Astro 243: Problem Set 8

1 Separate Universe Evolution

The spherical collapse of a linear density perturbation can be considered as the background FRW evolution of a universe that began with a slightly higher density. This technique is also useful for calculating higher order non-Gaussian effects and in that context is called the separate universe technique. The separate universe carries its own version of cosmological parameters and its own scale factor we will denote these with a tilde.

Derive this mapping in the regime where the density fluctuation $\delta(a) \ll 1$ is still small and the background universe is flat and matter dominated $\rho = \rho_m$.

1. The first step is to absorb the perturbation δ into a new background density $\rho(1+\delta) = \tilde{\rho}$. Since $\delta(a) \propto a$ it is negligible as $a \to 0$. Defining the scale factor to also coincide at that epoch $\lim_{a\to 0} \tilde{a} = a$ show that the matter density parameters are equal

$$\Omega_m H_0^2 = \tilde{\Omega}_m \tilde{H}_0^2 \tag{1}$$

2. Although the matter density parameters are equal, the matter density at a fixed time (rather than fixed scale factor) are not. Show that the relationship between the two scale factors is

$$\tilde{a} \approx a\left(1 - \frac{\delta}{3}\right) \tag{2}$$

3. The Hubble parameter in the separate universe is defined as $\tilde{H} = d\ln\tilde{a}/dt$. Using $\delta \propto a$ show that

$$\tilde{H} \approx H(1 - \frac{1}{3}\delta) \tag{3}$$

4. Use the Friedmann equation in the separate universe to express its Hubble parameter as

$$\tilde{H}^2 \approx H^2(1+\delta) + \frac{\tilde{H}_0^2 - H_0^2}{a^2} \tag{4}$$

argue that even though $\Omega_m = 1$ it is not in the separate universe. What does the $1 - \tilde{\Omega}_m$ component represent?

5. Equate the two relationships for $\tilde{H}$ in (3) and (4) at $a = 1$ to find

$$\tilde{H}_0 \approx H_0\left[1 - \frac{5}{6}\delta(a=1)\right] \tag{5}$$

What is $\tilde{\Omega}_m$ as a function of $\delta(a=1)$?