Set 4:

Cosmic Composition

Dark Energy

- Distance redshift relation depends on energy density components

$$H_0 D(z) = \int dz \frac{H_0}{H(a)}$$

- SN dimmer, distance further than in a matter dominated epoch
- Hence $H(a)$ must be smaller than expected in a matter only $w_c = 0$ universe where it increases as $(1+z)^{3/2}$

$$H_0 D(z) = \int dz e^{\int d\ln a \, \frac{3}{2}(1+w_c(a))}$$

- Distant supernova Ia as standard candles imply that $w_c < -1/3$ so that the expansion is accelerating

Cosmological Constant

- Consistent with a dark energy density that is constant as the universe expands
- $\Omega_\Lambda \approx 0.7$ in $w = -1$, with high precision after modeling out other other components measured independently
- Cosmological constant problem: why is there this constant energy density, extremely small compared to natural particle physics scales?
- Coincidence problem: different components of matter scale differently with a. Why are two components comparable today?

Vacuum Energy

- Vacuum energy provides a potential candidate for a cosmological constant
- In QFT, like the simple harmonic oscillator in ordinary quantum mechanics, there is a zero point energy to the ground state
- For bosons, $\hbar\omega/2 = E(q)/2$, so the most naive version of the cosmological constant problem is that integrating over all momentum states q with $E \approx q$ leads to $\rho \propto M^4$ where $M = M_{\rm Pl} = 1/\sqrt{8\pi G}$ if the theory applies out to the Planck scale
- The critical energy density $\rho_c = 3H_0^2/8\pi G \approx 8 \times 10^{-47} h^2 {\rm GeV}^4$ is more than 10^{120} off $M_{\rm Pl}^4 \approx 2 \times 10^{76}$ GeV^4.
- Note that $p_{\rm vac} \approx \rho_{\rm vac}/3$ so this fixed momentum cutoff calculation is a bit too naive since we know that $p_{\rm vac} = -\rho_{\rm vac}$

Vacuum Energy

- For advanced students: Lorentz invariant renormalization scheme corrects this to

$$\rho_{\rm vac} = \frac{m^4}{64\pi^2} \ln(m^2/\mu^2)$$

  where μ is some renormalization scale

- But even if there are no mass states above the known standard model bosons, e.g. Higgs boson of $m \approx 125$GeV, this is way off, even though it helps by some 68 orders of magnitude!
- Caveat: fermions contribute negatively to the vacuum energy so if supersymmetry is unbroken would cancel
- But supersymmetry is clearly broken at low energies and has yet to be seen at LHC - with this lower limit $m > 1$TeV, m^4 is still 60 orders of magnitude off.

Alternatives to Λ

- Alternatives to Λ attempt to have an energy density that is much like vacuum energy in that it remains constant as the universe expands
- For example the potential energy of a field stuck on a hill stays the same even as a grows
- But a field ϕ on a hill can roll converting some of the potential energy V to kinetic energy which does decay with a and make $w > -1$
- Quadratic approximation around minimum of potential: $V = m^2\phi^2/2$ but $m \ll H \sim 10^{-33}$ eV.
- Such a field would be a scalar (number at each spacetime position) we'll come back to this possibility in the early universe: inflation

Alternatives to Λ

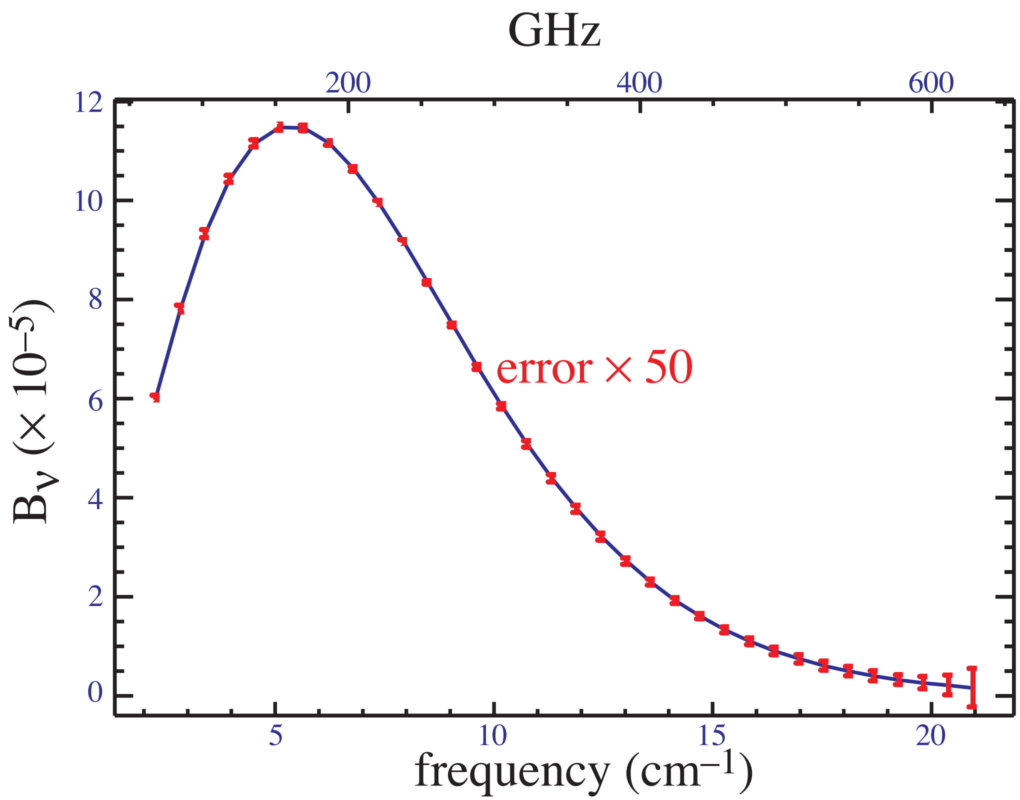
- A light scalar could potentially have exotic long range interactions and coupling to ordinary matter
- Such fifth forces and other modifications to Einstein gravity could also be the explanation of cosmic acceleration
- Very active and ongoing topic of research

Cosmic Microwave Radiation

- If we think instead of number rather than energy density, the dominant component is the CMB
- Existence of a $\sim$ 10K radiation background predicted by Gamow and Alpher in 1948 based on the formation of light elements in a hot big bang (BBN)
- Peebles, Dicke, Wilkinson & Roll in 1965 independently predicted this background and proceeded to build instrument to detect it
- Penzias & Wilson 1965 found unexplained excess isotropic noise in a communications antennae and learning of the Peebles et al calculation announced the discovery of the blackbody radiation
- Thermal radiation proves that the universe began in a hot dense state when matter and radiation was in equilibrium - ruling out a competing steady state theory

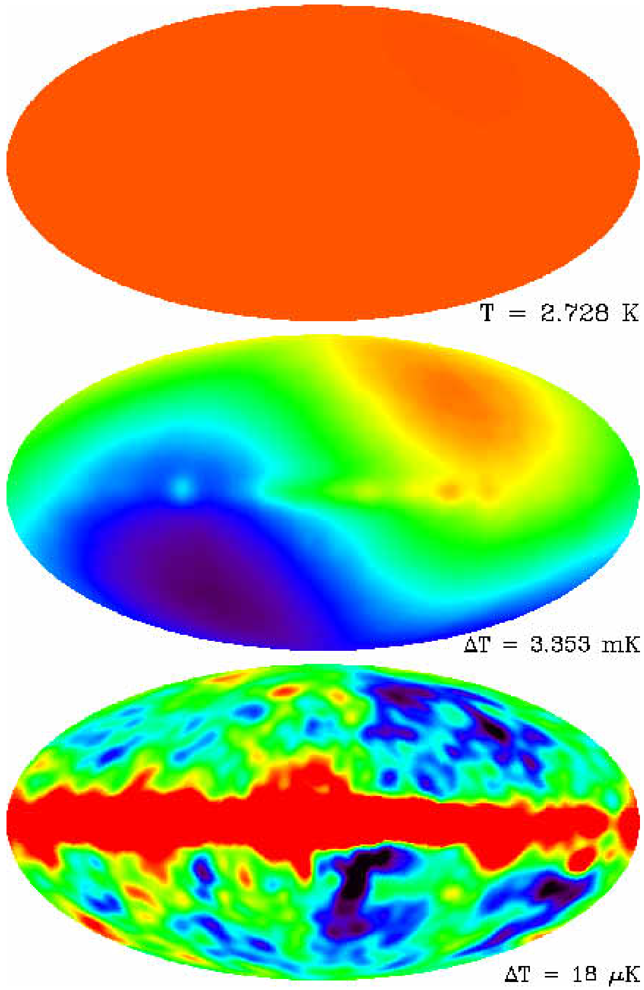
Cosmic Microwave Radiation

- Modern measurement from COBE satellite of blackbody spectrum. $T = 2.725\text{K}$ giving $\Omega_\gamma h^2 = 2.471 \times 10^{-5}$



