Set 6:

Inflation

Horizon Problem

- The horizon in a decelerating universe scales as $\eta \propto a^{(1+3w)/2}$, $w > -1/3$. For example in a matter dominated universe

$$\eta \propto a^{1/2}$$

- CMB decoupled at $a_* = 10^{-3}$ so subtends an angle on the sky

$$\frac{\eta_*}{\eta_0} = a_*^{1/2} \approx 0.03 \approx 2^\circ$$

- So why is the CMB sky isotropic to 10^{-5} in temperature if it is composed of $\sim 10^4$ causally disconnected regions
- If smooth by fiat, why are there 10^{-5} fluctuations correlated on superhorizon scales

Flatness & Relic Problems

- Flatness problem: why is the radius of curvature larger than the observable universe. (Before the CMB determinations, why is it at least comparable to observable universe $|\Omega_K| \lesssim \Omega_m$)
- Also phrased as a coincidence problem: since $\rho_K \propto a^{-2}$ and $\rho_m \propto a^{-3}$, why would they be comparable today – modern version is dark energy coincidence $\rho_\Lambda = \text{const.}$
- Relic problem – why don't relics like monopoles dominate the energy density

- Inflation is a theory that solves all three problems at once and also supplies a source for density perturbations

Accelerating Expansion

- In a matter or radiation dominated universe, comoving Hubble length $(1/aH)$ grows with a a so that there's no way to establish causal contact on larger scales, generally:

$$\eta = \int d\ln a \frac{1}{aH(a)}$$

- $H^2 \propto \rho \propto a^{-3(1+w)}$, $aH \propto a^{-(1+3w)/2}$, critical value of $w = -1/3$, the division between acceleration and deceleration determines whether as the universe expands comoving observers leave or come into causal contact
- Recall this is our fate in the current accelerating expansion – observers that were once in causal contact will no longer be able to communicate with each other due to the rapid expansion

Causal Contact

- True horizon always grows meaning that one always sees more and more of the universe. But the comoving Hubble length decreases: the difference in conformal time, the distance a photon can travel between two epochs denoted by the scale factor decreases. Regions that were in causal contact, leave causal contact.
- Horizon problem solved if the universe was in an acceleration phase up to η_i and the conformal time since then is shorter than the total conformal age

$$\eta_0 \gg \eta_0 - \eta_i$$

$$\text{total distance} \gg \text{distance traveled since inflation}$$

$$\text{apparent horizon}$$

Flatness & Relic

- Comoving radius of curvature is constant and can even be small compared to the full horizon $R \ll \eta_0$ yet still $\eta_0 \gg R \gg \eta_0 - \eta_i$
- In physical coordinates, the rapid expansion of the universe makes the current observable universe much smaller than the curvature scale
- Likewise, the number density of relics formed before the accelerating (or inflationary) epoch is diluted to make them rare in the current observable volume
- Common to place the zero point of (conformal) time at the end of inflation $\tilde{\eta} \equiv \eta - \eta_i$. Here conformal time is negative during inflation and size reflects the distance a photon can travel from that epoch to the end of inflation. To avoid confusion with the original zero point $\eta(a = 0) = 0$ let's call this $\tilde{\eta}$.

Sufficient Inflation

- If the accelerating component has equation of state $w = -1$, $\rho =$ const., $H = H_i$ const. so that $a \propto \exp(Ht)$

$$\tilde{\eta} = \int_{a_i}^{a} d\ln a \frac{1}{aH} = -\frac{1}{aH_i}\Big|_{a_i}^{a}$$
$$\approx -\frac{1}{aH_i} \quad (a_i \gg a)$$

- In particular, the current horizon scale $H_0\tilde{\eta}_0 \approx 1$ exited the horizon during inflation at

$$\tilde{\eta}_0 \approx H_0^{-1} = \frac{1}{a_H H_i}$$
$$a_H = \frac{H_0}{H_i}$$

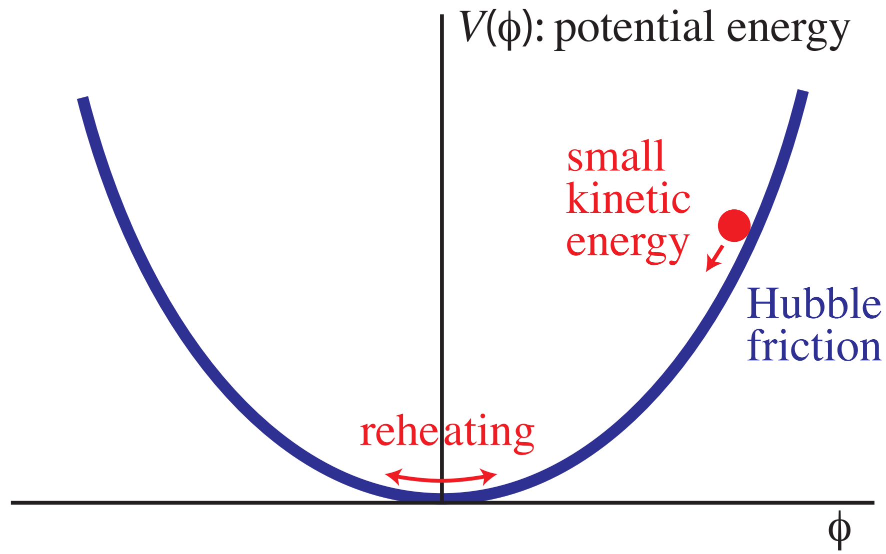
Sufficient Inflation

- Given some energy scale for inflation that defines H_i, this tells us what the scale factor a_H was when the current horizon left the horizon during inflation
- If we knew what the scale factor a_i was at the end of inflation, we could figure out the number of efolds $N = \ln(a_i/a_H)$ between these two epochs
- A rough way to characterize this is to quote it in terms of an effective temperature $T \propto T_{\rm CMB} a^{-1}$ at the end of inflation

$$\ln \frac{a_i}{a_H} = \ln \frac{T_{\rm CMB}}{T_i} \frac{H_i}{H_0} = 65 + 2 \ln \left(\frac{\rho_i^{1/4}}{10^{14} {\rm GeV}} \right) - \ln \left(\frac{T_i}{10^{10} {\rm GeV}} \right)$$

- So inflation lasted at least ~ 60efolds - a more detailed calculation would involve the epoch of reheating and g_* factors, so $T_i \neq T_{\rm reheat}$

