Astro 243: Problem Set 7

1 Sachs-Wolfe Effect and Inflationary Normalization

The COBE satellite in 1992 first measured the anisotropy of the cosmic microwave background to have an rms value of $\Delta T/T \approx 10^{-5}$ on $\sim 10^\circ$ angular scales. Recall from the notes that the Sachs Wolfe effect says $\Delta T/T = \Psi/3 = -\mathcal{R}/5$ in a matter dominated universe due to the relationship between $\delta a/a$, $\delta t/t$, the Newtonian potential $\Psi = -\Phi$ and the inflationary curvature perturbation $\mathcal{R}$. Suppose instead that recombination and the anisotropy observation were during an epoch where the equation of state were a constant but arbitrary w. Generalize the inflationary implications for the temperature perturbations for an arbitrary w.

1. What is the relationship between cosmic time t and scale factor a for an arbitrary w? Use this to derive the relationship between temporal fluctuations $\delta t/t$ and scale factor fluctuations $\delta a/a$.

2. The Sachs-Wolfe effect relates the temporal fluctuation and Newtonian potential as $\Psi = \delta t/t$ and to the temperature anisotropy as $\Delta T/T = -\delta a/a + \Psi$. Eliminate $\delta a/a$ and provide the general relationship between $\Delta T/T$ and Ψ.

3. Assuming that $\Phi = -\Psi$ take the general relationship of Φ to the inflationary curvature perturbation

$$\mathcal{R} = \frac{5+3w}{3+3w}\Phi \tag{1}$$

to infer $A_S = \langle \mathcal{R}^2 \rangle$ from the COBE measurement for a general w. Evaluate it for matter domination and radiation domination.

2 Acoustic Scale

The acoustic scale θ_* is the angular scale that the sound horizon $s = \int_0^{\eta(z_*)} d\eta c_s$ at recombination z_* subtends on the sky today

$$\theta_* = \frac{s}{D_A(z_*)} \tag{2}$$

Use the general definition of the sound speed $c_s^2 = \delta p/\delta\rho = \dot{p}/\dot{\rho}$ to derive the dependence of c_s on the baryon-photon momentum density ratio $R = 3\rho_b/4\rho_\gamma$. Discuss whether the multipole moment of the first peak would increase or decrease if R were raised but all else were equal.