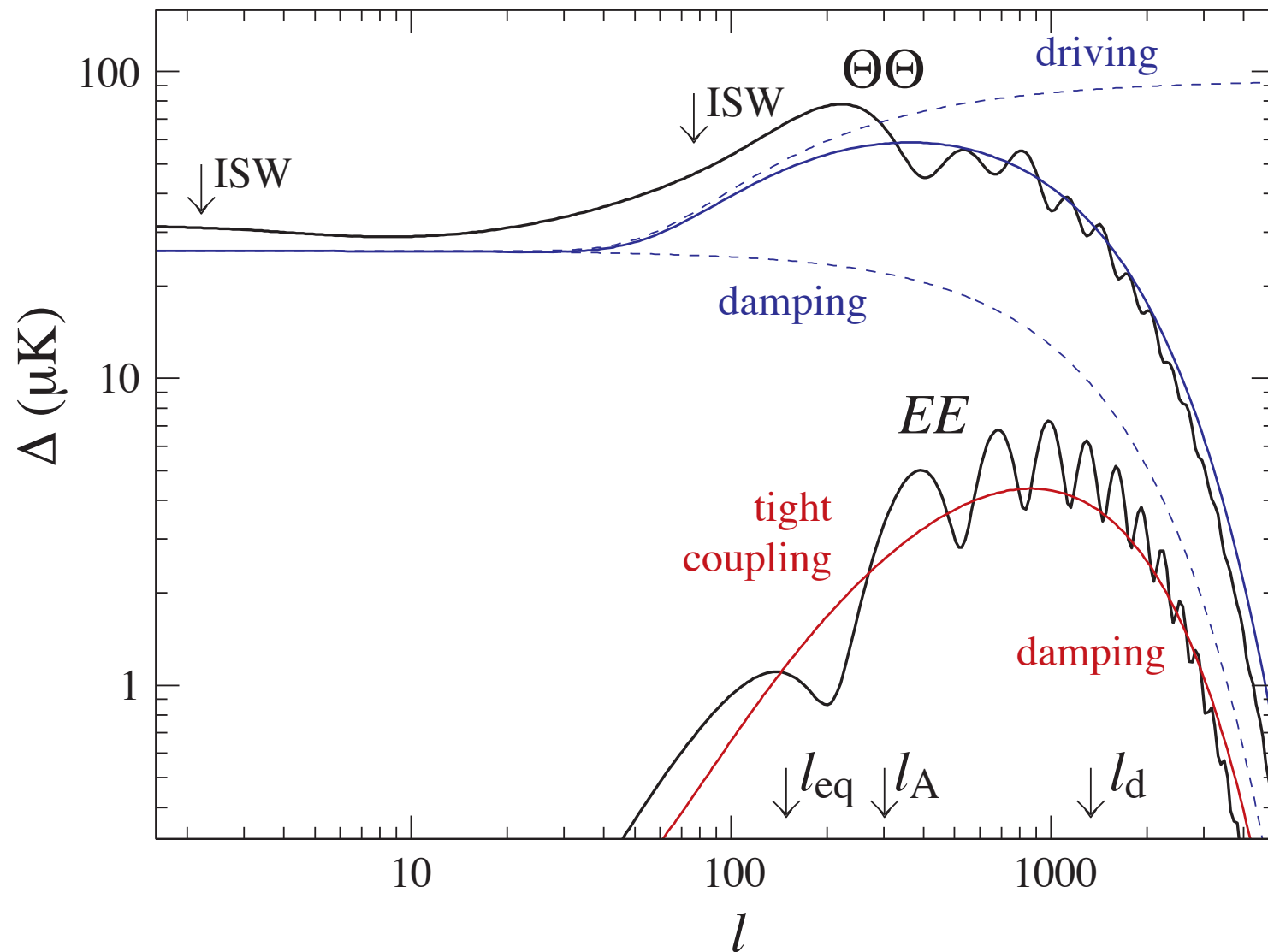
# Fluid Approximation

- Thomson scattering of photons and free electrons before recombination is sufficiently rapid that the bayrons and photons are in equilibrium and hence move together

- Mean free path of the photons for $z \approx 10^3$ and $\Omega_b h^2 \approx 0.02$

$$\lambda_C \equiv \frac{1}{n_e \sigma_T a} \sim 2.5 \mathrm{Mpc}$$

  small by cosmological standards!

- On scales $\lambda \gg \lambda_C$ photons are tightly coupled to the electrons by Thomson scattering which in turn are tightly coupled to the baryons by Coulomb interactions

- Specifically, their bulk velocities are defined by a single fluid velocity $v_\gamma = v_b$ and the photons carry no anisotropy in the rest frame of the baryons

# Zeroth Order Approximation

- **Momentum density** of a fluid is $(\rho + p)v$, where $p$ is the pressure

- **Neglect** the momentum density of the **baryons**

$$R \equiv \frac{(\rho_b + p_b)v_b}{(\rho_\gamma + p_\gamma)v_\gamma} = \frac{\rho_b + p_b}{\rho_\gamma + p_\gamma} = \frac{3\rho_b}{4\rho_\gamma}$$

$$\approx 0.6 \left(\frac{\Omega_b h^2}{0.02}\right)\left(\frac{a}{10^{-3}}\right)$$

since $\rho_\gamma \propto T^4$ is fixed by the CMB temperature $T = 2.73(1 + z)$K – OK substantially **before recombination**

- **Neglect** **radiation** in the **expansion**

$$\frac{\rho_m}{\rho_r} = 3.6 \left(\frac{\Omega_m h^2}{0.15}\right)\left(\frac{a}{10^{-3}}\right)$$

- **Neglect gravity**

# Fluid Equations

- Density $\rho_\gamma \propto T^4$ so define temperature fluctuation $\Theta$

$$\delta_\gamma = 4\frac{\delta T}{T} \equiv 4\Theta$$

- Real space continuity eqn.: the local number or energy density of photons changes if there is a divergence of the velocity field - a flow inwards or outwards from the volume

- Transformed to Fourier space $\nabla(e^{i\mathbf{k}\cdot\mathbf{x}}) \to i\mathbf{k}(e^{i\mathbf{k}\cdot\mathbf{x}})$ and $\nabla \cdot \mathbf{v} = -kv$

$$\dot{\delta}_\gamma = -(1 + w_\gamma)kv_\gamma$$

$$\dot{\Theta} = -\frac{1}{3}kv_\gamma$$

# Fluid Equations

- Euler equation (neglecting gravity for now): momentum conservation says that pressure gradients generate changes in momentum density $k\delta p_\gamma = k c_s^2 \delta \rho_\gamma$

$$\dot{v}_\gamma = \frac{k c_s^2}{1 + w_\gamma} \delta_\gamma$$

$$= k c_s^2 \frac{3}{4} \delta_\gamma = 3 c_s^2 k \Theta$$

  where the sound speed $c_s^2 = \delta p / \delta \rho$ is the pressure response to a density fluctuation

- So if you squeeze the photon gas to raise its density, its going to respond with a restoring force by raising the pressure and resisting compression $\rightarrow$ acoustic oscillations