Inflation: Acceleration from Scalar Field



- Unlike a true cosmological constant, the period of exponential expansion must end to produce the hot big bang phase
- A cosmological constant is like potential energy - so imagine a ball rolling slowly in into a valley eventually converting potential into kinetic energy
- Technically, this is a scalar field: where the position on the hill is ϕ and the height of the potential is $V(\phi)$
- In spacetime $\phi(\mathbf{x}, t)$ is a function of position: different spacetime points can be at different field positions

Scalar Fields

- Inflation ends when the field rolls sufficiently down the potential that its kinetic energy becomes comparable to its potential energy
- The field then oscillates at the bottom of the potential and small couplings to standard model particles "reheats" the universe converting the inflaton energy into particles
- Due to the uncertainty principle in quantum mechanics, the field cannot remain perfectly unperturbed
- The small field fluctuations mean that inflation ends at a slightly different time at different points in space - leaving fluctuations in the scale factor, which are curvature or gravitational potential fluctuations
- Gravitational attraction into these potential wells forms all of the structure in the universe

Scalar Fields

- Mathematically, the scalar field obeys the Klein-Gordon equation in an expanding universe

$$\frac{d^2\phi}{dt^2} + 3H\frac{d\phi}{dt} + V' = 0$$

where $V' = dV/d\phi$ is the slope of the potential - the first and third term look like the equations of motion of a ball rolling down a hill - acceleration = gradient of potential

- The second $d\phi/dt$ term is a friction term provided by the expansion - “Hubble friction” - just like particle numbers and energy density dilute with the expansion, so too does the kinetic energy of the scalar field.

Scalar Fields

- Kinetic energy is

$$\rho_{\rm kinetic} = \frac{1}{2}\left(\frac{d\phi}{dt}\right)^2$$

  so, without the V' forcing term, how does the energy density decay?

- Solve

$$\frac{d^2\phi}{dt^2} + 3H\frac{d\phi}{dt} = 0 \to \frac{d\phi}{dt} \propto a^{-3}$$

  so kinetic energy would decay as $\rho_{\rm kinetic} \propto a^{-6} = a^{-3(1+w_{\rm kinetic})}$, or $w_{\rm kinetic} = +1$

- Compare with the potential energy at fixed field position $w_{\rm potential} = -1$

Scalar Fields

- As the field rolls it slowly loses total energy to friction, which defines the slow roll parameter

$$\epsilon_H = -\frac{d\ln H}{d\ln a} = \frac{3}{2}(1+w_\phi)$$

- Requirement that inflation last for the sufficient ∼60 efolds requires that $\epsilon_H \lesssim 1/60 \ll 1$
- This requirement also means that ϵ_H must also be slowly varying so as not to grow much during these 60 efolds

$$\delta_1 = \frac{1}{2}\frac{d\ln \epsilon_H}{d\ln a} - \epsilon_H$$

with $|\delta_1| \ll 1$ (advanced students: its defined this way since it also determines how close the roll is to friction dominated $3H d\phi/dt \approx -V'$)

Perturbation Generation

- Horizon scale $\tilde{\eta}$ during inflation acts like an event horizon - things that are separated by more than this distance leave causal contact
- Result of treating field fluctuations as a quantum simple harmonic oscillator (advanced students: see supplement) is that the uncertainty principle leads to inevitable fluctuations
- Fluctuations freeze in when the comoving wavelength $\lambda = 2\pi/k$ becomes larger than the comoving horizon $1/aH$, so that parts of the fluctuation are no longer in causal contact with itself, i.e. when $k \approx aH$

$$\delta\phi \approx \frac{H}{2\pi}$$

- We can also view this as an "origins" problem. Quantum fluctuations behave as a simple harmonic oscillator with frequency or rate $\omega \approx k/a$ and freezeout occurs when $\omega = H$, so $k/a = H$

Perturbation Generation

- Interpretation: universe is expanding quickly enough that various parts of the wave cannot "find" each other to maintain "equilibrium" (continue oscillating)
- Can heuristically understand the freezout value in the same way as Hawking radiation from a black hole - virtual particles become real when separated by the horizon
- Here H defines the horizon area (or in black hole language the Hawking temperature) and dimensional analysis says the field fluctuation must scale with H, the only dimensionful quantity
- Because H remains roughly constant during inflation the result is a scale invariant spectrum of fluctuations

Curvature Fluctuation

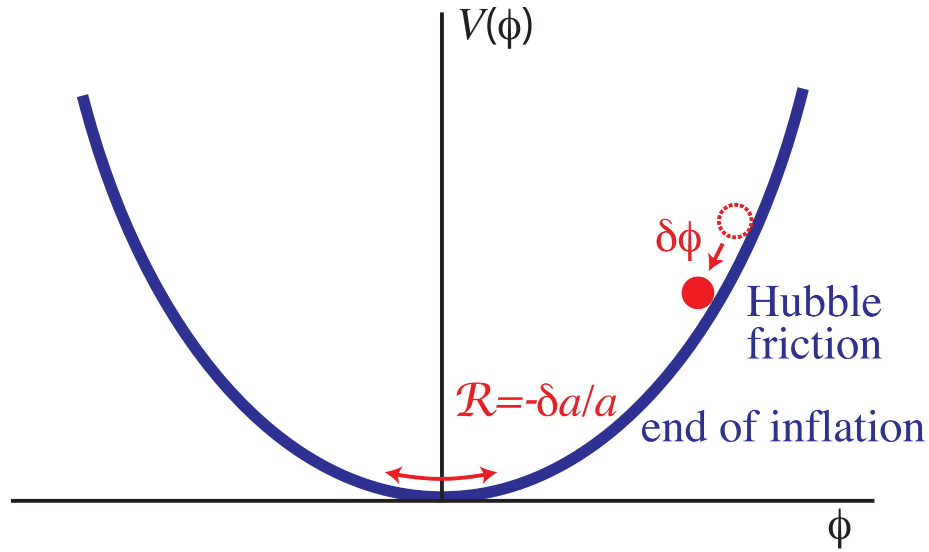
- Field fluctuations change the scale factor at which inflation ends

$$\mathcal{R} = -\delta \ln a = -\frac{d \ln a}{dt}\frac{dt}{d\phi}\delta\phi = -\frac{H^2}{2\pi}\frac{dt}{d\phi}$$

- Using the equation of state of ϕ we can convert $d\phi/dt$ to ϵ_H

$$\begin{aligned} w_\phi &= \frac{p_\phi}{\rho_\phi} \\ &= \frac{(d\phi/dt)^2/2 - V}{(d\phi/dt)^2/2 + V} \\ &\approx \frac{(d\phi/dt)^2}{V} - 1 \end{aligned}$$

and $H^2 \approx 8\pi G V/3$ from Friedmann



Curvature Fluctuation

- So

$$\epsilon_H \approx \frac{3}{2}\frac{(d\phi/dt)^2}{V} \approx 4\pi G \frac{(d\phi/dt)^2}{H^2}$$

and the variance of fluctuations per log wavenumber $d\ln k$

$$\Delta^2_{\mathcal{R}} \equiv \langle \mathcal{R}^2 \rangle \approx \frac{H^4}{4\pi^2}\frac{4\pi G}{H^2 \epsilon_H} \approx \frac{G}{\pi}\frac{H^2}{\epsilon_H}$$

- Remember this: $\Delta^2_{\mathcal{R}} \propto H^2/\epsilon_H$!

