Ast 243:

Cosmological Physics

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Course text book: Ryden, Introduction to Cosmology, 2nd edition
• Olber’s paradox, expansion of the universe: Ch 2
• Cosmic geometry, expansion rate, acceleration: Ch 3,6
• Cosmic dynamics and composition: Ch 4,5
• Dark matter and dark energy: Ch 5,7
• Hot big bang and origin of species: Ch 9
• Inflation: Ch 10
• Cosmic microwave background: Ch 8
• Gravitational instability and structure formation: Ch 11,12

Final (30%), reading project format

HW (70% TA Meng-Xiang Lin, mxlin@uchicago.edu)
Set 1:
Expansion of the Universe
Observables

- Most cosmological inferences are based on interpreting the radiation we receive on Earth from astrophysical objects: stars, galaxies, clusters of galaxies, cosmic microwave background...

- Largely electromagnetic radiation but now also neutrinos, cosmic rays, and most recently, gravitational waves

- How do we convert this basic observable into a 3D model?

- Basic observable properties of radiation (recall radiative processes course)
Sky Maps

- Map radiation from directions on the sky
- Background: stars in our Milky Way galaxy
- Color overlay, furthest source: microwave background
Observables

- Energy flux $F$ given a luminosity $L$ of the source $L = \Delta E/\Delta t$ (energy/time)

  $$F = \frac{\Delta E}{\Delta t \Delta A} = \frac{L}{4\pi r^2}$$

- Astro units: often in cgs erg s$^{-1}$ cm$^{-2}$ (mks W m$^{-2}$)

- Energy conservation says rate of energy passing through the shell at $r_1$ must be the same as $r_2$

- Thus flux decreases as $1/r^2$ from the source

  $$F(r_1)4\pi r_1^2 = F(r_2)4\pi r_2^2$$

  $$F \propto r^{-2}$$
From 2D to 3D

- So even without knowing the luminosity of a set of standard objects, we can judge relative distance from flux:
  \[ \frac{r_2}{r_1} = \left( \frac{F_1}{F_2} \right)^{1/2} \]

- This is the idea of a standard candle, star of the same type, supernovae etc.

- Aside: certain variable stars called Cepheids are excellent standard candles – pulsation period is linked to the luminosity so we can pick out objects of the same luminosity and use the measured flux to put place their relative position on a map.

- 2D map becomes a 3D map and we can start to talk about the physical structure of the universe.

- We can calibrate the absolute distance to the nearby ones by other methods, ultimately parallax - change in angle on the sky as earth orbits the sun.
Cosmological Units

- Astro and cosmo units often look bizarre at first sight - however they are useful in that they tell you something about the observation behind the inference.

- Ground based optical telescopes are limited by the atmosphere to $\sim 1$ arcsec resolution - so direct parallax measurements before satellites were limited in distance of objects whose parallax is measurable.

- Parsec (parallax arcsec) is the distance at which the parallax of an object would give an angular displacement of 1''.

![Diagram showing the concept of parsec and AU](image)
Cosmological Units

- Parsec is a typical distance between stars in the galaxy but cosmologists favorite unit is the megaparsec

\[ 1 \text{Mpc} = 3.0856 \times 10^{24} \text{cm} \]

because it is the typical separation between galaxies

- In other astrophysical contexts, use length scales appropriate to the system - Mpc is based on the AU - earth-sun distance
  - 1 AU subtends 1 arcsec at 1 pc – parallax to close objects allows us to convert relative distance to absolute distance
  - 1 pc: nearest stars, 1 kpc distances in the galaxy, 1 Mpc distance between galaxies, 1 Gpc distances across observable universe
Received Light: Received History

- Because of the finite light travel time, light from distant objects is emitted at earlier times so sky maps become snapshots of the history of the universe

\[ \Delta t = \frac{\Delta x}{c} \]

- Speed of light can be thought of as a conversion factor between time and distance (see problem set)

- In this class we will often measure time and space in the same units, i.e. we set \( c = 1 \) and measure time in Mpc or distance in (light) years

- We shall see that the earliest light we can measure (from the most distant sources) comes from the cosmic microwave background, the afterglow of the big bang - many Gpc or Gyr away
Observables: Surface Brightness

- If the 2D map resolves the object in question, we can do better: measure surface brightness
- Direction: colimate in an acceptance angle $d\Omega$ normal to $dA \rightarrow$ surface brightness

$$S(\Omega) = \frac{\Delta E}{\Delta t \Delta A \Delta \Omega}$$

- Units: for example in cgs, erg s$^{-1}$ cm$^{-2}$ sr$^{-1}$ or mks, W m$^{-2}$ sr$^{-1}$
Surface Brightness Conservation

- Surface brightness $\Delta \Omega = (\lambda/d)^2$ for a region with fixed size $\lambda$

$$S = \frac{F}{\Delta \Omega} = \frac{L}{4\pi d^2} \frac{d^2}{\lambda^2}$$

- In a non-expanding geometry, these two distances cancel

$$S = \text{const.}$$

- Aside: since $S = L/4\pi \lambda^2$, astro/cosmo units for galactic scale objects are also often quoted in $L_\odot/\text{pc}^2$
Olber’s Paradox