Cosmic Microwave Radiation

- Radiation is isotropic to 10^{-5} in temperature $\rightarrow$ horizon problem



Total Radiation

- Adding in neutrinos to the radiation gives the total radiation (next lecture set) content as $\Omega_r h^2 = 4.153 \times 10^{-5}$
- Since radiation redshifts faster than matter by one factor of $1+z$ even this small radiation content will dominate the total energy density at sufficiently high redshift
- Matter-radiation equality

$$1 + z_{\rm eq} = \frac{\Omega_m h^2}{\Omega_r h^2}$$

$$1 + z_{\rm eq} = 3130 \frac{\Omega_m h^2}{0.13}$$

- Equivalently the temperature of CMB increases as $(1+z)$ leading to the hot big bang model

Dark Matter

- Since Zwicky in the 1930's non-luminous or dark matter has been known to dominate over luminous matter in stars (and hot gas)
- Arguments based on internal motion holding system up against gravitational force
- Equilibrium requires a balance pressure of internal motions

  rotation velocity of spiral galaxies

  velocity dispersion of galaxies in clusters

  gas pressure or thermal motion in clusters

  radiation pressure in CMB acoustic oscillations
- On dimensional grounds alone we would expect $M \approx v^2 r/G$ for bound systems and to some extent this dimensional reasoning applies to the final evidence using gravitational lensing where $v = c$ but orbits are unbound (with small deflections)

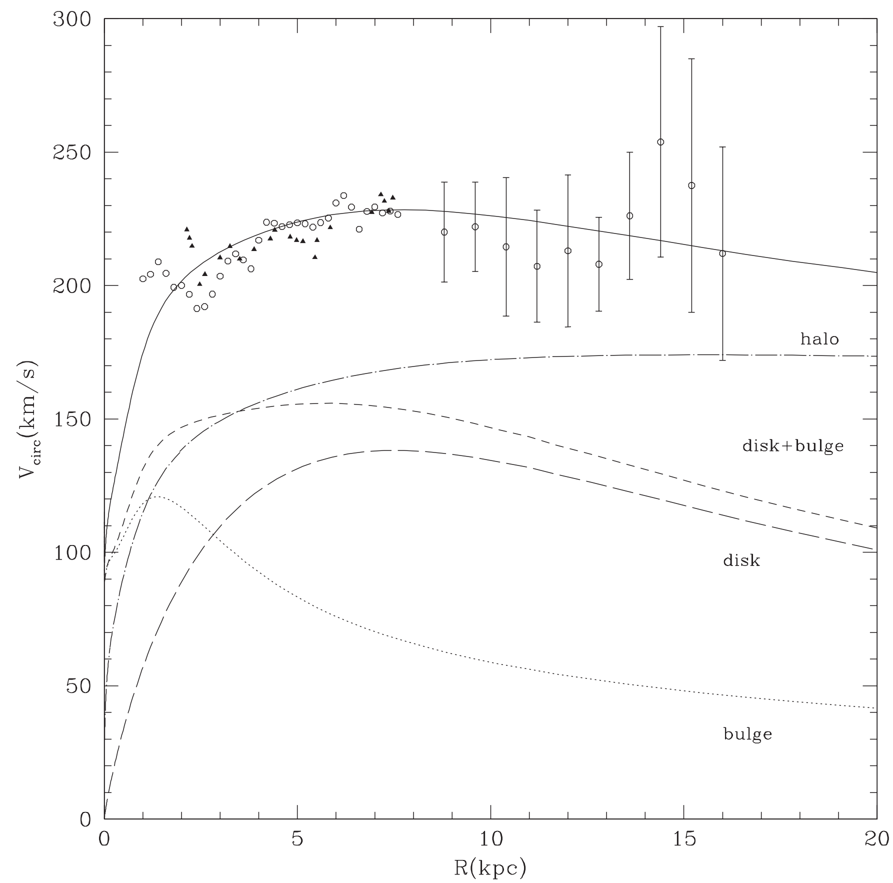
Dark Matter: Rotation Curves

- Flat rotation curves:

$$GM/r^2 \approx v^2/r$$
$$M \approx v^2 r/G$$

  so $M \propto r$ out to tens of kpc

- Dark mass required to keep rotation curves flat much larger than implied by stars and gas $M/L > 30h(M_\odot/L_\odot)$



- Given M/L, if Milky Way is typical and rotation curves are flat out to 1Mpc then dark matter approaches critical density
- Rubin & Ford showed that it is not just the Milky Way that has evidence for dark matter but all spiral galaxies with disks

Rotation Curves

- Also consistent with the NFW profile predicted by cold dark matter (e.g. weakly interacting massive particles or WIMPs)

$$\rho(r) = \frac{\rho_0}{(r/a)(1 + r/a)^2}$$

  with possible modification in the central regions which are dominated by ordinary matter

- Notice increase in density $\rho \propto r^{-1}$ at center – high density makes the central regions of our galaxy and our satellite galaxies a good place to look for dark matter annihilation

- However the complicated astrophysics of the center of the galaxy makes it difficult to make a robust detection there - dark matter dominated dwarf satellites are easier to interpret

From Objects to Mean Density

- Dark matter that is associated with objects such as galaxies and galaxy clusters is well established
- To establish the mean density we need to know how representative such an object is in its dark to luminous matter
- Classical argument for measuring total amount of dark matter

From Objects to Mean Density

- Assuming that the object is somehow typical in its non-luminous to luminous density: "mass-to-light ratio"
- Convert to dark matter density as $M/L\times$ luminosity density
- From galaxy surveys the luminosity density in solar units is

$$\rho_L = 2 \pm 0.7 \times 10^8 h \, L_\odot \mathrm{Mpc}^{-3}$$

(h's: $L \propto F d^2$ so $\rho_L \propto L/d^3 \propto d^{-1}$ and d in h^{-1} Mpc

- Critical density in solar units is

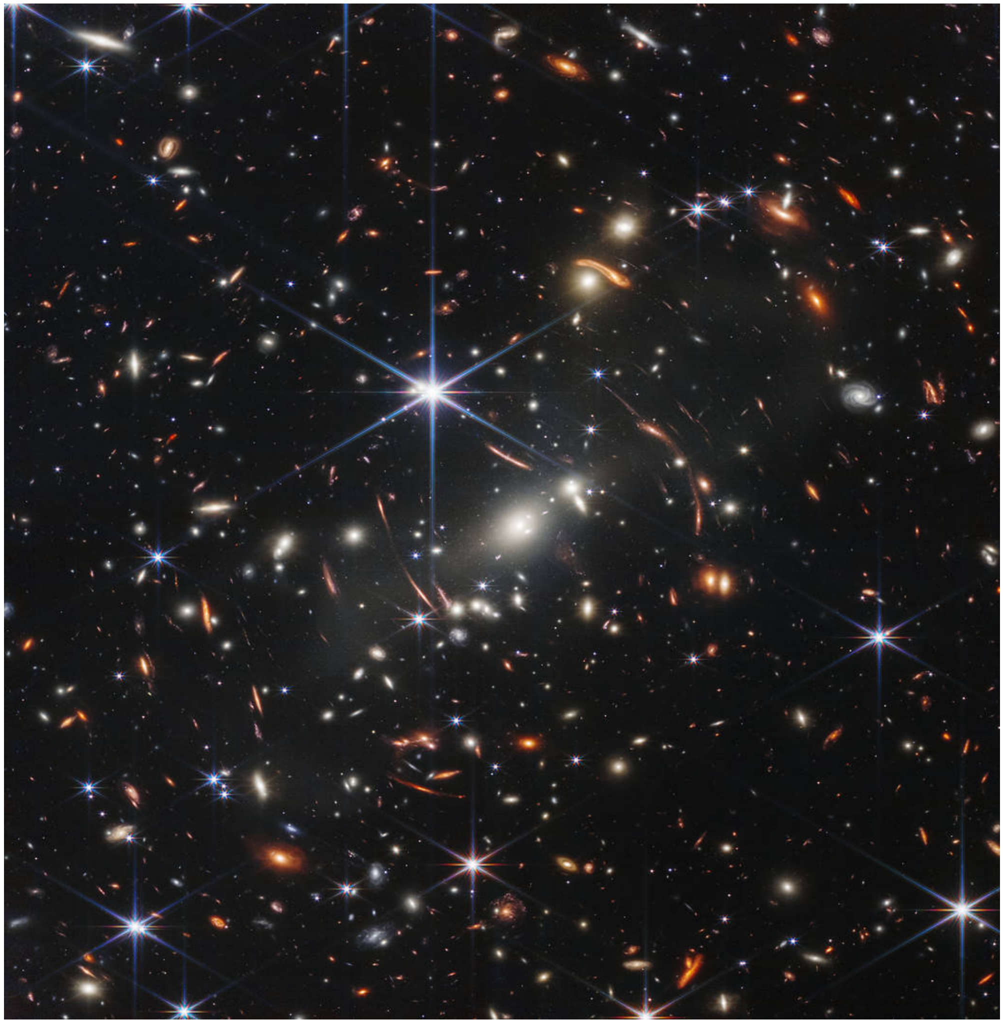
$$\rho_c = 2.7754 \times 10^{11} h^2 \, M_\odot \mathrm{Mpc}^{-3}$$

so that the critical mass-to-light ratio in solar units is

$$M/L \approx 1400 h (M_\odot / L_\odot)$$

Clusters of Galaxies

- Largest bound structures: mass to light approaches that required for $\rho_m \sim \rho_c/3$ (JWST: SMACS 0723)