Tilt

- Curvature power spectrum is scale invariant to the extent that H and ϵ_H are constant
- Scalar spectral index

$$\frac{d\ln\Delta^2_{\mathcal{R}}}{d\ln k} \equiv n_S - 1 = 2\frac{d\ln H}{d\ln k} - \frac{d\ln\epsilon_H}{d\ln k}$$

- Evaluate at horizon crossing where fluctuation freezes $k = aH$

$$\frac{d\ln H}{d\ln k} \approx \frac{d\ln H}{d\ln a} = -\epsilon_H$$

$$\frac{d\ln\epsilon}{d\ln k} \approx \frac{d\ln\epsilon}{d\ln a} = 2(\delta_1 + \epsilon_H)$$

- Tilt in the slow-roll approximation

$$n_S - 1 = -4\epsilon_H - 2\delta_1$$

Gravitational Waves

- Gravitational wave amplitude satisfies Klein-Gordon equation $(K = 0)$, same as scalar field

$$\frac{d^2 h_{+,\times}}{dt^2} + 3H\frac{dh_{+,\times}}{dt} + \frac{k^2}{a^2}h_{+,\times} = 0\,.$$

- Acquires quantum fluctuations in same manner as ϕ. Canonical normalization (Lagrangian) sets the normalization
- Scale-invariant gravitational wave amplitude

$$\Delta^2_{+,\times} = 16\pi G\frac{H^2}{(2\pi)^2}$$

- Gravitational wave power $\propto H^2 \propto V \propto E_i^4$ where E_i is the energy scale of inflation

Gravitational Waves

- Tensor-scalar ratio is therefore generally small

$$r \equiv 4\frac{\Delta_+^2}{\Delta_{\mathcal{R}}^2} = 16\epsilon_H$$

- Tensor tilt:

$$\frac{d\ln\Delta_+^2}{d\ln k} \equiv n_T = 2\frac{d\ln H}{d\ln k} = -2\epsilon_H$$

- Consistency relation between tensor-scalar ratio and tensor tilt

$$r = 16\epsilon = -8n_T$$

- Measurement of scalar tilt and gravitational wave amplitude constrains inflationary model in the slow roll context
- Comparision of tensor-scalar ratio and tensor tilt tests the idea of slow roll itself

Observability

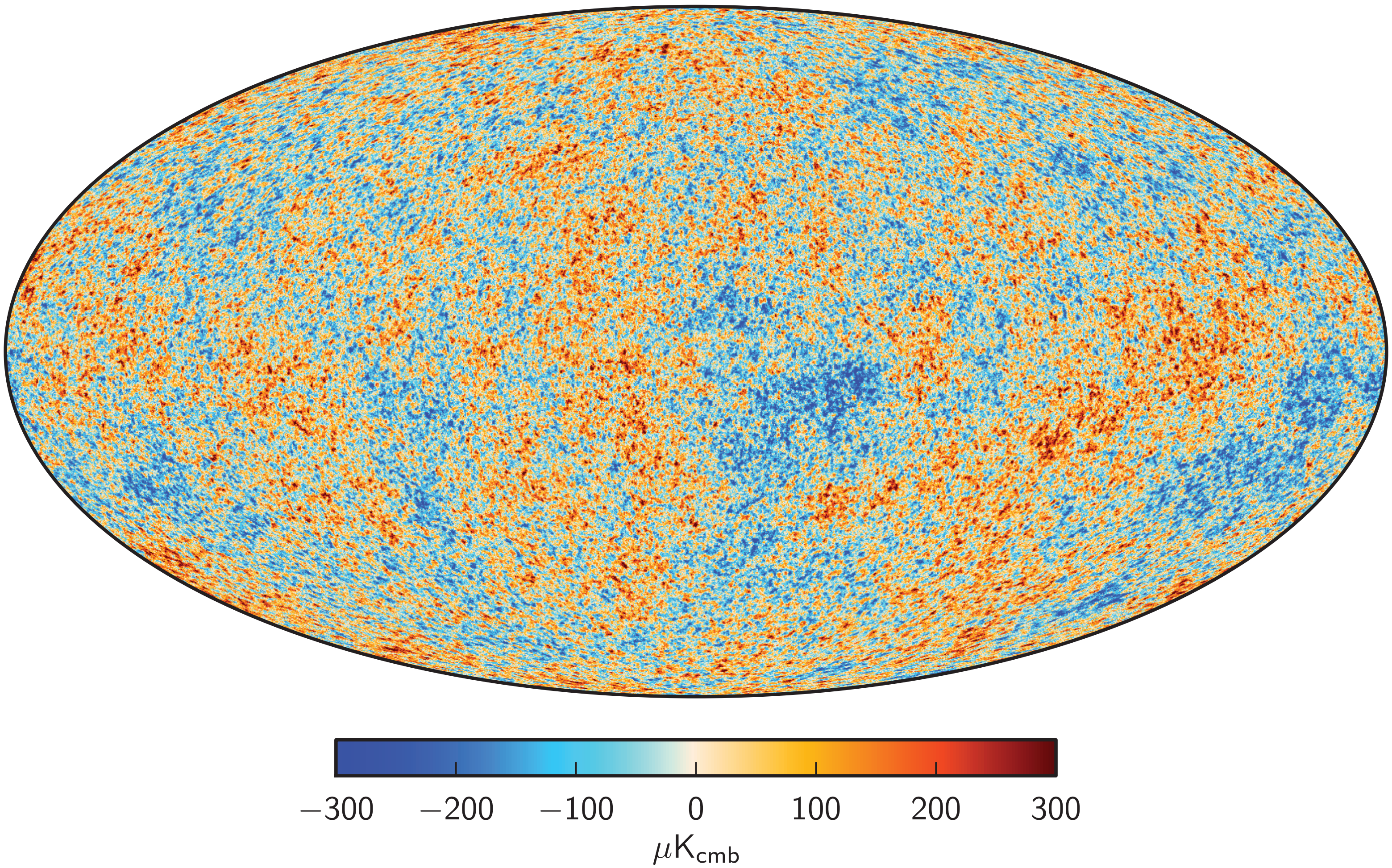
- Gravitational waves from inflation can be measured via its imprint on the polarization of the CMB…