# Oscillator: Take One

- Combine these to form the simple harmonic oscillator equation

$$\ddot{\Theta} + c_s^2 k^2 \Theta = 0$$

  where the sound speed is adiabatic

$$c_s^2 = \frac{\delta p}{\delta \rho} = \frac{\dot{p}_\gamma}{\dot{\rho}_\gamma}$$

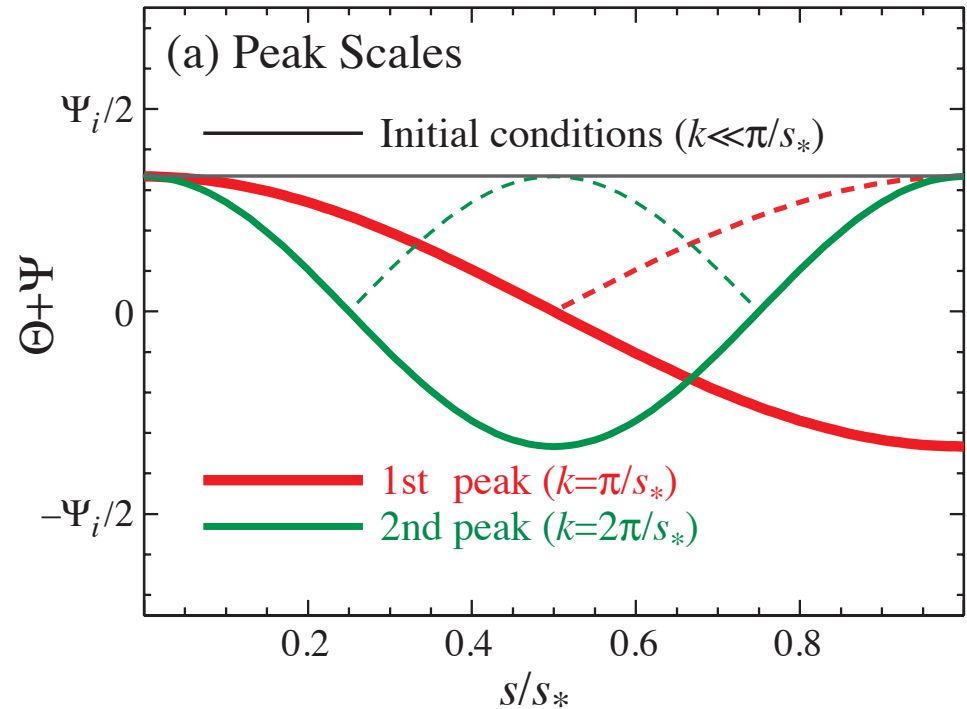  here $c_s^2 = 1/3$ since we are photon-dominated

- General solution:

$$\Theta(\eta) = \Theta(0)\cos(ks) + \frac{\dot{\Theta}(0)}{kc_s}\sin(ks)$$

  where the sound horizon is defined as $s \equiv \int c_s d\eta$

# Harmonic Extrema

- All modes begin at end of inflation and are frozen in at recombination (denoted with a subscript $*$)

- Temperature perturbations of different amplitude for different modes.



(a) Peak Scales

Initial conditions ($k \ll \pi/s_*$)

1st peak ($k=\pi/s_*$)

2nd peak ($k=2\pi/s_*$)

- For the adiabatic (curvature mode) initial conditions

$$\dot{\Theta}(0) = 0$$

- So solution

$$\Theta(\eta_*) = \Theta(0)\cos(ks_*)$$

# Harmonic Extrema

- Modes caught in the extrema of their oscillation will have enhanced fluctuations

$$k_n s_* = n\pi$$

yielding a fundamental scale or frequency, related to the inverse sound horizon

$$k_A = \pi/s_*$$

and a harmonic relationship to the other extrema as $1 : 2 : 3...$

# Peak Location

- The fundmental physical scale is translated into a fundamental angular scale by simple projection according to the angular diameter distance $D_A$

$$\theta_A = \lambda_A / D_A$$
$$\ell_A = k_A D_A$$

- In a flat universe, the distance is simply $D_A = D \equiv \eta_0 - \eta_* \approx \eta_0$, the horizon distance, and $k_A = \pi/s_* = \sqrt{3}\pi/\eta_*$ so
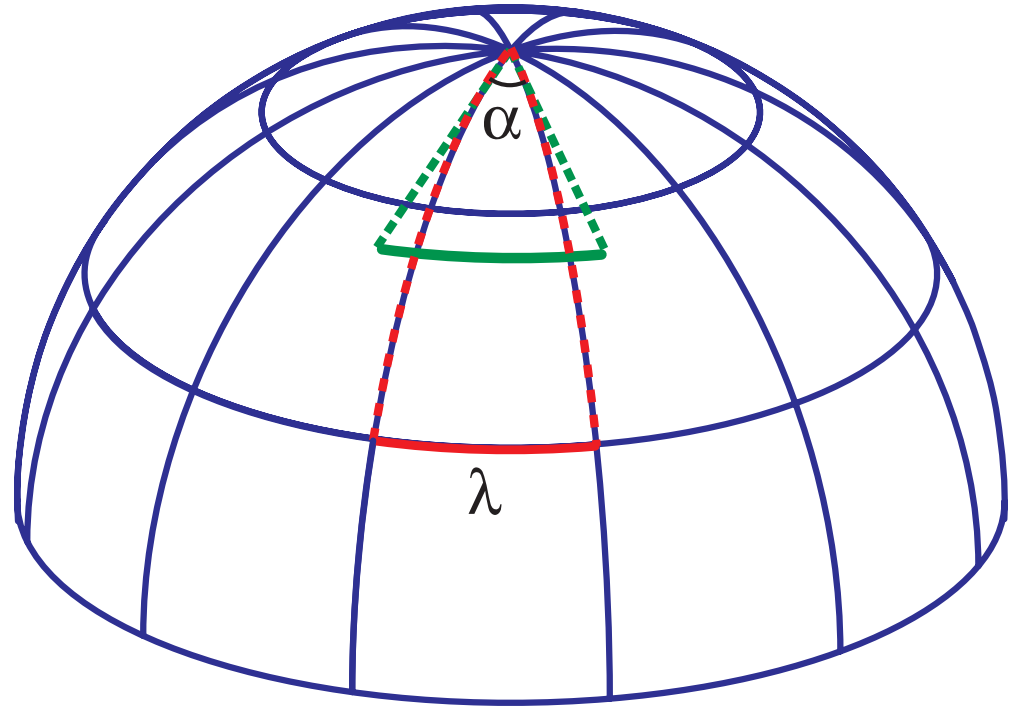
$$\theta_A \approx \frac{\eta_*}{\eta_0}$$

- In a matter-dominated universe $\eta \propto a^{1/2}$ so $\theta_A \approx 1/30 \approx 2°$ or

$$\ell_A \approx 200$$

# Curvature



- In a curved universe, the apparent or angular diameter distance is no longer the conformal distance $D_A = R\sin(D/R) \neq D$

- Objects in a closed universe are further than they appear! gravitational lensing of the background...