Gravitational Lensing

- If a source lies directly behind a massive galaxy, it can be gravitationally lensed and produce multiple images
- In general relativity, masses curve space and bend the trajectory of photons - for this discussion lets restore the different units of t and x by restoring c - but note that is the vacuum, not the local coordinate speed of light
- Newtonian approximation to the line element

$$ds^2 = (1 + 2\Phi/c^2)c^2 dt^2 - (1 - 2\Phi/c^2)dx^2$$

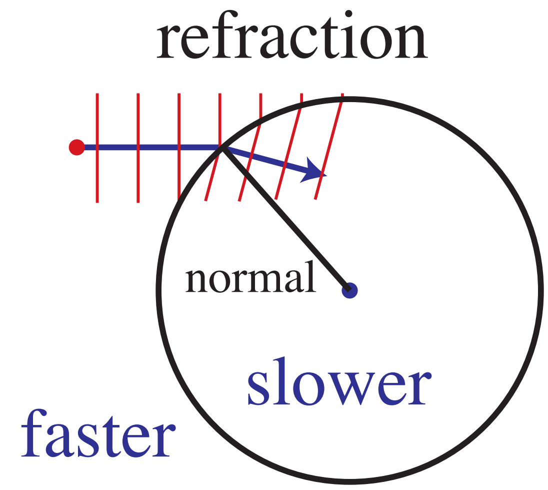
- Photons travel on null geodesics ($ds^2 = 0$) - so the coordinate speed of light is ($\Phi \ll 1$)

$$v = \frac{dx}{dt} \approx c\sqrt{\frac{1 + 2\Phi/c^2}{1 - 2\Phi/c^2}} \approx c(1 + 2\Phi/c^2) = c\left(1 - \frac{2GM}{rc^2}\right)$$

Gravitational Lensing

- Coordinate speed of light slows in the presence of mass due to the warping of spacetime as quantified by the gravitational potential

  Can be modelled as an optics problem, defines an effective index of refraction



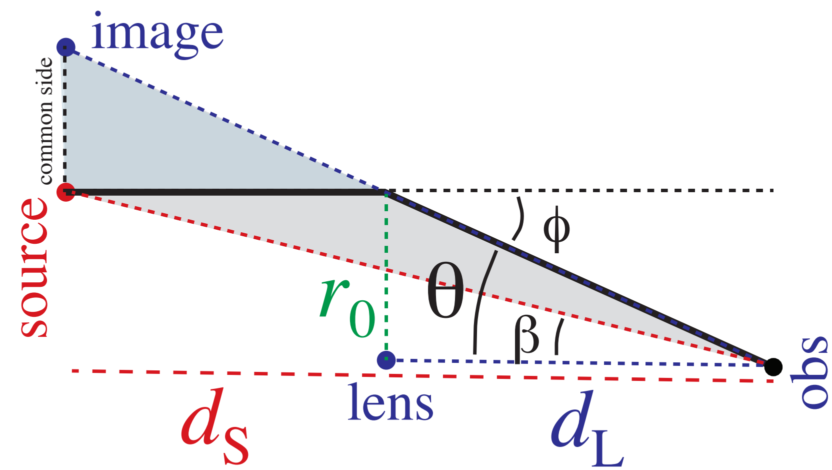
$$n = \frac{c}{v} = \left(1 - \frac{2GM}{rc^2}\right)^{-1} \approx 1 + \frac{2GM}{rc^2}$$

- As light passes by the object, the change in the index of refraction or delay of the propagation of wavefronts bends the trajectory

$$\nabla n = -\frac{2GM}{r^2c^2}\hat{\mathbf{r}}$$

Gravitational Lensing of Quasars

- Calculation would take the same form if we took a nonrelativistic particle of mass m and used Newtonian mechanics - general relativity just doubles it the deflection for light due to space curvature



- Deflection is small so integrate the transverse ($\perp$) deflection on the unperturbed trajectory

$$\phi = -\int_{-\infty}^{\infty} dx \nabla_\perp n = \int_{-\infty}^{\infty} dx \frac{2GMr_0}{(r_0^2 + x^2)^{3/2} c^2} = \frac{4GM}{r_0 c^2}$$

Lens Equation

- Given the thin lens deflection formula, the lens equation follows from simple geometry
- Solve for the image position θ with respect to line of sight. Small angle approximation

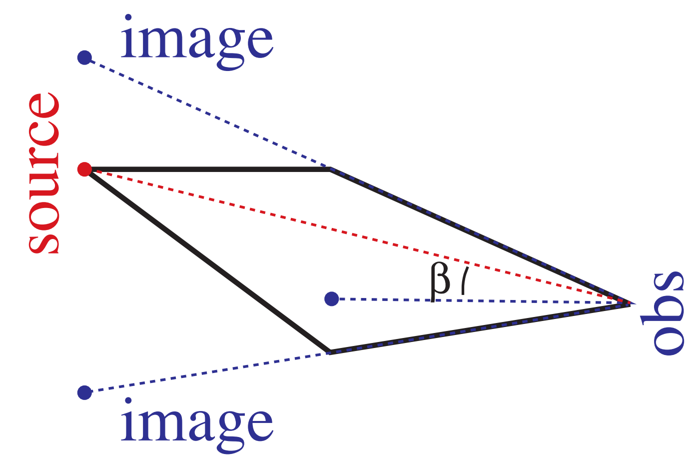
$$\text{common side: } (d_S - d_L)\phi \approx d_S(\theta - \beta)$$

- Substitute in deflection angle

$$(d_S - d_L)\frac{4GM}{r_0 c^2} \approx d_S(\theta - \beta)$$

- Eliminate $r_0 = d_L \sin\theta \approx d_L\theta$
- For cosmological distances replace d's with angular diameter distances D_A

Lens Equation



- Solve for θ to obtain the lens equation

$$\theta^2 - \beta\theta - \frac{4GM}{c^2}\left(\frac{d_S - d_L}{d_S d_L}\right) = 0$$