Set 7:

CMB and Large Scale Structure

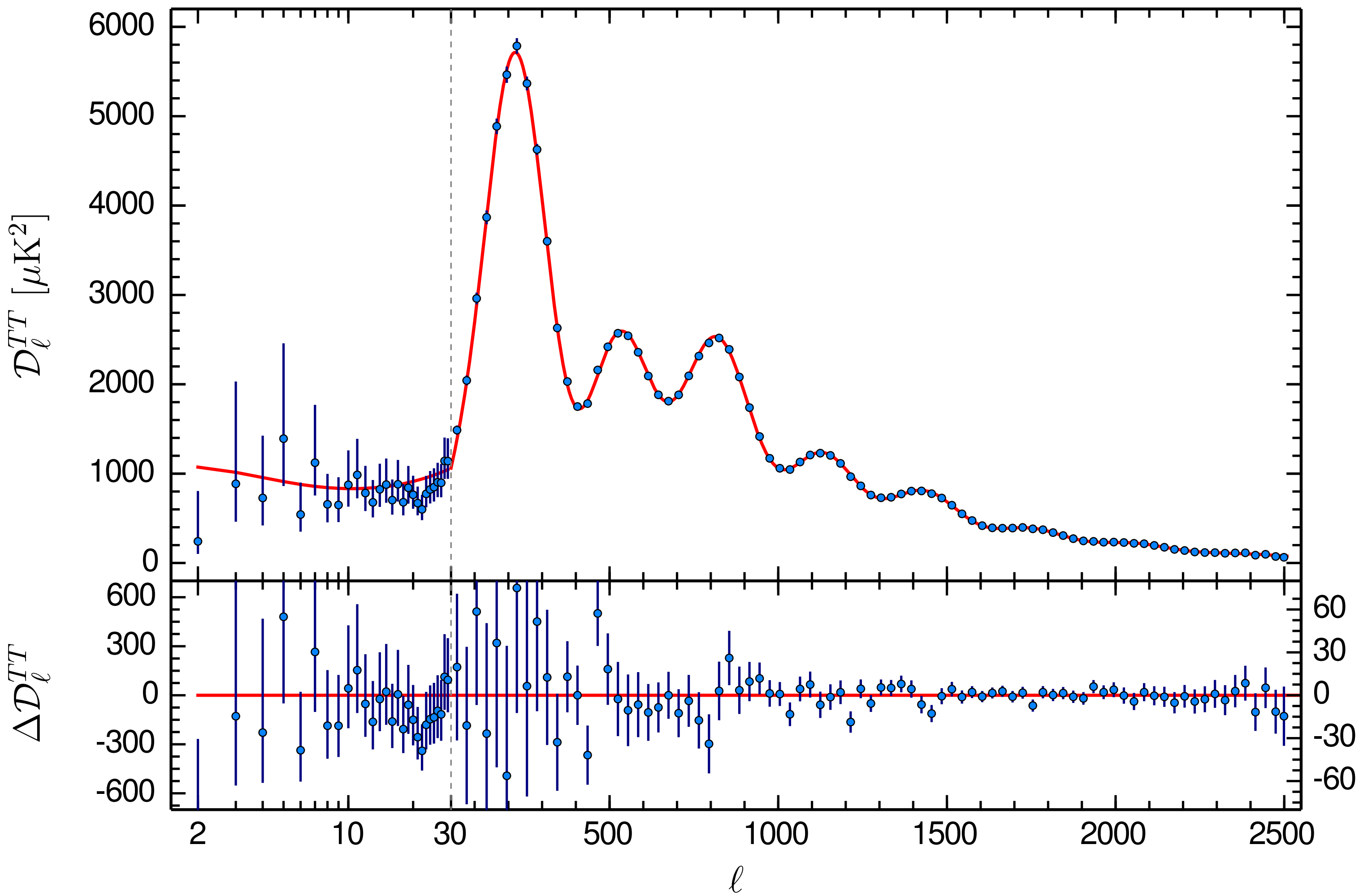
CMB Temperature Anisotropy

- Planck 2015 map of the temperature anisotropy (first discovered by COBE) from recombination:



CMB Temperature Anisotropy

- Power spectrum shows characteristic scales where the intensity of variations peak - reveals geometry and contents of the universe:



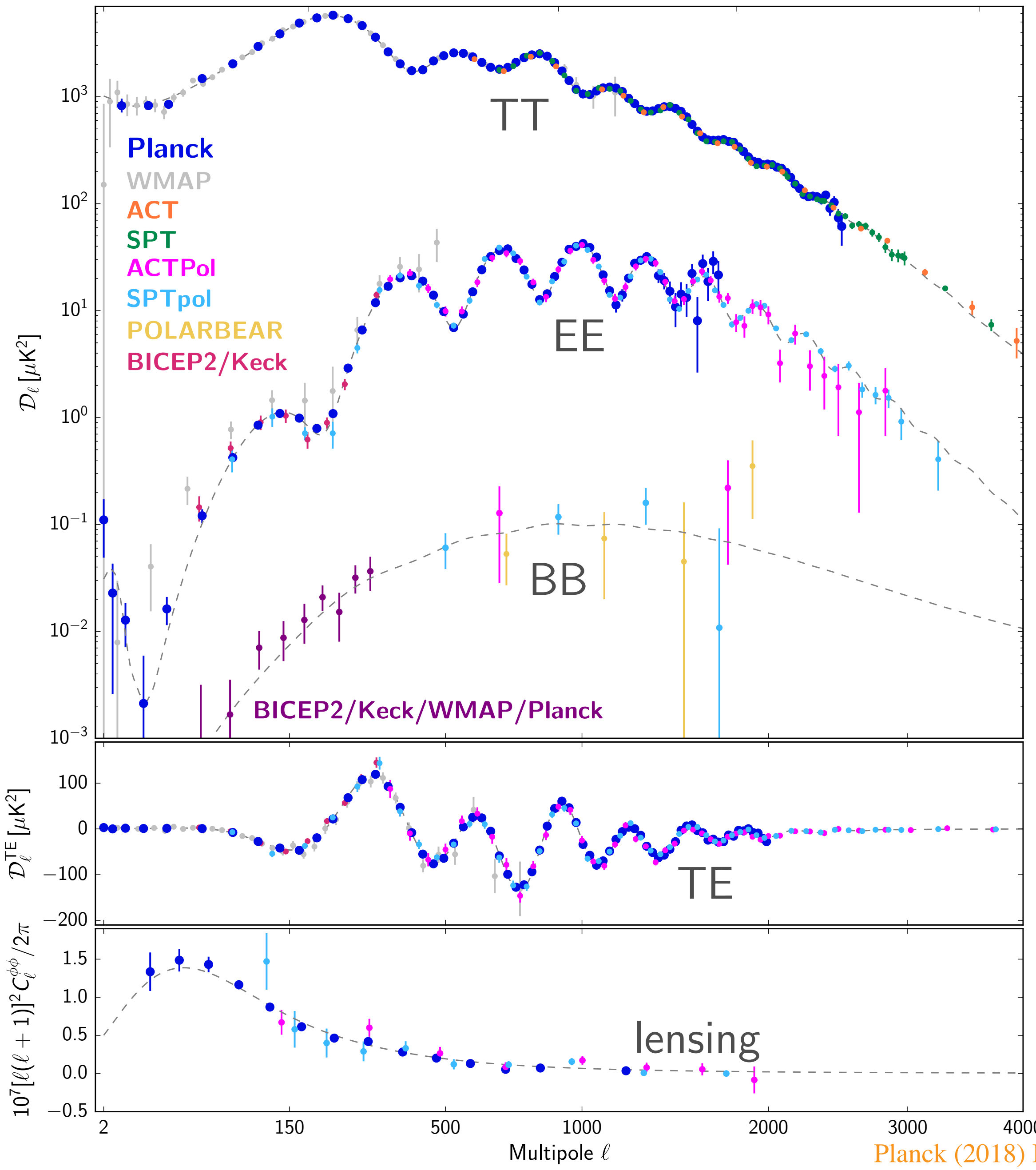
CMB Parameter Inferences

- Spectrum constrains the matter-energy contents of the universe
- Planck 2018 results [arXiv:1807.06209]

Parameter	TT+lowE 68% limits	TE+lowE 68% limits	EE+lowE 68% limits	TT,TE,EE+lowE 68% limits	TT,TE,EE+lowE+lensing 68% limits	TT,TE,EE+lowE+lensing+BAO 68% limits
$\Omega_b h^2$	0.02212 ± 0.00022	0.02249 ± 0.00025	0.0240 ± 0.0012	0.02236 ± 0.00015	0.02237 ± 0.00015	0.02242 ± 0.00014
$\Omega_c h^2$	0.1206 ± 0.0021	0.1177 ± 0.0020	0.1158 ± 0.0046	0.1202 ± 0.0014	0.1200 ± 0.0012	0.11933 ± 0.00091
$100\theta_{MC}$	1.04077 ± 0.00047	1.04139 ± 0.00049	1.03999 ± 0.00089	1.04090 ± 0.00031	1.04092 ± 0.00031	1.04101 ± 0.00029
τ	0.0522 ± 0.0080	0.0496 ± 0.0085	0.0527 ± 0.0090	$0.0544^{+0.0070}_{-0.0081}$	0.0544 ± 0.0073	0.0561 ± 0.0071
$\ln(10^{10} A_s)$	3.040 ± 0.016	$3.018^{+0.020}_{-0.018}$	3.052 ± 0.022	3.045 ± 0.016	3.044 ± 0.014	3.047 ± 0.014
n_s	0.9626 ± 0.0057	0.967 ± 0.011	0.980 ± 0.015	0.9649 ± 0.0044	0.9649 ± 0.0042	0.9665 ± 0.0038
H_0 [km s^{-1} Mpc^{-1}] . .	66.88 ± 0.92	68.44 ± 0.91	69.9 ± 2.7	67.27 ± 0.60	67.36 ± 0.54	67.66 ± 0.42

CMB Power Spectra

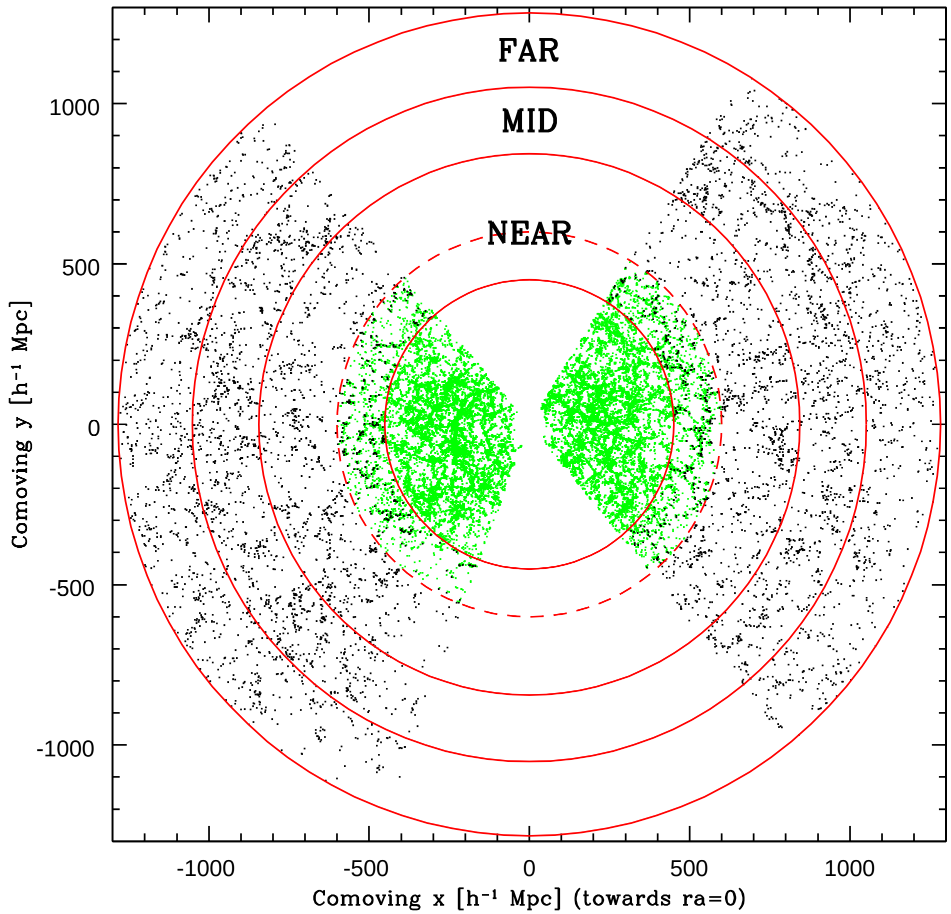
- Power spectra of CMB
  - temperature
  - polarization
  - lensing