- Curvature scale of the universe must be substantially larger than current horizon

# Curvature

- Flat universe indicates critical density and implies missing energy given local measures of the matter density "dark energy"

- $D$ also depends on dark energy density $\Omega_{\mathrm{DE}}$ and equation of state $w = p_{\mathrm{DE}}/\rho_{\mathrm{DE}}$.

- Expansion rate at recombination

or matter-radiation ratio enters into calculation of $k_A$.
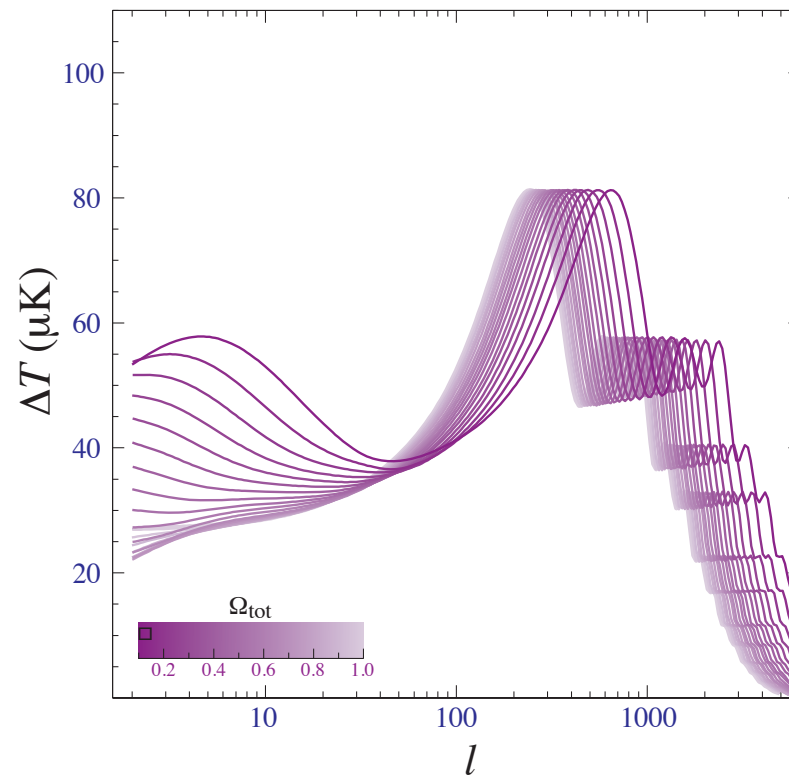
# Restoring Gravity

- Take a simple photon dominated system with gravity

- Continuity altered since a gravitational potential represents a stretching of the spatial fabric that dilutes number densities – formally a spatial curvature perturbation

- Think of this as a perturbation to the scale factor $a \to a(1 + \Phi)$ so that the cosmogical redshift is generalized to

$$\frac{\dot{a}}{a} \to \frac{\dot{a}}{a} + \dot{\Phi}$$

so that the continuity equation becomes

$$\dot{\Theta} = -\frac{1}{3}kv_\gamma - \dot{\Phi}$$

# Restoring Gravity

- **Gravitational force** in momentum conservation $\mathbf{F} = -m\nabla\Psi$ generalized to momentum density modifies the Euler equation to

$$\dot{v}_\gamma = k(\Theta + \Psi)$$

- General relativity says that $\Phi$ and $\Psi$ are the relativistic analogues of the Newtonian potential and that $\Phi \approx -\Psi$.

- In our matter-dominated approximation, $\Phi$ represents matter density fluctuations through the cosmological Poisson equation

$$k^2\Phi = 4\pi G a^2 \rho_m \Delta_m$$

where the difference comes from the use of comoving coordinates for $k$ ($a^2$ factor), the removal of the background density into the background expansion $(\rho\Delta_m)$ and finally a coordinate subtlety that enters into the definition of $\Delta_m$

# Constant Potentials

- In the matter dominated epoch potentials are constant because infall generates velocities as $v_m \sim k\eta\Psi$

- Velocity divergence generates density perturbations as
  $\Delta_m \sim -k\eta v_m \sim -(k\eta)^2\Psi$

- And density perturbations generate potential fluctuations

$$\Phi = \frac{4\pi G a^2 \rho \Delta}{k^2} \approx \frac{3}{2}\frac{H^2 a^2}{k^2}\Delta \sim \frac{\Delta}{(k\eta)^2} \sim -\Psi$$

keeping them constant. Note that because of the expansion, density perturbations must grow to keep potentials constant.

# Constant Potentials

- More generally, if stress perturbations are negligible compared with density perturbations ( $\delta p \ll \delta\rho$ ) then potential will remain roughly constant

- More specifically a variant called the Bardeen or comoving curvature is strictly constant

$$\mathcal{R} = \text{const} \approx \frac{5 + 3w}{3 + 3w}\Phi$$

where the approximation holds when $w \approx$ const.

# Oscillator: Take Two

- Combine these to form the simple harmonic oscillator equation

$$\ddot{\Theta} + c_s^2 k^2 \Theta = -\frac{k^2}{3}\Psi - \ddot{\Phi}$$

- In a CDM dominated expansion $\dot{\Phi} = \dot{\Psi} = 0$. Also for photon domination $c_s^2 = 1/3$ so the oscillator equation becomes

$$\ddot{\Theta} + \ddot{\Psi} + c_s^2 k^2 (\Theta + \Psi) = 0$$

- Solution is just an offset version of the original

$$[\Theta + \Psi](\eta) = [\Theta + \Psi](0) \cos(ks)$$

- $\Theta + \Psi$ is also the observed temperature fluctuation since photons lose energy climbing out of gravitational potentials at recombination

# Effective Temperature

- Photons climb out of potential wells at last scattering

- Lose energy to gravitational redshifts

- Observed or effective temperature

$$\Theta + \Psi$$

- Effective temperature oscillates around zero with amplitude given by the initial conditions

- Note: initial conditions are set when the perturbation is outside of horizon, need inflation or other modification to matter-radiation FRW universe.

