1 Lyth Bound

Recall from class that the tensor to scalar ratio $r$ is related to the slow roll parameter $\epsilon_H$ as $r = 16\epsilon_H$ and

$$\epsilon_H = 4\pi G \frac{(d\phi/dt)^2}{H^2}$$  \hspace{1cm} (1)

(1) Estimate $\Delta \phi$, i.e. how far $\phi$ must have rolled in $N = \Delta \ln a$ e-folds of the expansion as a function of $r$ and express your answer in units of the reduced Planck mass $M_{\text{pl}} = 1/\sqrt{8\pi G}$. (2) Recall that inflation is supposed to last for $N \sim 60$ e-folds. If $r \sim 10^{-1}$, near the current upper limits, does the roll exceed a Planck mass range $\Delta \phi > M_{\text{pl}}$ and therefore violate the so called Lyth bound?

2 $m^2 \phi^2$ Inflation

Suppose inflation was described by a simple mass term $m$ so that the potential $V = m^2 \phi^2 / 2$. Recall that in a friction dominated roll $3Hd\phi/dt = -V'$ and $H^2 \approx 8\pi GV/3$. (1) Give the expression for $\epsilon_H(\phi)$. Where is the field in $M_{\text{pl}}$ units when inflation ends ($\epsilon_H = 1$)? (2) In a friction dominated roll

$$\delta_1 = \epsilon_H - \frac{1}{8\pi G} \frac{V''}{V}$$  \hspace{1cm} (2)

Give the expression for $\delta_1(\phi)$. (3) Give the scalar tilt $n_s(\phi) - 1$ (see notes for the slow-roll formula). The observed value of the tilt in the CMB is $n_s - 1 \approx -0.04$. What is the field value that corresponds to CMB scales? Is the Lyth bound violated given that the inflaton must roll between these two field points in (1) and (3)?