• Surface brightness of an object is independent of its distance

• So since each site line in universe full of stars will eventually end on surface of star

• Olber’s Paradox: why isn’t night sky as bright as sun (not infinite)

• We shall see that the resolution lies in the expansion of the universe:
  – finite “horizon” distance for the observable universe
  – and that the two distance factors don’t cancel

• To understand the expansion, we need not just a map of the universe but a measurement of motion
Observables: Redshift

• We can also measure the frequency of radiation from objects

• If emission contains atomic lines with a natural rest frequency $\nu_{\text{rest}}$ we can measure the redshift or velocity by the ratio of observed to rest frequency

$$1 + z = \frac{\nu_{\text{rest}}}{\nu_{\text{obs}}} = \frac{1 + v/c}{\sqrt{1 - v^2/c^2}}$$

and for $v \ll c$, $z = v/c$

• Here $v$ is the recession velocity - i.e. light is shifted to the red if the object is receding from us

• In this class we’ll often use units where $c = \hbar = 1$ which means time and length are energy are measured in the same units

• If units don’t make sense add $c, \hbar$ until they do (see problem set)!
Expansion of the Universe

- Now let’s put together these observational tools.
- Given a standard candle with known luminosity $L$, we measure its distance $d$ away from us from the measured flux $F = L / 4\pi d^2$.
- Given atomic line transitions, we measure the redshift $z$ or equivalently the recession velocity $v$.
- Hubble then plotted out recession velocity as a function of distance....
Hubble Law

- Hubble in 1929 used the Cepheid period luminosity relation to infer distances to nearby galaxies thereby discovering the expansion of the universe.

- Hubble actually inferred too large a Hubble constant of $H_0 \sim 500 \text{km/s/Mpc}$ due to a miscalibration of the Cepheid distance scale.

- $H_0$ now measured as $74.03 \pm 1.42 \text{km/s/Mpc}$ by combining a suite of distance measurements [arXiv:1903.07603]
Fundamental Properties

- Hubble law: objects are receding from us at a velocity that is proportional to their distance
- Universe is highly isotropic at sufficiently large distances
- Universe is homogeneous on large scales
- Let’s see why the first property, along with the implications of the second two that we are not in a special position, imply the universe is expanding

- The Hubble law sounds much like we are at the center of an explosion outwards but that would violate homogeneity and put us in a special place
- To be consistent with both, we posit space itself is expanding...
Expansion of the Universe

- Consider a 1 dimensional expansion traced out by galaxies

\[ \Delta x = 2a \quad a \quad \Delta x/\Delta t = H_0 d \]

- From the perspective of the central galaxy the others are receding with a velocity proportional to distance

- Proportionality constant is called the *Hubble Constant* \( H_0 \)

- Each observer in the expansion will see the same relative recession of galaxies
Expansion of the Universe

• Generalizes to a three dimensional expansion. Consider the observer at the origin and two galaxies at position \(d_A\) and \(d_B\).

• Recession velocities according to the observer

\[
v_A = H_0 d_A, \quad v_B = H_0 d_B
\]

• According to galaxy \(B\), the recession velocity of galaxy \(A\) is

\[
v_B - v_A = H_0 d_B - H_0 d_A = H_0 d_{AB}
\]

so that \(B\) will see the same expansion rate as the observer at the origin given the linearity of Hubble’s law.

• Hubble’s law is best thought of as an expansion of space itself, with galaxies carried along the “Hubble flow”.
Olber’s Paradox Redux

• In an expanding universe Olber’s paradox is resolved

• First piece: *age finite* so there is a finite distance light can travel called the horizon distance - even if stars exist in the early universe, not all site lines end on stars

• But even as *age* goes to infinity and the number of site lines goes to 100%, *surface brightness* of distant objects (of fixed physical size) goes to *zero*
  – Angular size increases
  – Redshift of energy and arrival time

we’ll see in the next set of lectures

\[ S \propto (1 + z)^{-4} \]
Supernovae as Standard Candles

- Type 1A supernovae are white dwarfs that reach Chandrashekar mass where electron degeneracy pressure can no longer support the star, hence a very regular explosion.

- Moreover, the scatter in luminosity (absolute magnitude) is correlated with the shape of the light curve - the rate of decline from peak light, empirical “Phillips relation”.

- Higher $^{56}$Ni, brighter SN, higher opacity, longer light curve duration.
Beyond Hubble’s Law

- Type 1A are therefore “standardizable” candles leading to a very low scatter $\delta m \sim 0.15$ and visible out to high redshift $z \sim 1$

- Two groups in 1999 found that SN more distant at a given redshift than expected

- Cosmic acceleration discovery won the 2011 Nobel Prize in Physics

- Requires more on cosmic geometry to understand...