- GR says that initial temperature is given by initial potential

# Sachs-Wolfe Effect and the Magic 1/3

- A gravitational potential is a perturbation to the temporal coordinate [formally a gauge transformation]

$$\frac{\delta t}{t} = \Psi$$

- Convert this to a perturbation in the scale factor, in a matter dominated expansion $a \propto t^{2/3}$ so

$$\frac{\delta a}{a} = \frac{2}{3}\frac{\delta t}{t}$$

- CMB temperature is cooling as $T \propto a^{-1}$ so

$$\Theta + \Psi \equiv \frac{\delta T}{T} + \Psi = -\frac{\delta a}{a} + \Psi = \frac{1}{3}\Psi$$

# Sachs-Wolfe Normalization

- Use measurements of $\Delta T/T \approx 10^{-5}$ in the Sachs-Wolfe effect to infer $\Delta_{\mathcal{R}}^2$

- Recall in matter domination $\Psi = -3\mathcal{R}/5$ and so $\Delta T/T = -\mathcal{R}/5$

- So that the amplitude of initial curvature fluctuations is
  $\Delta_R \approx 5 \times 10^{-5}$

- This then determines the amplitude of the inflationary power spectrum $A_S = \Delta_{\mathcal{R}}^2$ in the previous lecture set

# Baryon Loading

- Baryons add extra mass to the photon-baryon fluid

- Controlling parameter is the momentum density ratio:

$$R \equiv \frac{p_b + \rho_b}{p_\gamma + \rho_\gamma} \approx 30\Omega_b h^2 \left(\frac{a}{10^{-3}}\right)$$

  of order unity at recombination

- Momentum density of the joint system is conserved

$$(\rho_\gamma + p_\gamma)v_\gamma + (\rho_b + p_b)v_b \approx (p_\gamma + p_\gamma + \rho_b + \rho_\gamma)v_\gamma$$
$$= (1 + R)(\rho_\gamma + p_\gamma)v_{\gamma b}$$

- Momentum density ratio enters as

$$[(1 + R)v_{\gamma b}]^{\cdot} = k\Theta + (1 + R)k\Psi$$

# New Euler Equation

- Photon continuity remains the same

$$\dot{\Theta} = -\frac{k}{3} v_{\gamma b} - \dot{\Phi}$$

- Modification of oscillator equation

$$\frac{d}{d\eta}[(1+R)\dot{\Theta}] + \frac{1}{3}k^2\Theta = -\frac{1}{3}k^2(1+R)\Psi - \frac{d}{d\eta}[(1+R)\dot{\Phi}]$$

- In a CDM dominated expansion $\dot{\Phi} = \dot{\Psi} = 0$ and the adiabatic approximation where the sound speed evolves slowly
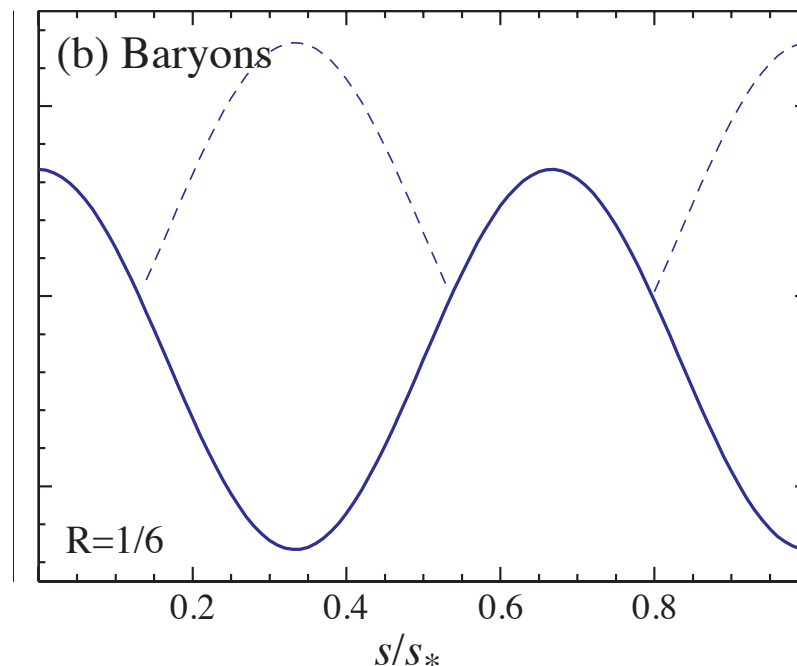
$$c_s = \sqrt{\frac{1}{3}\frac{1}{1+R}}$$

$$[\Theta + (1+R)\Psi](\eta) = [\Theta + (1+R)\Psi](0)\cos(ks)$$

# Baryon Peak Phenomenology

- Photon-baryon ratio enters in three ways

- Overall larger amplitude:

$$[\Theta + (1 + R)\Psi](0) = \frac{1}{3}(1 + 3R)\Psi(0)$$

- Even-odd peak modulation of effective temperature

$$[\Theta + \Psi]_{\text{peaks}} = [\pm(1 + 3R) - 3R]\frac{1}{3}\Psi(0)$$

$$[\Theta + \Psi]_1 - [\Theta + \Psi]_2 = [-6R]\frac{1}{3}\Psi(0)$$

- Shifting of the sound horizon down or $\ell_A$ up

$$\ell_A \propto \sqrt{1 + R}$$

(b) Baryons

R=1/6

$s/s_*$

# Photon Baryon Ratio Evolution

- Actual effects smaller since $R$ evolves

- Oscillator equation has time evolving mass

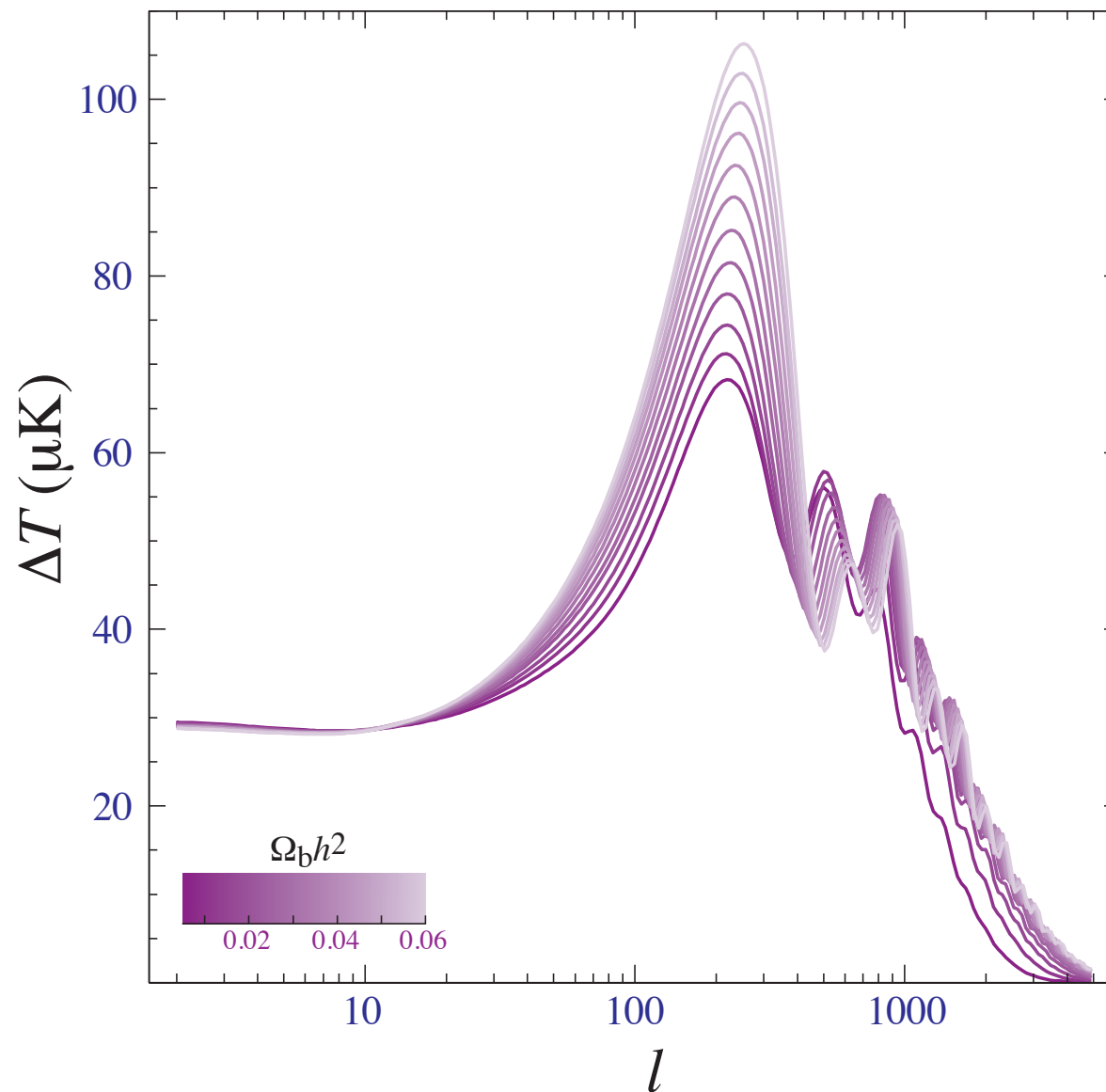$$c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Theta}) + c_s^2 k^2 \Theta = 0$$

