1 Predict the Temperature of the CMB

Deuterium ($D$) is formed by the reaction

$$n + p \leftrightarrow D + \gamma$$

Recall that big bang nucleosynthesis occurs when the universe was about $t \sim 3$ minutes old and at a temperature of $T = 10^9$K. To have an observable abundance of deuterium today, this reaction must have fallen out of equilibrium at about the time of big bang nucleosynthesis. If it happened much earlier there would be no light elements at all, if it happened much later then all of the deuterium would be burned into helium and heavier elements.

1. Given the thermally averaged cross section for this process is $\langle \sigma v \rangle = 4.6 \times 10^{-20}$ cm$^3$ s$^{-1}$, estimate the baryon number density at big bang nucleosynthesis. (Hint: remember how age $t$ and expansion rate $H$ are related - you may drop order unity factors.)

2. If $\Omega_b h^2 = 0.02$, what is the baryon number density today. Since baryons are conserved what is the scale factor at big bang nucleosynthesis.

3. From the knowledge that big bang nucleosynthesis occurs when the photons have a temperature of $10^9$K, predict the temperature of the CMB today. How close is your prediction to the measured value of 2.725K?

2 Helium Abundance (Ryden 9.1,9.4 and followup)

1. Assuming all available neutrons are incorporated into $^4$He nuclei at a time associated with helium formation shortly after deuterium formation (assume $t \approx 4$ minutes), write down an expression for the Helium mass fraction $Y_{He}$ in terms of the neutron to proton number density fraction $f = n_n/n_p$ at that time.

2. Suppose the neutron decay time were $\tau_n = 88$ s instead of the correct decay time $\tau_n = 880$ s, with all other physical parameters unchanged. Estimate $Y_{He}$ taking into account neutron decay between freezeout and helium formation.

3. Use the correct decay time estimate of $Y_{He}$ and $n_b$ today in the first problem to determine the number density of hydrogen today $n_H$ assuming that the universe is now composed of mostly hydrogen and helium.