- A quadratic equation with two solutions for the image position - two images

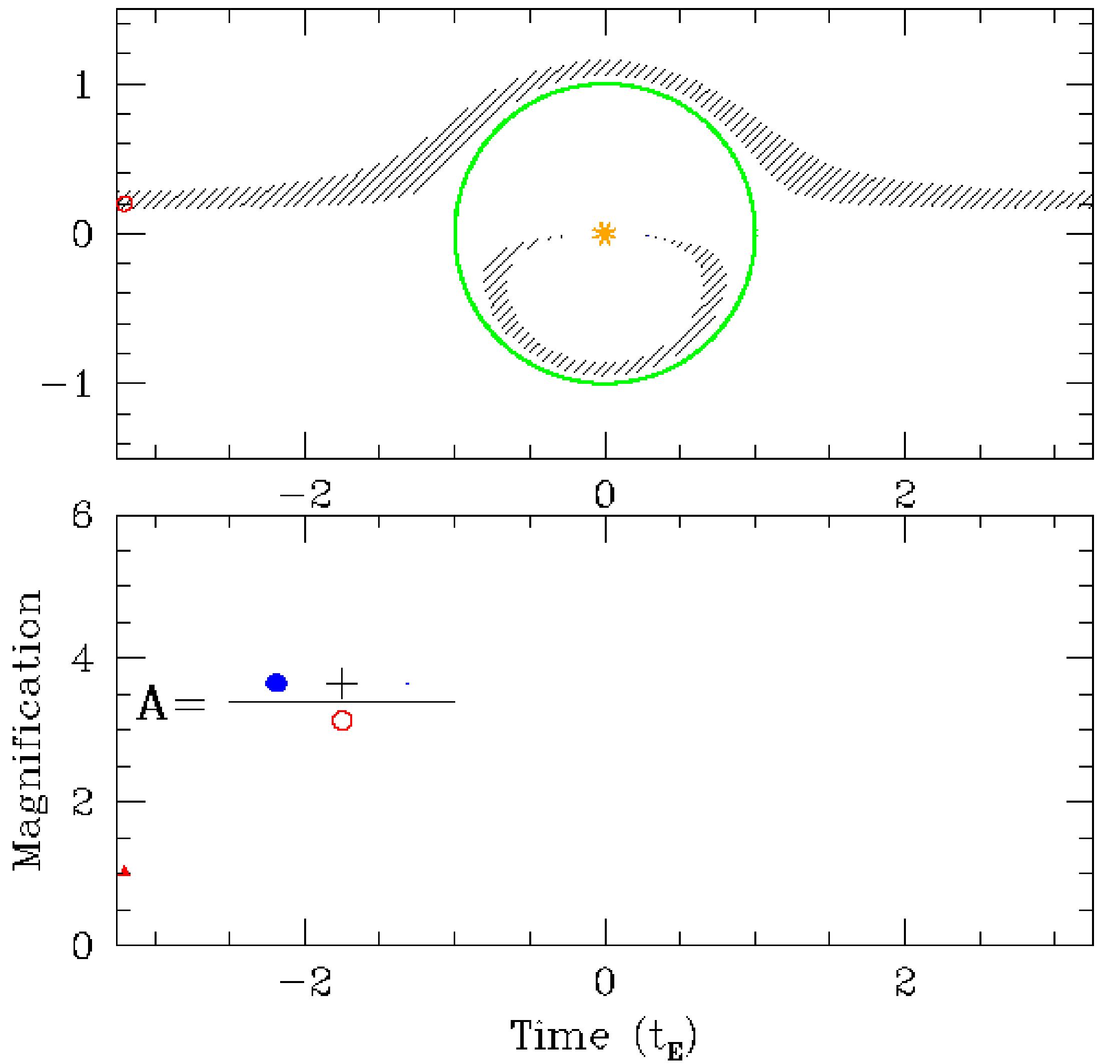
$$\theta_\pm = \frac{\beta}{2} \pm \frac{1}{2}\sqrt{\beta^2 + 16\frac{GM}{c^2}\left(\frac{d_S - d_L}{d_S d_L}\right)}$$

- Sum of angles - second image has negative angle - opposite side of lens

$$\theta_+ + \theta_- = \beta$$

Two Images

- Two images on opposite side of lens (magnified, sheared and time delayed)



Lens Equation

- Measure both images and measure the mass:

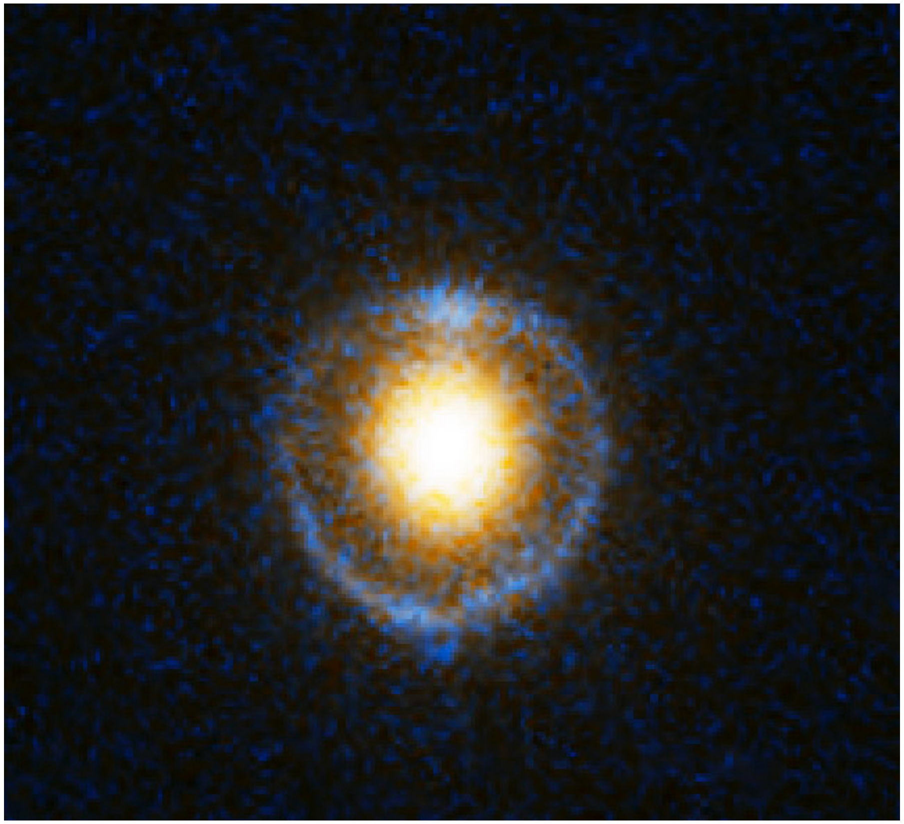
$$\theta_+\theta_- = \frac{\beta^2}{4} - \frac{1}{4}\left(\beta^2 + 16\frac{GM}{c^2}\left(\frac{d_S - d_L}{d_S d_L}\right)\right)$$

$$\theta_+\theta_- = -4\frac{GM}{c^2}\left(\frac{d_S - d_L}{d_S d_L}\right)$$

$$M = -\frac{\theta_+\theta_- c^2}{4G}\left(\frac{d_S d_L}{d_S - d_L}\right)$$

- Dimensionally of the form $M \propto v^2 r_0/G$ but reduced because light is not on a bound orbit

Einstein Ring



- If source
  is aligned right behind the
  lens $\beta = 0$ and the two images
  merge into a ring - Einstein
  ring - at an angular separation

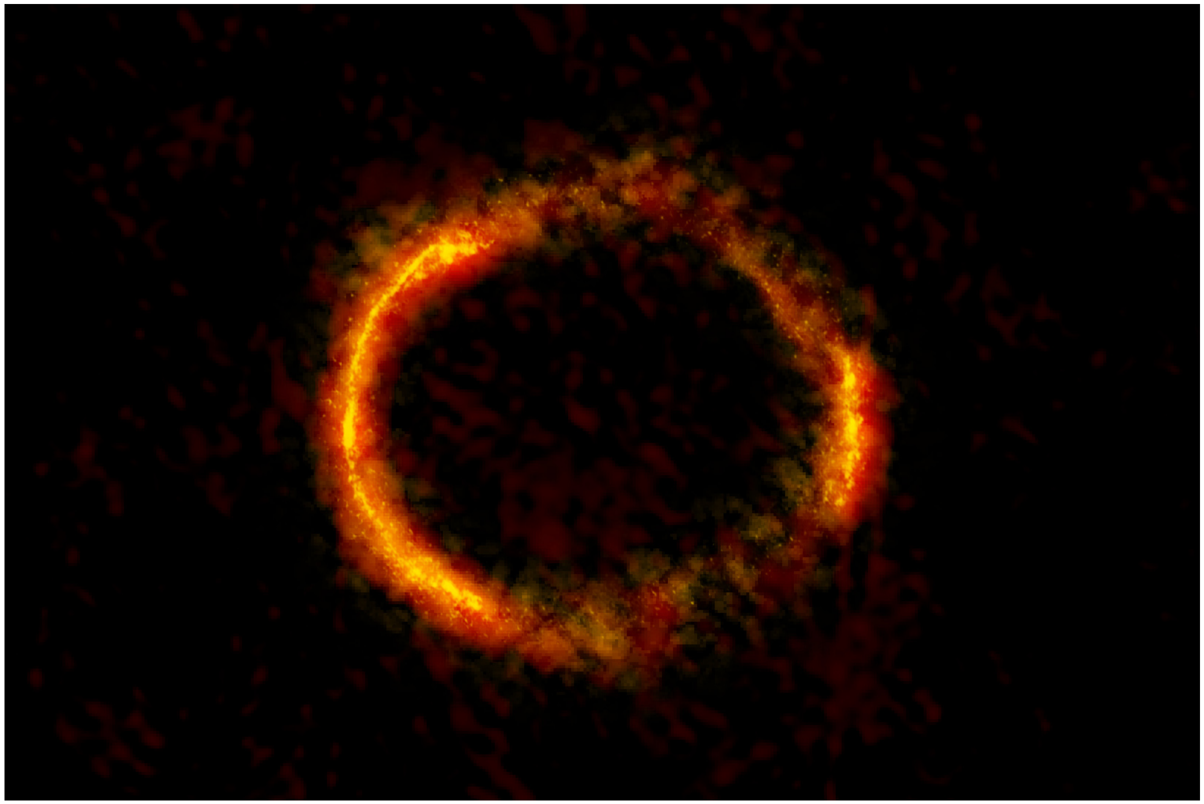
$$\theta_E = \sqrt{\frac{4GM}{c^2}\left(\frac{d_S - d_L}{d_S d_L}\right)}$$