- Effective mass is is $m_{\text{eff}} = 3c_s^{-2} = (1+R)$

- Adiabatic invariant

$$\frac{E}{\omega} = \frac{1}{2}m_{\text{eff}}\omega A^2 = \frac{1}{2}3c_s^{-2}kc_s A^2 \propto A^2(1+R)^{1/2} = const.$$

- Amplitude of oscillation $A \propto (1+R)^{-1/4}$ decays adiabatically as the photon-baryon ratio changes

# Baryons in the Power Spectrum

- Relative heights of peaks

# Oscillator: Take Three and a Half

- The not-quite-so simple harmonic oscillator equation is a forced harmonic oscillator

$$c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Theta}) + c_s^2 k^2 \Theta = -\frac{k^2}{3}\Psi - c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Phi})$$

changes in the gravitational potentials alter the form of the acoustic oscillations

- If the forcing term has a temporal structure that is related to the frequency of the oscillation, this becomes a driven harmonic oscillator

- Term involving $\Psi$ is the ordinary gravitational force

- Term involving $\Phi$ involves the $\dot{\Phi}$ term in the continuity equation as a (curvature) perturbation to the scale factor

# Potential Decay

- Matter-to-radiation ratio

$$\frac{\rho_m}{\rho_r} \approx 24\Omega_m h^2 \left(\frac{a}{10^{-3}}\right)$$

  of order unity at recombination in a low $\Omega_m$ universe

- Radiation is not stress free and so impedes the growth of structure
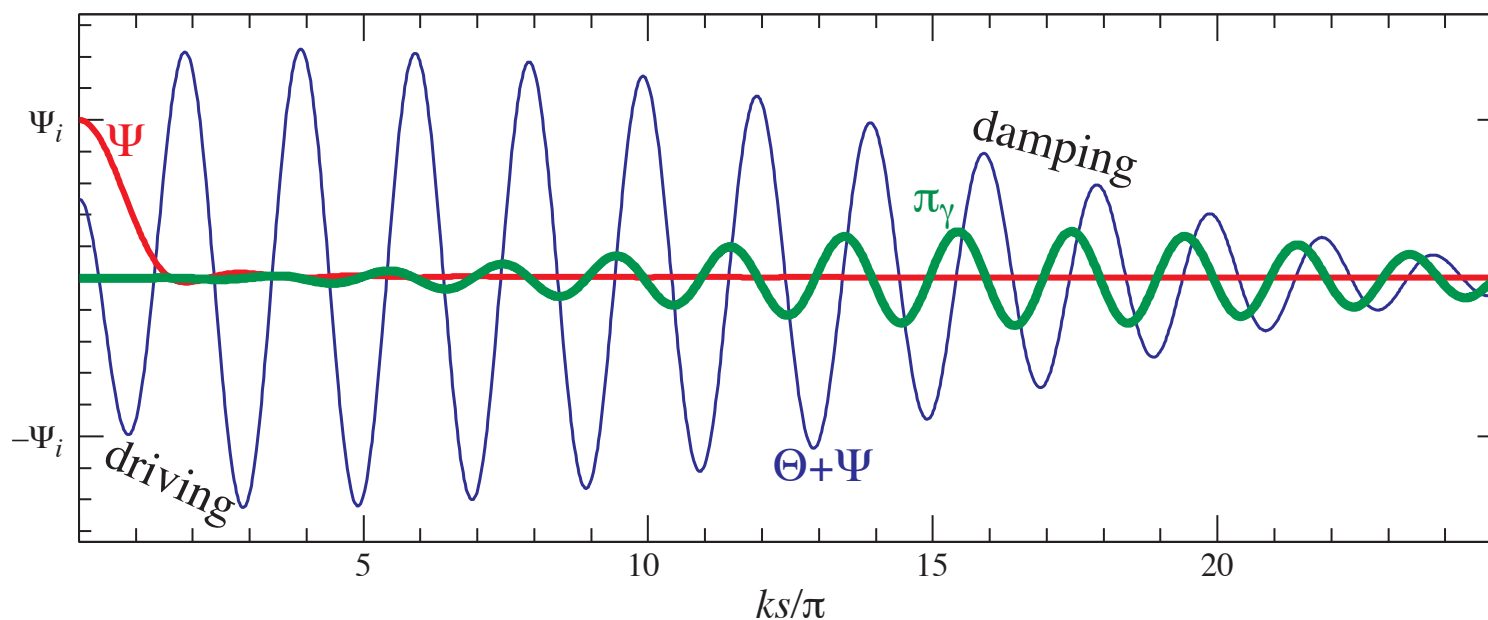
$$k^2\Phi = 4\pi G a^2 \rho_r \Delta_r$$

$\Delta_r \sim 4\Theta$ oscillates around a constant value, $\rho_r \propto a^{-4}$ so the Netwonian curvature decays.

- General rule: potential decays if the dominant energy component has substantial stress fluctuations, i.e. below the generalized sound horizon or Jeans scale

# Radiation Driving

- Decay is timed precisely to drive the oscillator - close to fully coherent

$$|[\Theta + \Psi](\eta)| = |[\Theta + \Psi](0) + \Delta\Psi - \Delta\Phi|$$

$$= |\frac{1}{3}\Psi(0) - 2\Psi(0)| = |\frac{5}{3}\Psi(0)|$$



- $5\times$ the amplitude of the Sachs-Wolfe effect!