Planck (2018) I

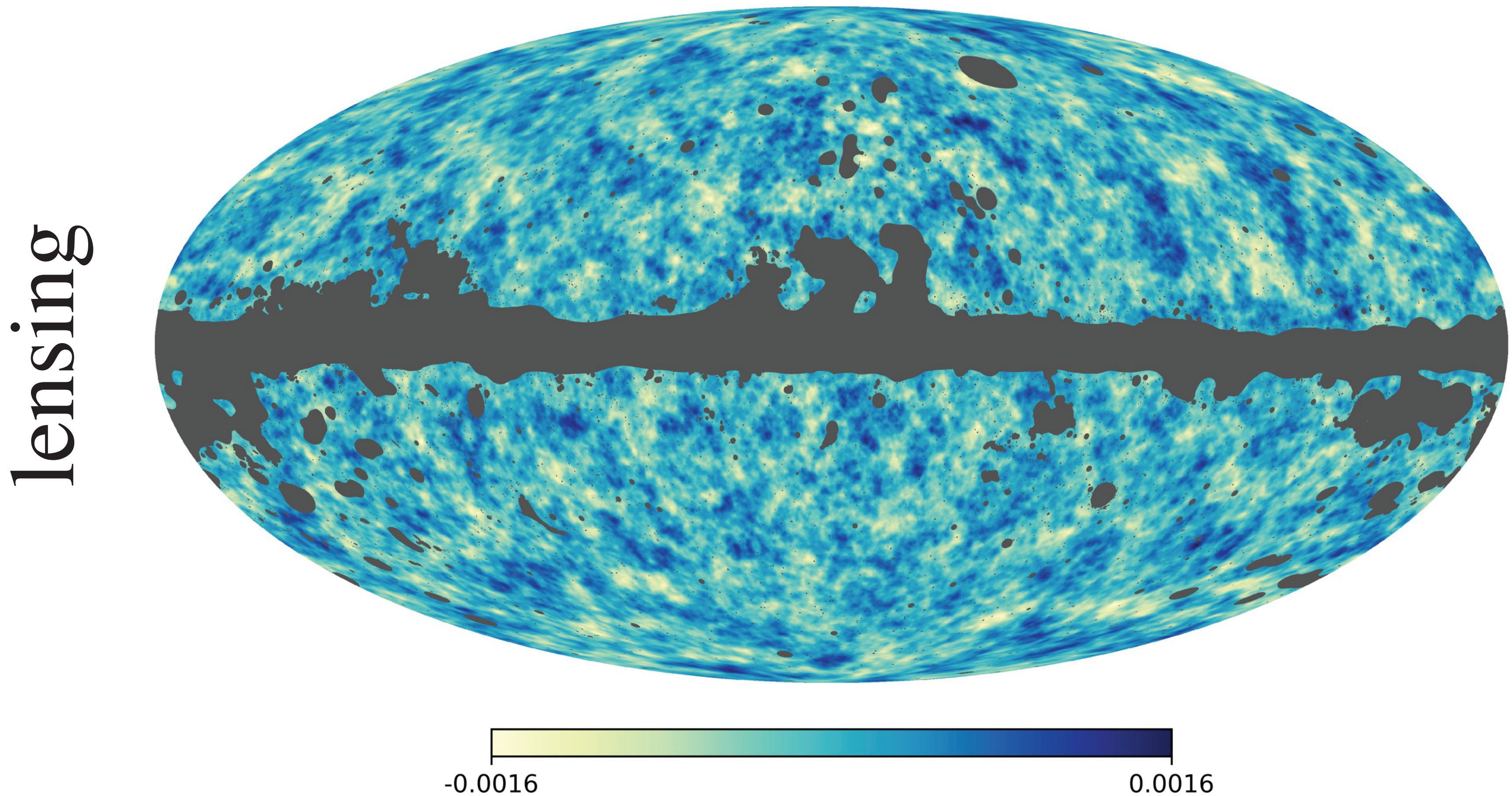
Galaxy Redshift Surveys

- Galaxy redshift surveys (e.g. 2dF and SDSS) measure the three dimensional distribution of galaxies today:



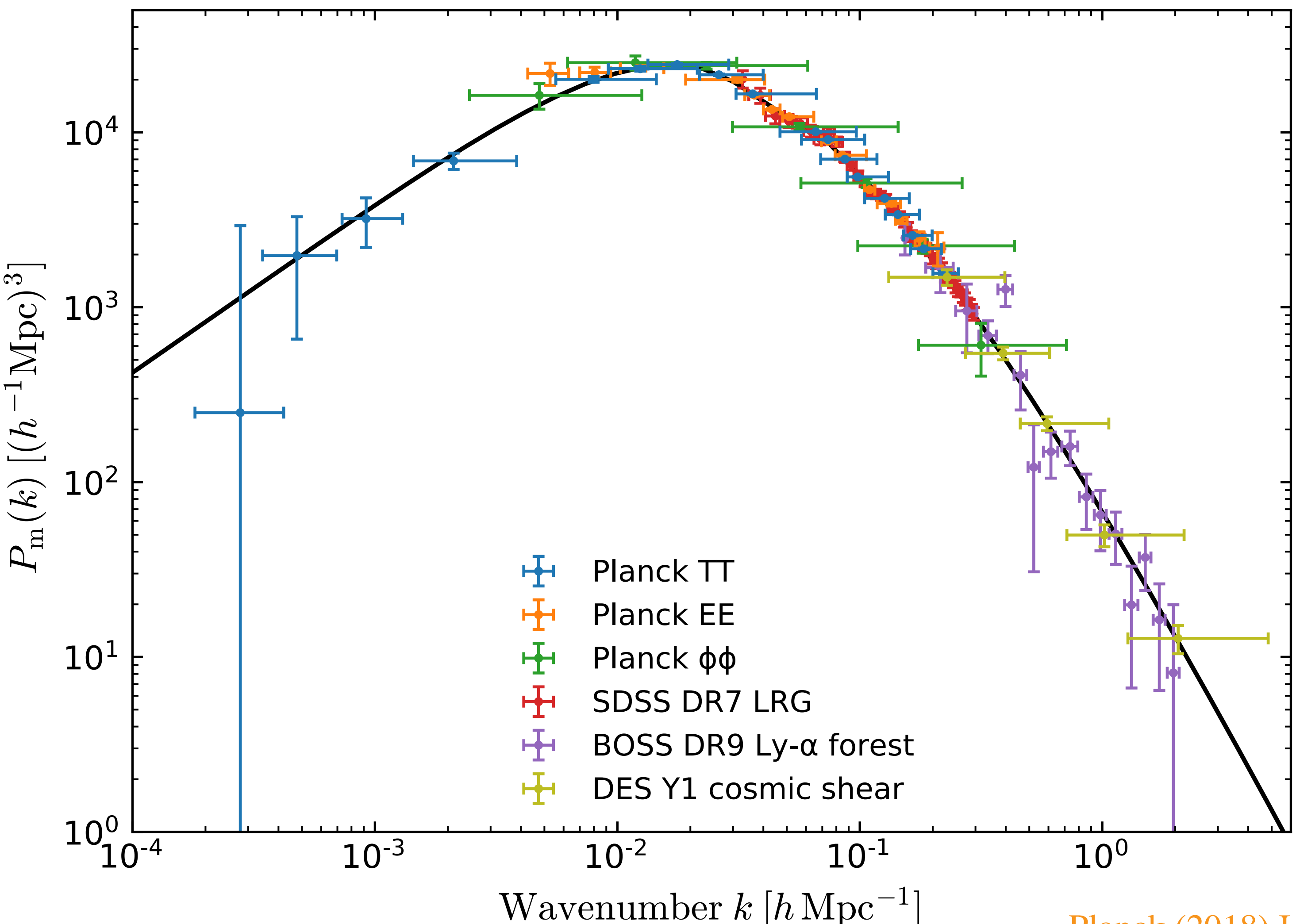
Gravitational Lensing

- Gravitational Lensing measures projected mass
- Planck CMB lensing map



Matter Power Spectrum

- Compilation of Redshift Surveys, Lensing, CMB



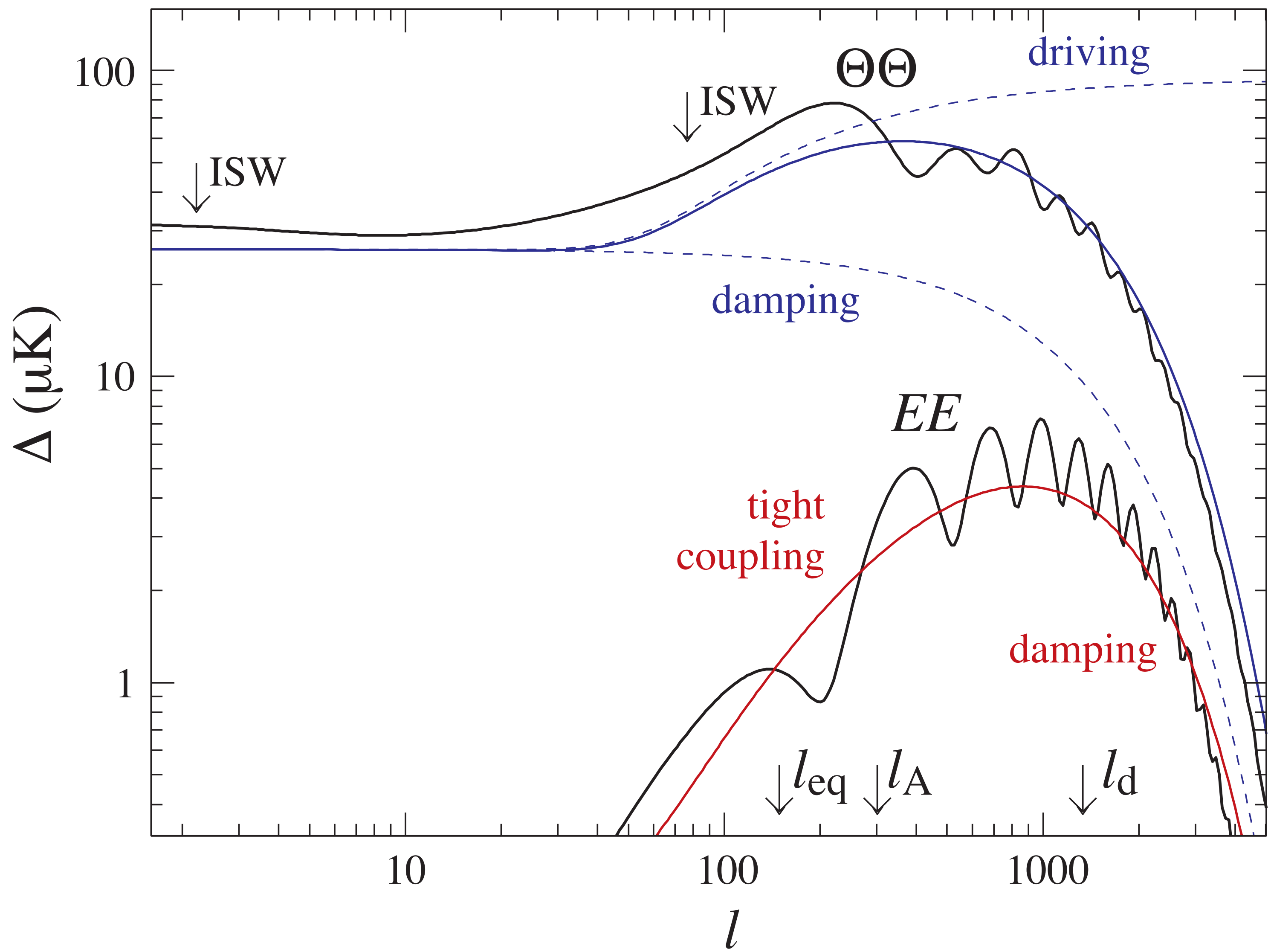
Planck (2018) I

Structure Formation

- Small perturbations from inflation over the course of the 14Gyr life of the universe are gravitationally enhanced into all of the structure seen today
- Cosmic microwave background shows a snapshot at a few hundred thousand years old at recombination
- Discovery in 1992 of cosmic microwave background anisotropy provided the observational breakthrough - convincing support for adiabatic initial density fluctuations of amplitude 10^{-5}
- Combine with galaxy clustering - large scale structure seen in galaxy surveys - right amplitude given cold dark matter

Schematic CMB Spectrum

- Take apart features in the power spectrum



Fluid Approximation

- Thomson scattering of photons and free electrons before recombination is sufficiently rapid that the bayrons and photons are in equilibrium and hence move together
- Mean free path of the photons for $z \approx 10^3$ and $\Omega_b h^2 \approx 0.02$

$$\lambda_C \equiv \frac{1}{n_e \sigma_T a} \sim 2.5\mathrm{Mpc}$$

small by cosmological standards!