- Image of a quasar lensed by a
  giant elliptical galaxy

- Some differences: extended mass of a galaxy that is not perfectly
  axially symmetric or singular at the center

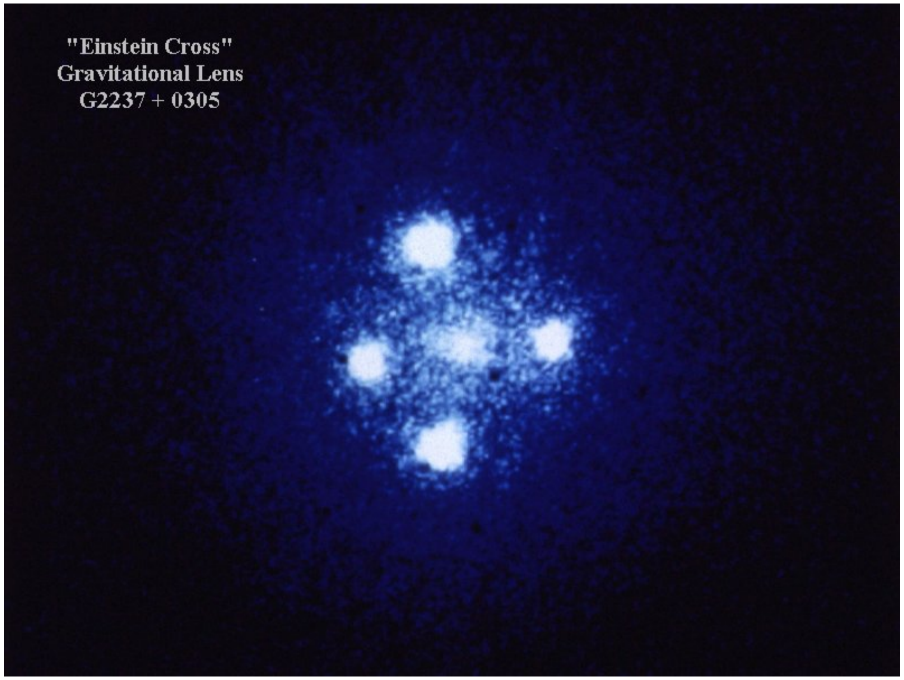
Einstein Ring

- ALMA (sub/millimeter) image of a galaxy lensing a galaxy



Einstein Cross

- With nonsingular, nonsymmetric extended potential multiple images form
- Mathematically an odd number of images but with an even number of bright detectable images
- Potential of an elliptical galaxy: 4 image system is an Einstein cross



Time Delay

- Arrival times of images differ so for a variable source like a quasar, time delay is another observable
- Deflected path is longer geometrically

$$
\begin{aligned}
t_{\rm geom} &\approx [1-\cos(\theta-\beta)]d_L + (1-\cos\beta)(d_S-d_L) \\
&\approx \frac{1}{2}(\theta-\beta)^2 d_L + \frac{1}{2}\beta^2(d_S-d_L) \\
&\approx \frac{d_L d_S}{2(d_S-d_L)}(\theta-\beta)^2
\end{aligned}
$$

where we have used $d_L(\theta-\beta) \approx (d_S-d_L)\beta$

- Light propagates more slowly in the gravitational potential Φ: Shapiro time delay

$$
t_{\rm Shapiro} = \int dx(1/v - 1/c) \approx -\frac{2}{c^3}\int dx\Phi
$$

- So full time delay $t_{\rm delay} = t_{\rm geom} + t_{\rm Shapiro}$

Time Delay

- For the 2 image, point mass case

$$\begin{aligned} t_{\rm Shapiro} &\approx \frac{2GM}{c^3}\int_{-d_L}^{d_S-d_L}\frac{dx}{\sqrt{x^2+r_0^2}} \\ &= -\frac{2GM}{c^3}\ln\frac{\sqrt{(d_S-d_L)^2+r_0^2}-(d_S-d_L)}{\sqrt{d_L^2+r_0^2}+d_L} \end{aligned}$$

- Again take the small angle approximation $r_0 \approx d_L\theta$ and expand $\theta \ll 1$ dropping subdominant ln factors

$$t_{\rm Shapiro} \approx -\frac{4GM}{c^3}\ln\theta$$

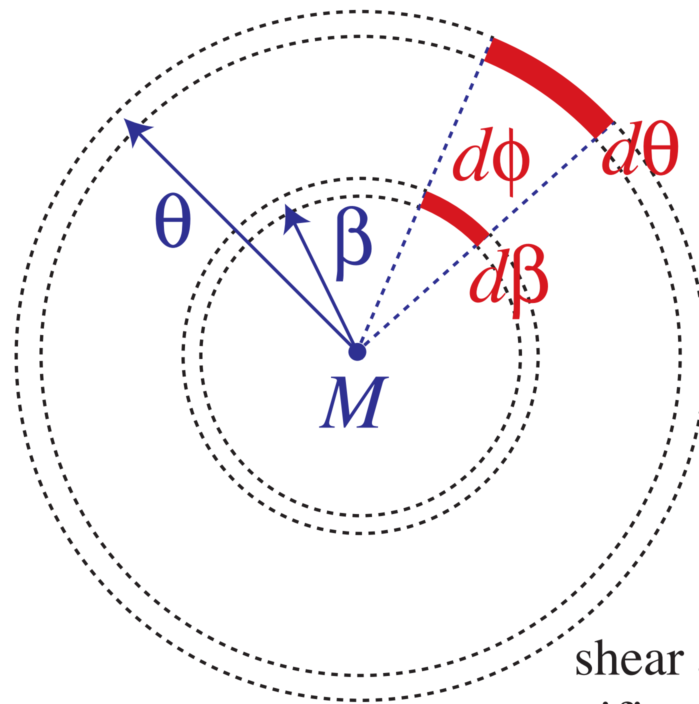
- Notice that if we measure the time delay and image positions we break the degeneracy between mass and cosmological distance factors and can measure the absolute distance scale, i.e H_0

Magnification

- Lens equation in terms of Einstein radius

$$\theta^2 - \beta\theta - \theta_E^2 = 0$$

- Given an extended source that covers an angular distance $d\beta$ will have an image cover an angular distance $d\theta_\pm$ related by the derivative $d\theta_\pm/d\beta$



shear and magnification

- The displacement in the image is purely radial so that the angular scale of arc $d\phi$ remains unchanged.
- The surface area of the source $\beta d\beta d\phi$ thus becomes $\theta_\pm d\theta_\pm d\phi$.