# Matter-Radiation in the Power Spectrum

- Coherent approximation is exact for a photon-baryon fluid but reality is reduced to $\sim 4\times$ because neutrino contribution is free streaming not fluid like

- Neutrinos drive the oscillator less efficiently and also slightly change the phase of the oscillation



- Actual initial conditions are $\Theta + \Psi = \Psi/2$ for radiation domination but comparison to matter dominated SW correct

- With 3 peaks, it is possible to solve for both the baryons and dark matter densities, providing a calibration for the sound horizon

- Higher peaks check consistency with assumptions: e.g. extra relativistic d.o.f.s
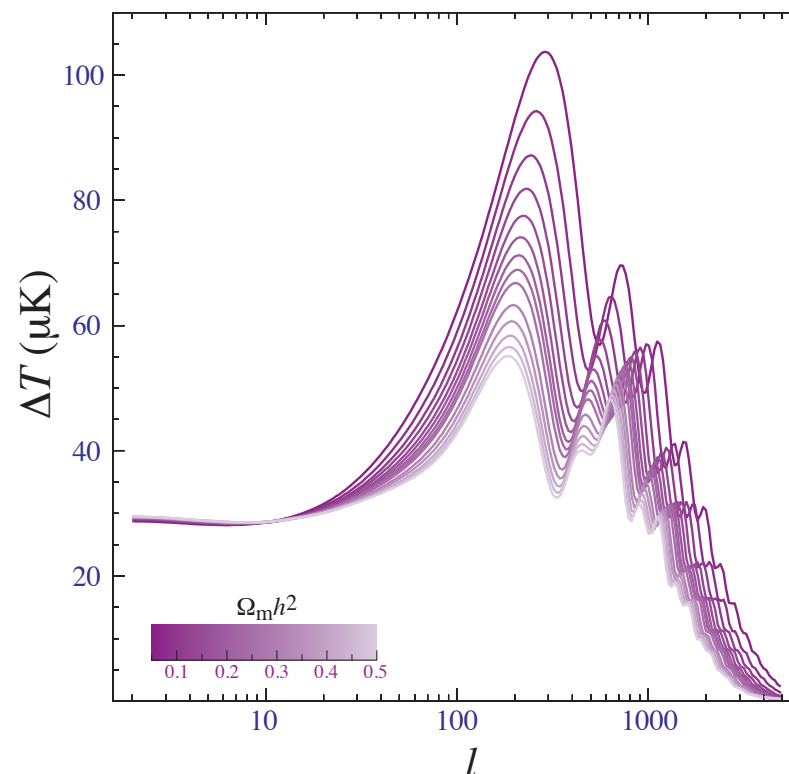
# Damping

- Tight coupling equations assume a perfect fluid: no viscosity, no heat conduction

- Fluid imperfections are related to the mean free path of the photons in the baryons

$$\lambda_C = \dot{\tau}^{-1} \quad \text{where} \quad \dot{\tau} = n_e \sigma_T a$$

  is the conformal opacity to Thomson scattering

- Dissipation is related to the diffusion length: random walk approximation

$$\lambda_D = \sqrt{N}\lambda_C = \sqrt{\eta/\lambda_C}\,\lambda_C = \sqrt{\eta\lambda_C}$$

  the geometric mean between the horizon and mean free path

- $\lambda_D/\eta_* \sim$ few %, so expect the peaks :> 3 to be affected by dissipation

# Near Perfection in 6 Numbers

- All this precision data described by 6 ΛCDM parameters

  - $\Omega_c h^2$: CDM
  - $\Omega_b h^2$: baryons
  - $\theta_s$: sound scale
  - $A_s$: amplitude
  - $n_s$: tilt
  - $\tau$: reionization

- Measured to sub percent precision (except $\tau$)
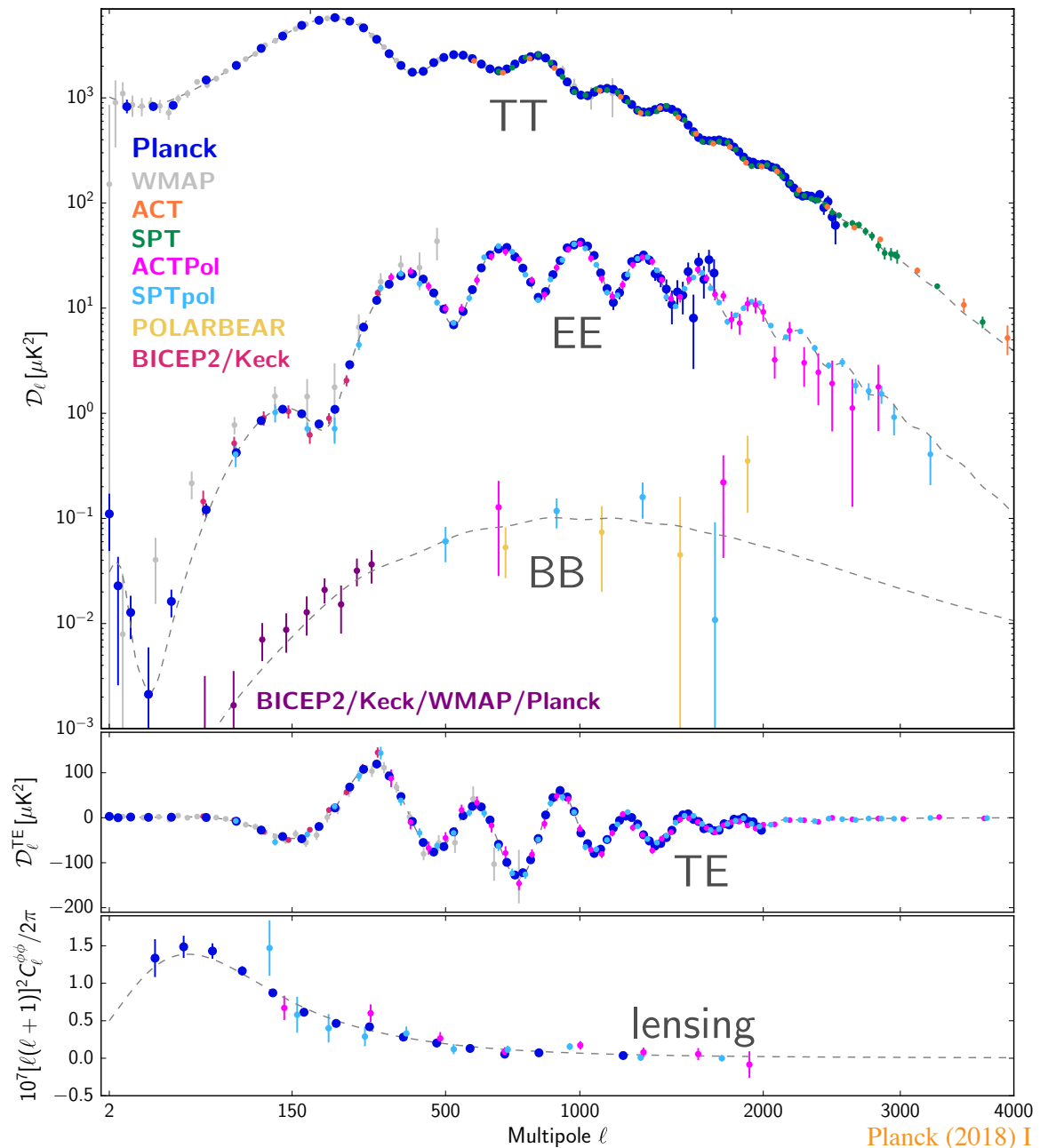
# Predictive Power

