1 Sachs-Wolfe Effect and Inflationary Normalization

The COBE satellite in 1992 first measured the anisotropy of the cosmic microwave background to have an rms value of $\Delta T/T \approx 10^{-5}$ on $\sim 10^\circ$ angular scales. Recall from the notes that the Sachs Wolfe effect says $\Delta T/T = \Psi/3 = -R/5$ in a matter dominated universe due to the relationship between $\delta a/a$, $\delta t/t$, the Newtonian potential $\Psi = -\Phi$ and the inflationary curvature perturbation $\mathcal{R}$. Suppose instead that recombination and the anisotropy observation were during an epoch where the equation of state were a constant but arbitrary $w$. Generalize the inflationary implications for the temperature perturbations for an arbitrary $w$.

1. What is the relationship between cosmic time $t$ and scale factor $a$ for an arbitrary $w$? Use this to derive the relationship between temporal fluctuations $\delta t/t$ and scale factor fluctuations $\delta a/a$.

2. The Sachs-Wolfe effect relates the temporal fluctuation and Newtonian potential as $\Psi = \delta t/t$ and to the temperature anisotropy as $\Delta T/T = -\delta a/a + \Psi$. Eliminate $\delta a/a$ and provide the general relationship between $\Delta T/T$ and $\Psi$.

3. Assuming that $\Phi = -\Psi$ take the general relationship of $\Phi$ to the inflationary curvature perturbation

$$\mathcal{R} = \frac{5 + 3w}{3 + 3w} \Phi$$

(1)

to infer $A_S = \langle \mathcal{R}^2 \rangle$ from the COBE measurement for a general $w$. Evaluate it for matter domination and radiation domination.

2 Acoustic Scale

The acoustic scale $\theta_*$ is the angular scale that the sound horizon $s = \int_0^{\eta_*(z_*)} d\eta c_s$ at recombination $z_*$ subtends on the sky today

$$\theta_* = \frac{s}{D_A(z_*)}$$

(2)

Use the general definition of the sound speed $c_s^2 = \delta p/\delta \rho = \dot{p}/\dot{\rho}$ to derive the dependence of $c_s$ on the baryon-photon momentum density ratio $R = 3\rho_b/4\rho_\gamma$. Discuss whether the multipole moment of the first peak would increase or decrease if $R$ were raised but all else were equal.