Magnification

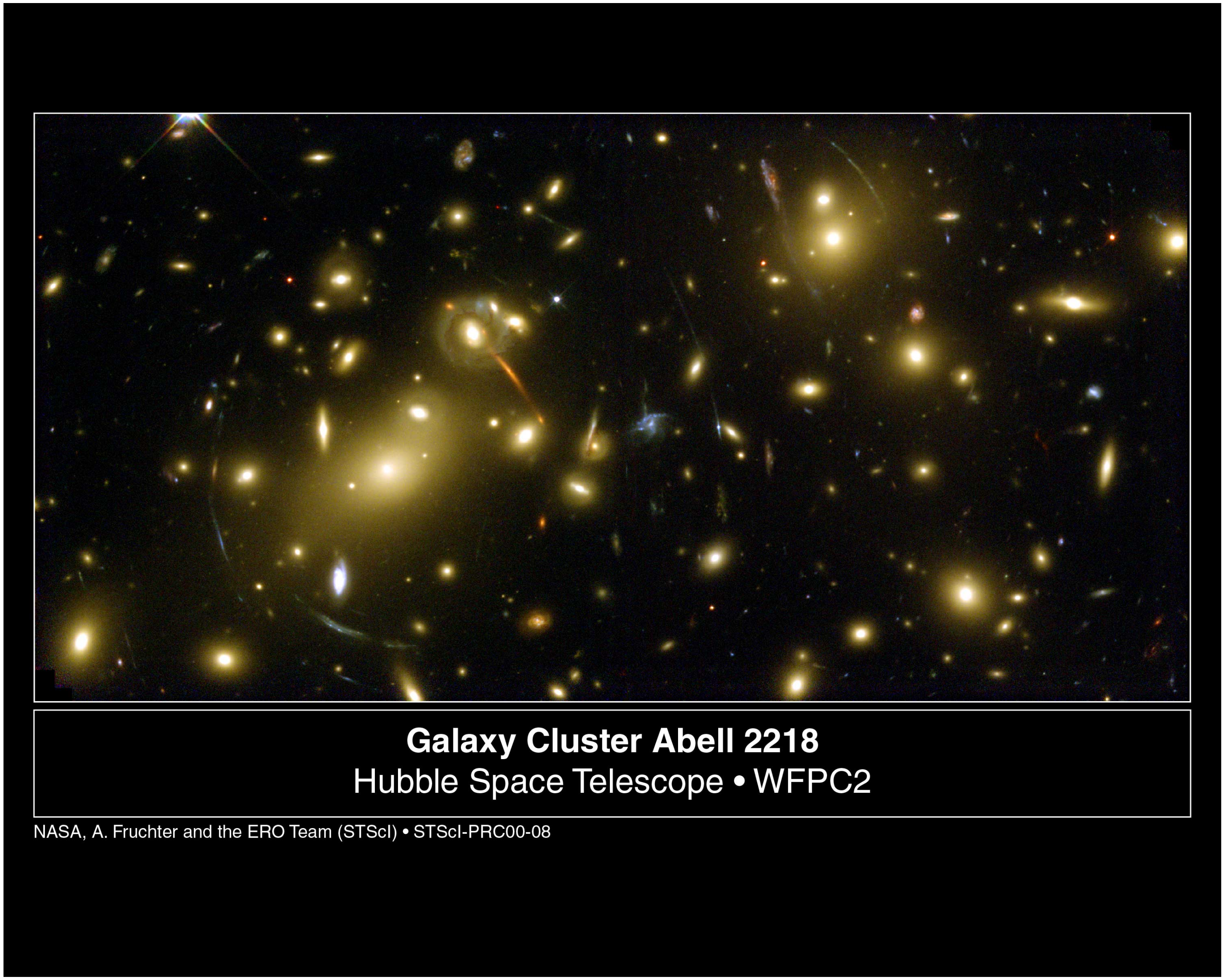
- Summing the two images yields

$$\frac{A_{\rm images}}{A_{\rm source}} = \sum_{\pm} \left| \frac{\theta_\pm}{\beta} \frac{d\theta_\pm}{d\beta} \right| = \frac{(\beta/\theta_E)^2 + 2}{(\beta/\theta_E)\sqrt{(\beta/\theta_E)^2 + 4}}$$

- Surface brightness is conserved so the area ratio gives the magnification
- If two images are not resolved this gives the microlensing magnification formula, see next few slides
- Magnification and conservation of $d\phi$ implies that image is distorted - stretched out into the tangential direction or "sheared"

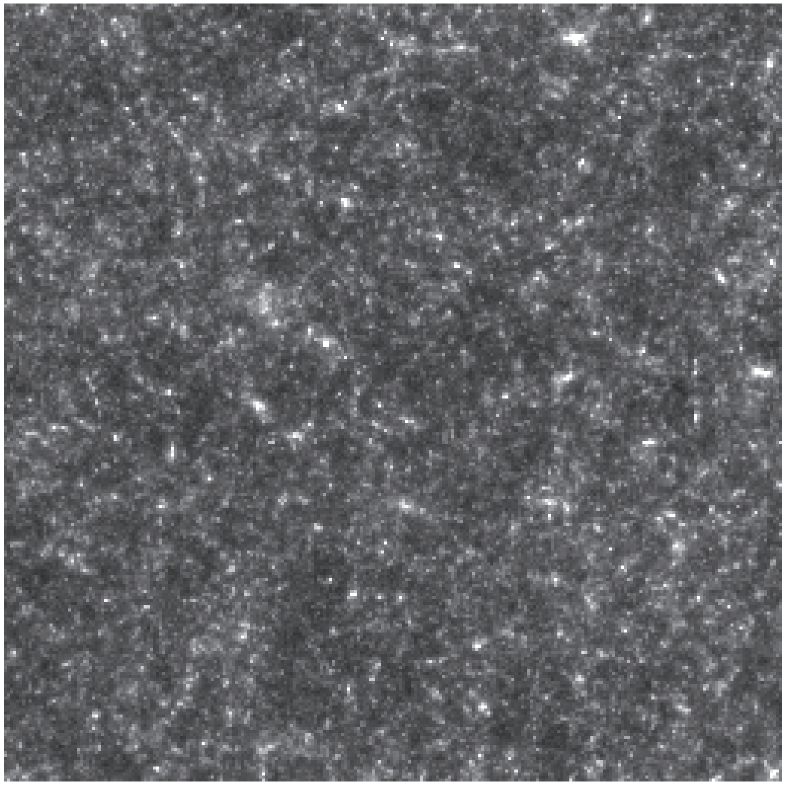
Giant Arcs

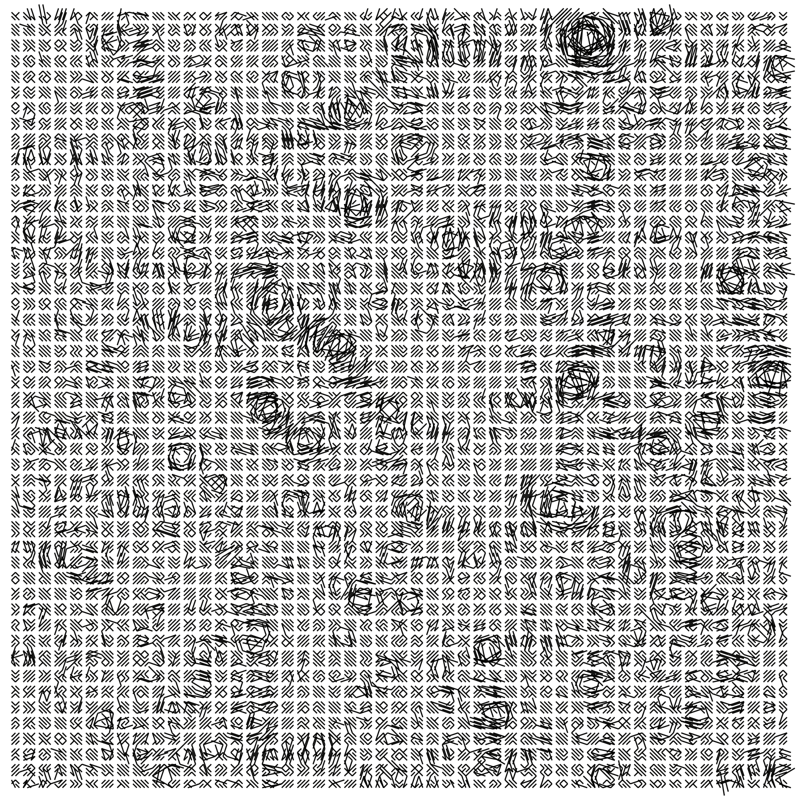
- Giant arcs in galaxy clusters: galaxies, source; cluster, lens



Cosmic Shear

- On even larger scales, the large-scale structure weakly shears background images: weak lensing