- Predicts all other observables, which direct measurements test



BBN

SNIa

BAO

$P(k)$

growth rate

Planck (2018) I,VI
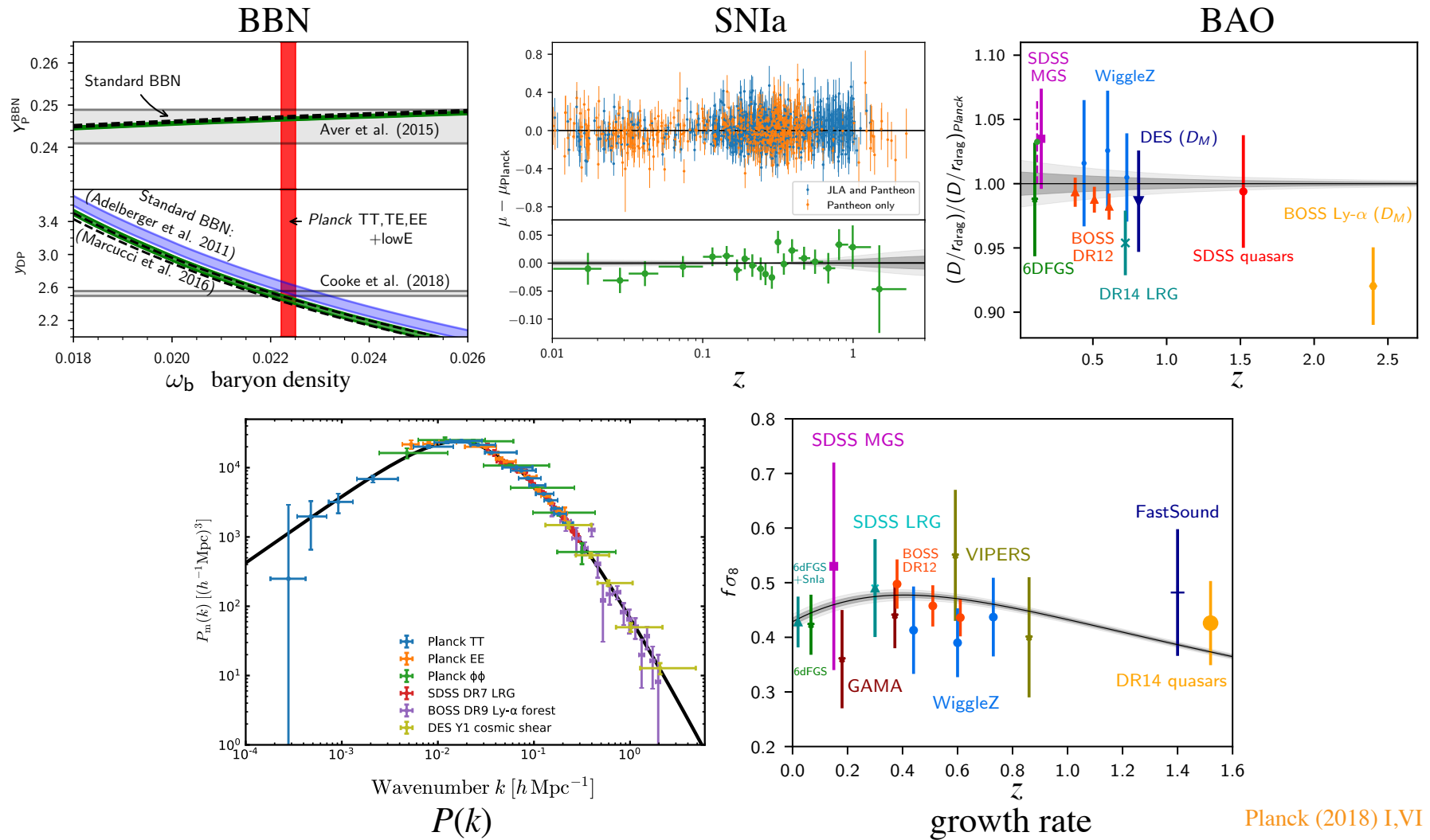
- Good agreement, even weak lensing, clusters, and yes $H_0$ (< 10%)

# Transfer Function

- Transfer function transfers the initial Newtonian curvature to its value today (linear response theory)

$$T(k) = \frac{\Phi(k, a = 1)}{\Phi(k, a_{\mathrm{init}})} \frac{\Phi(k_{\mathrm{norm}}, a_{\mathrm{init}})}{\Phi(k_{\mathrm{norm}}, a = 1)}$$

- Conservation of Bardeen curvature: Newtonian curvature is a constant when stress perturbations are negligible: above the horizon during radiation and dark energy domination, on all scales during matter domination

- When stress fluctuations dominate, perturbations are stabilized by the Jeans mechanism

- Hybrid Poisson equation: Newtonian curvature, comoving density perturbation $\Delta \equiv (\delta\rho/\rho)_{\mathrm{com}}$ implies $\Phi$ decays

$$(k^2 - 3K)\Phi = 4\pi G a^2 \rho \Delta \sim \eta^{-2}\Delta$$

# Transfer Function

- **Freezing** of $\Delta$ stops at $\eta_{\mathrm{eq}}$

$$\Phi \sim (k\eta_{\mathrm{eq}})^{-2}\Delta_H \sim (k\eta_{\mathrm{eq}})^{-2}\Phi_{\mathrm{init}}$$

- Transfer function has a $k^{-2}$ fall-off beyond $k_{\mathrm{eq}} \sim \eta_{\mathrm{eq}}^{-1}$

$$\eta_{\mathrm{eq}} = 15.7(\Omega_m h^2)^{-1}\left(\frac{T}{2.7K}\right)^2 \mathrm{Mpc}$$

- Small correction since growth with a smooth radiation component is logarithmic not frozen