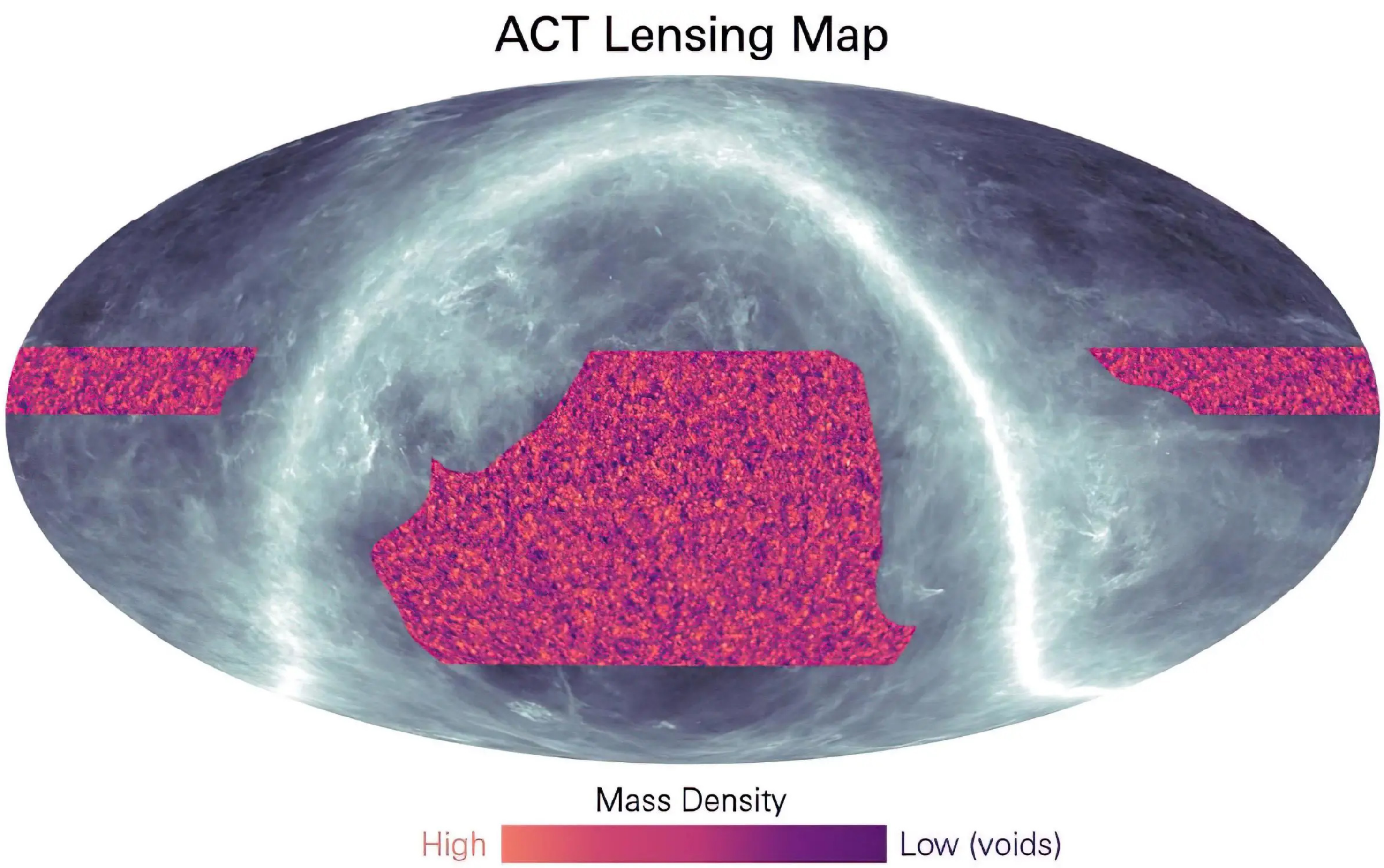


Gravitational Lensing

- Mass can be directly measured in the gravitational lensing of sources behind the cluster
- Strong lensing (giant arcs) probes central region of clusters
- Weak lensing (1-10%) elliptical distortion to galaxy image probes outer regions of cluster and total mass

Weak lensing of the CMB

- 2023 ACT mass reconstruction



Microlensing

- Rotation curves leave open the question of what dark matter is
- Alternate hypothesis: dead stars or black holes - massive astrophysical compact halo object "MACHO"
- MACHOs have their mass concentrated into objects with mass comparable to the sun or large planet
- A MACHO at an angular distance $u = \theta/\theta_E$ from the line of sight to the star will gravitationally lens or magnify the star by a factor of

$$A(u) = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}}$$

where θ_E is the Einstein ring radius in projection

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{d_S - d_L}{d_S d_L}}$$

Gravitational Lensing

- Again masses related to a physical scale $r_E = \theta_E d_L$ and speed c

$$M \sim \frac{d_S \theta_E}{4(d_S - d_L)} \frac{c^2 r_E}{G}$$

  e.g. for a typical lens half way to the source the prefactor is $\theta_E/2$, dimensionless but not order unity since light is not bound to system

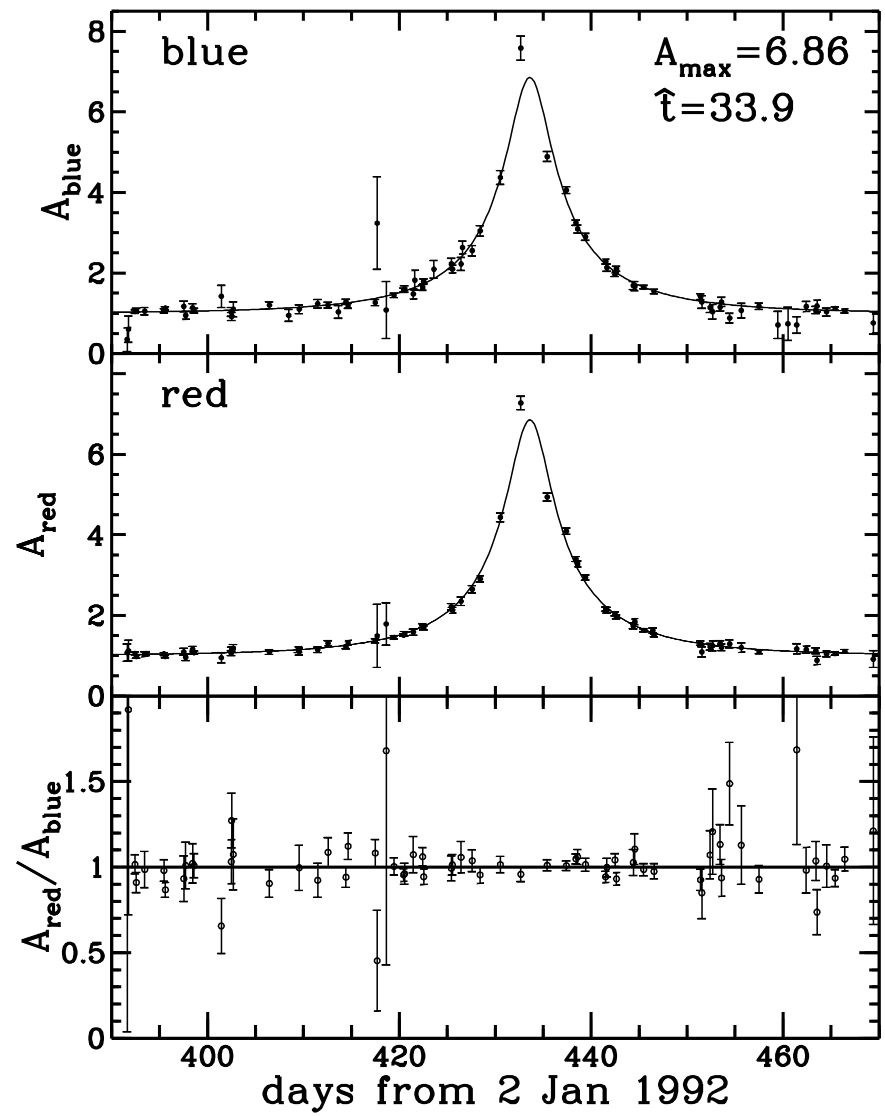
- A MACHO would move at a velocity typical of the disk and halo $v \sim 200$km/s and so the star behind it would brighten as it crossed the line of sight to a background star. With $u_{\rm min}$ as the distance of closest approach at $t = 0$

$$u^2(t) = u^2_{\rm min} + \left(\frac{vt}{d_L \theta_E} \right)^2$$

- Paczynski (1986) proposed monitoring 10^6 LMC stars to see this characteristic brightening.

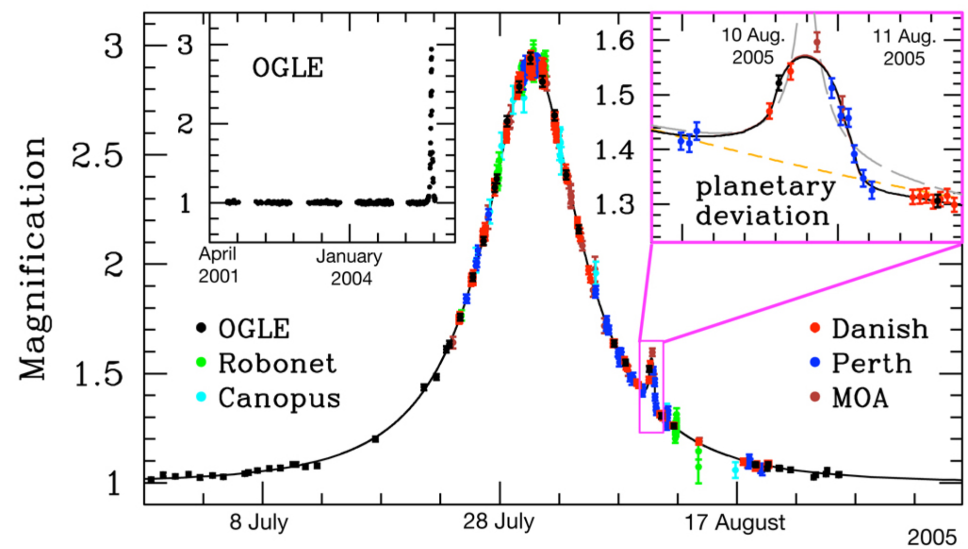
Gravitational Lensing

- In the 1990's
  large searches measured
  the rate of microlensing
  in the halo and
  bulge and determined that
  only a small fraction of its
  mass could be in MACHOs