- Transfer function is a direct output of an Einstein-Boltzmann code

# Fitting Function

- Alternately accurate fitting formula exist, e.g. pure CDM form:

$$T(k(q)) = \frac{L(q)}{L(q) + C(q)q^2}$$
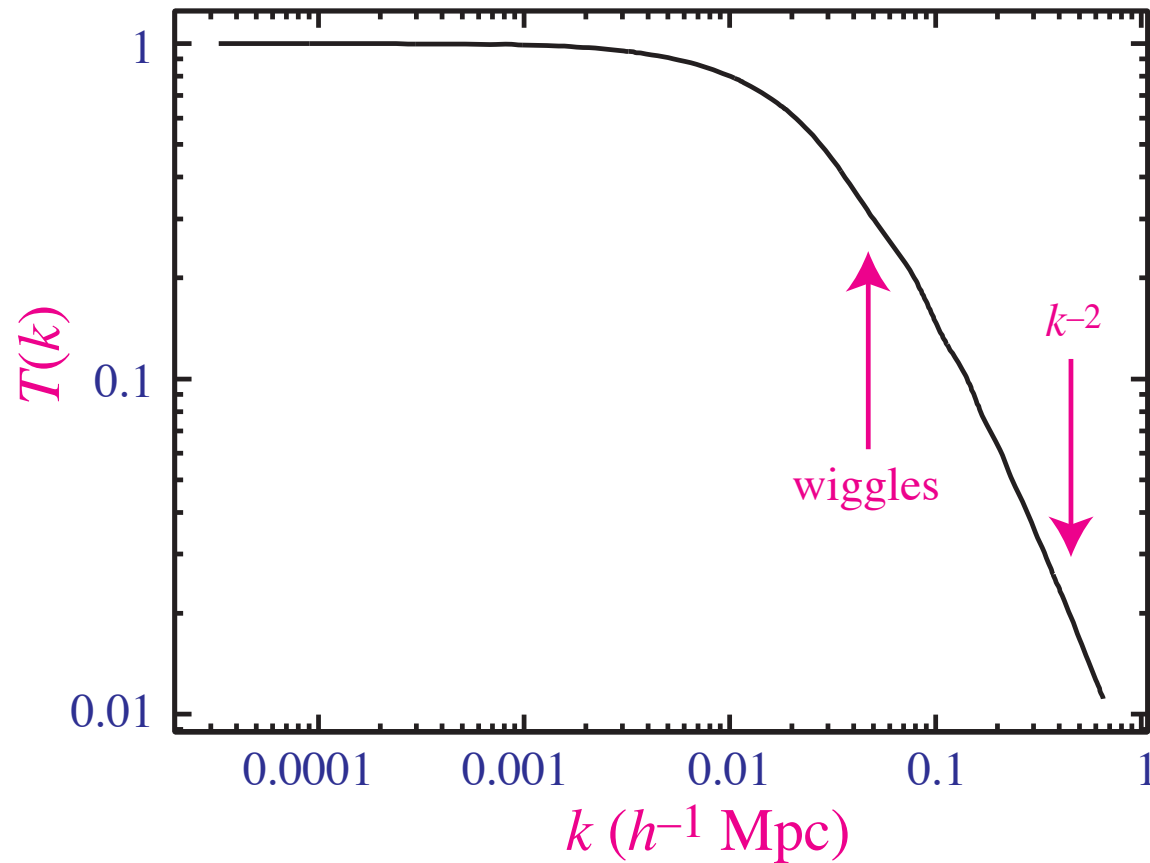
$$L(q) = \ln(e + 1.84q)$$

$$C(q) = 14.4 + \frac{325}{1 + 60.5q^{1.11}}$$

$$q = k/\Omega_m h^2 \mathrm{Mpc}^{-1}(T_{\mathrm{CMB}}/2.7K)^2$$

- In $h\,\mathrm{Mpc}^{-1}$, the critical scale depends on $\Gamma \equiv \Omega_m h$ also known as the shape parameter

# Transfer Function

- Numerical calculation

# Baryon Acoustic Oscillations

- Baryons caught up in the acoustic oscillations of the CMB and impart acoustic oscillations to the transfer function. Density enhancements are produced kinematically through the continuity equation $\delta_b \sim (k\eta)v_b$ and hence are out of phase with CMB temperature peaks

- Dissipation of the acoustic oscillations eliminates both the CMB and baryon perturbations – known as Silk damping for the baryons. This suppression and the general fact that baryons are caught up with photons was one of the main arguments for CDM

- Detected first in the SDSS LRG survey.

- An excellent standard ruler for angular diameter distance $D_A(z)$ since it does not evolve in redshift in linear theory

- Radial extent of BAO gives $H(z)$

# Power Spectrum

- SDSS data
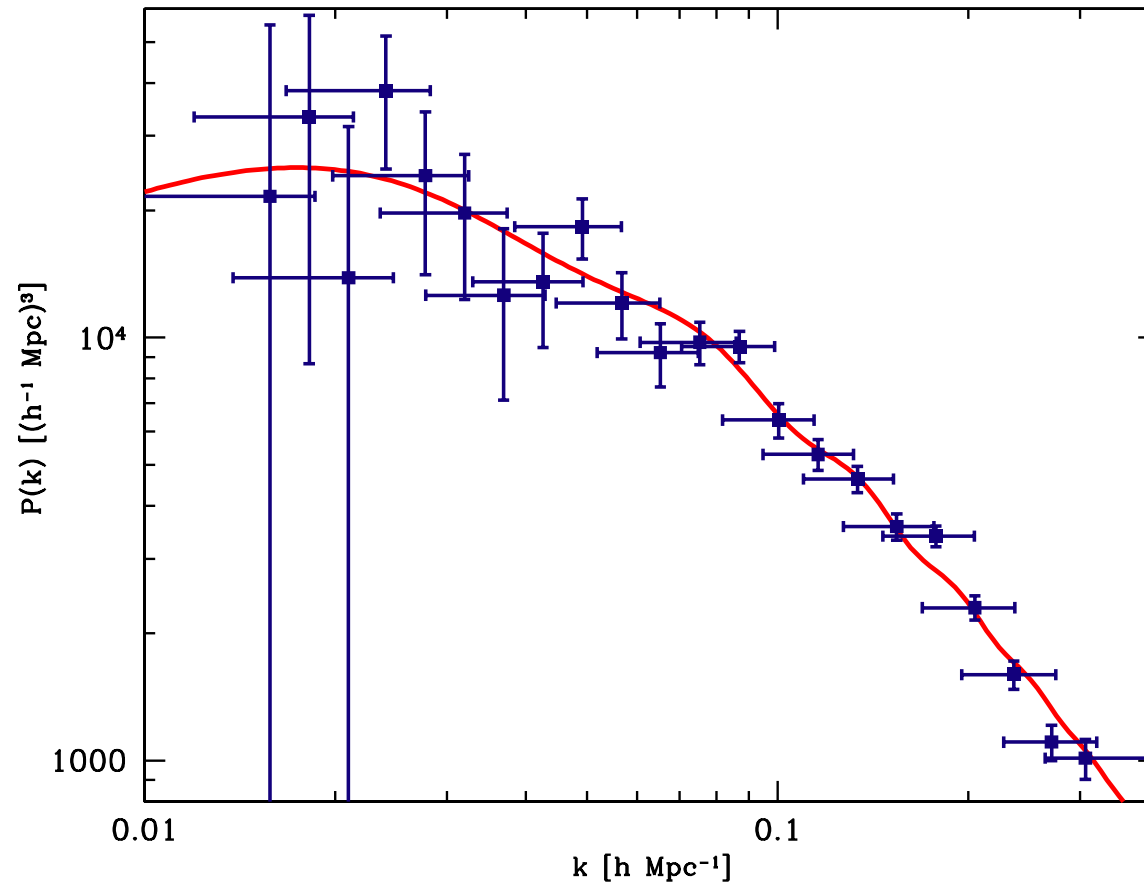


- Power spectrum defines large scale structure observables: galaxy clustering, velocity field, Ly$\alpha$ forest clustering, cosmic shear