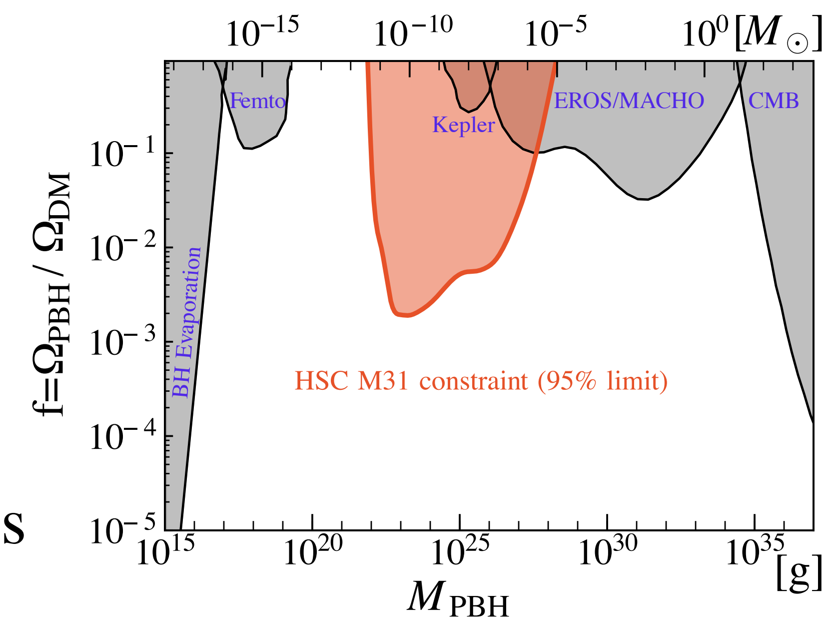


Gravitational Lensing

- Current searches (toward the bulge) used to find planets
- Enhanced microlensing by planet around star leads to a blip in the brightening.



- Searches with different temporal cadence rule out other compact objects like primordial black holes as the dark matter all the way down to asteroid sized masses



Hydrostatic Equilibrium

- Evidence for dark matter in X-ray clusters also comes from direct hydrostatic equilibrium inference from the gas: balance radial pressure gradient with gravitational potential gradient
- Infinitesimal volume of area dA and thickness dr at radius r and interior mass $M(r)$: pressure difference supports the gas

$$[p_g(r) - p_g(r+dr)]dA = \frac{GmM}{r^2} = \frac{G\rho_g M}{r^2}dV$$

$$\frac{dp_g}{dr} = -\rho_g \frac{GM}{r^2}$$

with $p_g = \rho_g T_g/m$ becomes

$$\frac{GM}{r} = -\frac{T_g}{m}\left(\frac{d\ln\rho_g}{d\ln r} + \frac{d\ln T_g}{d\ln r}\right)$$

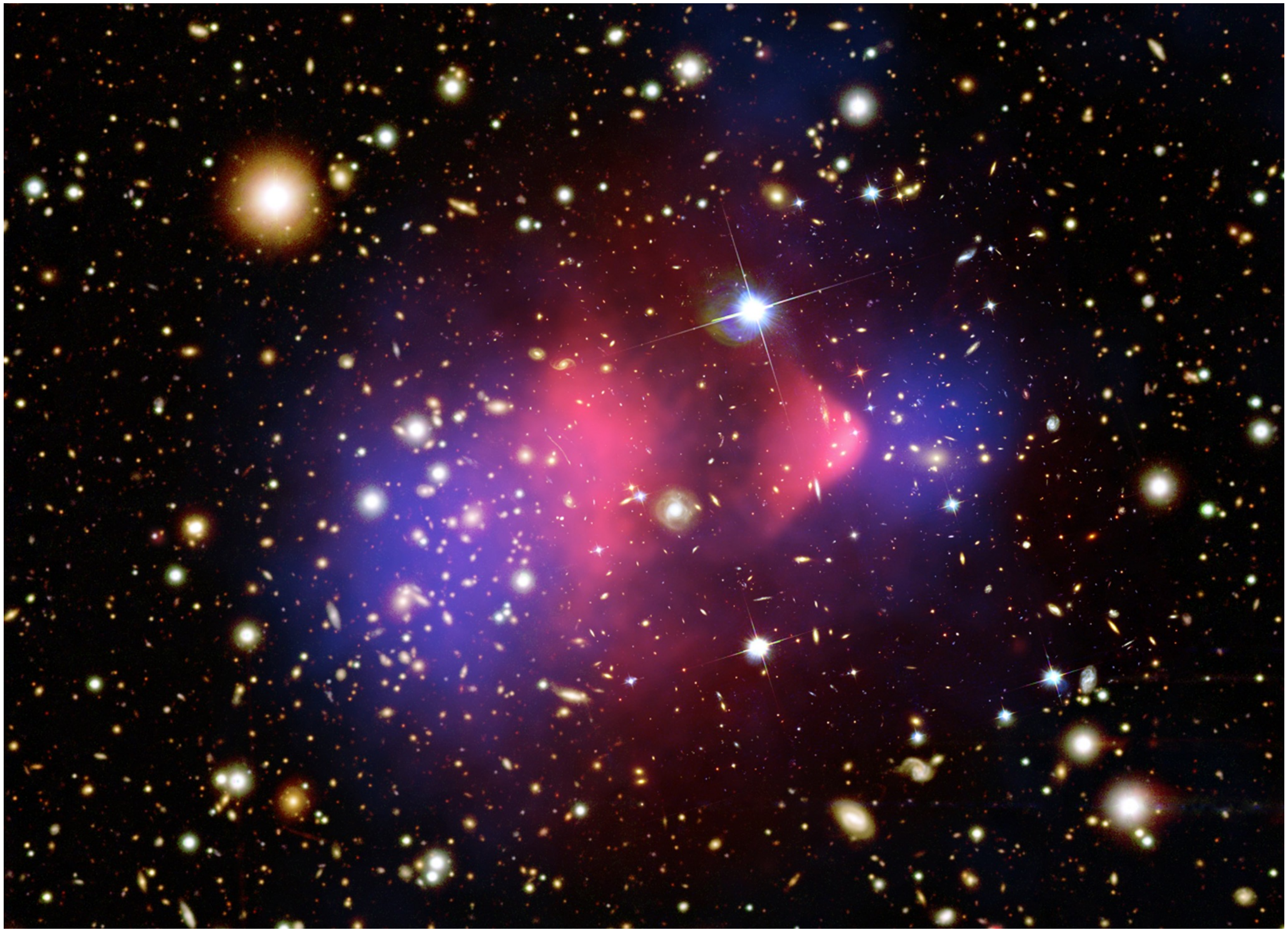
- ρ_g from X-ray luminosity; T_g sometimes taken as isothermal

Dark Matter: Clusters

- Notice that to order of magnitude $M \sim v^2 r/G$ with v being the velocity dispersion associated with the gas
- Centripetal force is replaced by pressure gradient $T/m = \sigma^2$ and $p = \rho T/m = \rho\sigma^2$
- Generalization to the gas distribution also gives evidence for dark matter

Dark Matter: Bullet Cluster

- Merging clusters: gas (visible matter) collides and shocks (X-rays), dark matter measured by gravitational lensing passes through



CMB Hydrostatic Equilibrium

- Same argument in the CMB with radiation pressure in the gas balancing the gravitational potential gradients of linear fluctuations
- Best measurement of the dark matter density to date (Planck 2018): $\Omega_c h^2 = 0.1198 \pm 0.0012$, $\Omega_b h^2 = (2.233 \pm 0.015) \times 10^{-2}$.
- Unlike other techniques, measures the physical density of the dark matter rather than contribution to critical since the CMB temperature sets the physical density and pressure of the photons

Cosmic Census

- With $h = 0.68$ and CMB $\Omega_m h^2 = 0.14$, $\Omega_m = 0.30$ - consistent with other, less precise, dark matter measures
- CMB provides a test of $D_A \neq D$ through the standard rulers of the acoustic peaks and shows that the universe is close to flat $\Omega \approx 1$
- Consistency has lead to the standard model for the cosmological matter budget:
  - 70% dark energy
  - 30% non-relativistic matter (with 84% of that in dark matter)
  - 0% spatial curvature
- Next we take these components today and wind back the clock and cover the thermal history and origin of the particle components