- On scales $\lambda \gg \lambda_C$ photons are tightly coupled to the electrons by Thomson scattering which in turn are tightly coupled to the baryons by Coulomb interactions
- Specifically, their bulk velocities are defined by a single fluid velocity $v_\gamma = v_b$ and the photons carry no anisotropy in the rest frame of the baryons

Zeroth Order Approximation

- Momentum density of a fluid is $(\rho + p)v$, where p is the pressure
- Neglect the momentum density of the baryons

$$R \equiv \frac{(\rho_b + p_b)v_b}{(\rho_\gamma + p_\gamma)v_\gamma} = \frac{\rho_b + p_b}{\rho_\gamma + p_\gamma} = \frac{3\rho_b}{4\rho_\gamma}$$

$$\approx 0.6 \left(\frac{\Omega_b h^2}{0.02}\right)\left(\frac{a}{10^{-3}}\right)$$

since $\rho_\gamma \propto T^4$ is fixed by the CMB temperature $T = 2.73(1+z)$K – OK substantially before recombination

- Neglect radiation in the expansion

$$\frac{\rho_m}{\rho_r} = 3.6 \left(\frac{\Omega_m h^2}{0.15}\right)\left(\frac{a}{10^{-3}}\right)$$

- Neglect gravity

Fluid Equations

- Density $n_\gamma \propto T^3$ so define temperature fluctuation Θ

$$\delta n_\gamma = 3\frac{\delta T}{T} n_\gamma \equiv 3\Theta n_\gamma$$

- Real space continuity eqn.: the local number or energy density of photons changes if there is a divergence of the velocity field - a flow inwards or outwards or a change in the volume
- We know in the background expansion $n_\gamma \propto a^{-3}$ so continuity:

$$[a^3 \delta n_\gamma]^{\cdot} \;\; = \;\; -a^3 n_\gamma \nabla \cdot \mathbf{v}_\gamma$$

which we transform to Fourier space $\nabla(e^{i\mathbf{k}\cdot\mathbf{x}}) \to i\mathbf{k}(e^{i\mathbf{k}\cdot\mathbf{x}})$

$$\dot\Theta = -\frac{1}{3} k v_\gamma$$

Fluid Equations

- Euler equation (neglecting gravity for now): momentum conservation says that pressure gradients generate changes in momentum density $k\delta p_\gamma = kc_s^2\delta\rho_\gamma$

$$\begin{aligned}\dot{v}_\gamma &= \frac{kc_s^2}{1+w_\gamma}\delta_\gamma \\ &= kc_s^2\frac{3}{4}\delta_\gamma = 3c_s^2 k\Theta\end{aligned}$$

  where the sound speed $c_s^2 = \delta p/\delta\rho$ is the pressure response to a density fluctuation

- So if you squeeze the photon gas to raise its density, its going to respond with a restoring force by raising the pressure and resisting compression $\rightarrow$ acoustic oscillations

Oscillator: Take One

- Combine these to form the simple harmonic oscillator equation

$$\ddot{\Theta} + c_s^2 k^2 \Theta = 0$$

where the sound speed is adiabatic

$$c_s^2 = \frac{\delta p}{\delta \rho} = \frac{\dot{p}_\gamma}{\dot{\rho}_\gamma}$$

here $c_s^2 = 1/3$ since we are photon-dominated

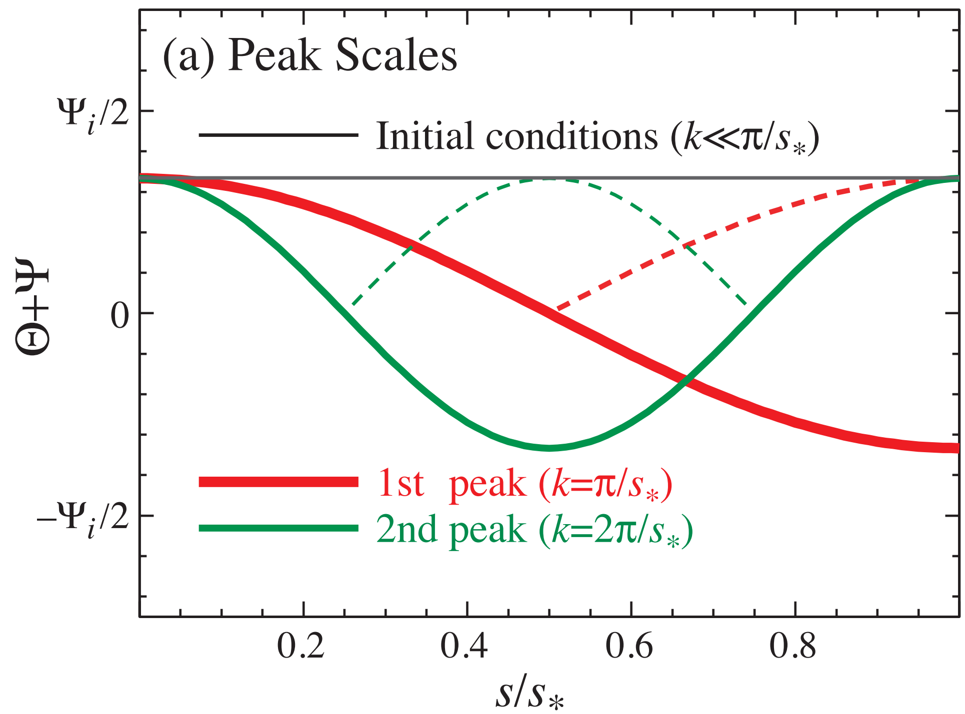
- General solution:

$$\Theta(\eta) = \Theta(0)\cos(ks) + \frac{\dot{\Theta}(0)}{kc_s}\sin(ks)$$

where the sound horizon is defined as $s \equiv \int c_s d\eta$

Harmonic Extrema

- All modes begin at end of inflation and are frozen in at recombination (denoted with a subscript $*$)
- Temperature perturbations of different amplitude for different modes.



- For the adiabatic (curvature mode) initial conditions

$$\dot{\Theta}(0) = 0$$

- So solution

$$\Theta(\eta_*) = \Theta(0)\cos(ks_*)$$

Harmonic Extrema

- Modes caught in the extrema of their oscillation will have enhanced fluctuations

$$k_n s_* = n\pi$$

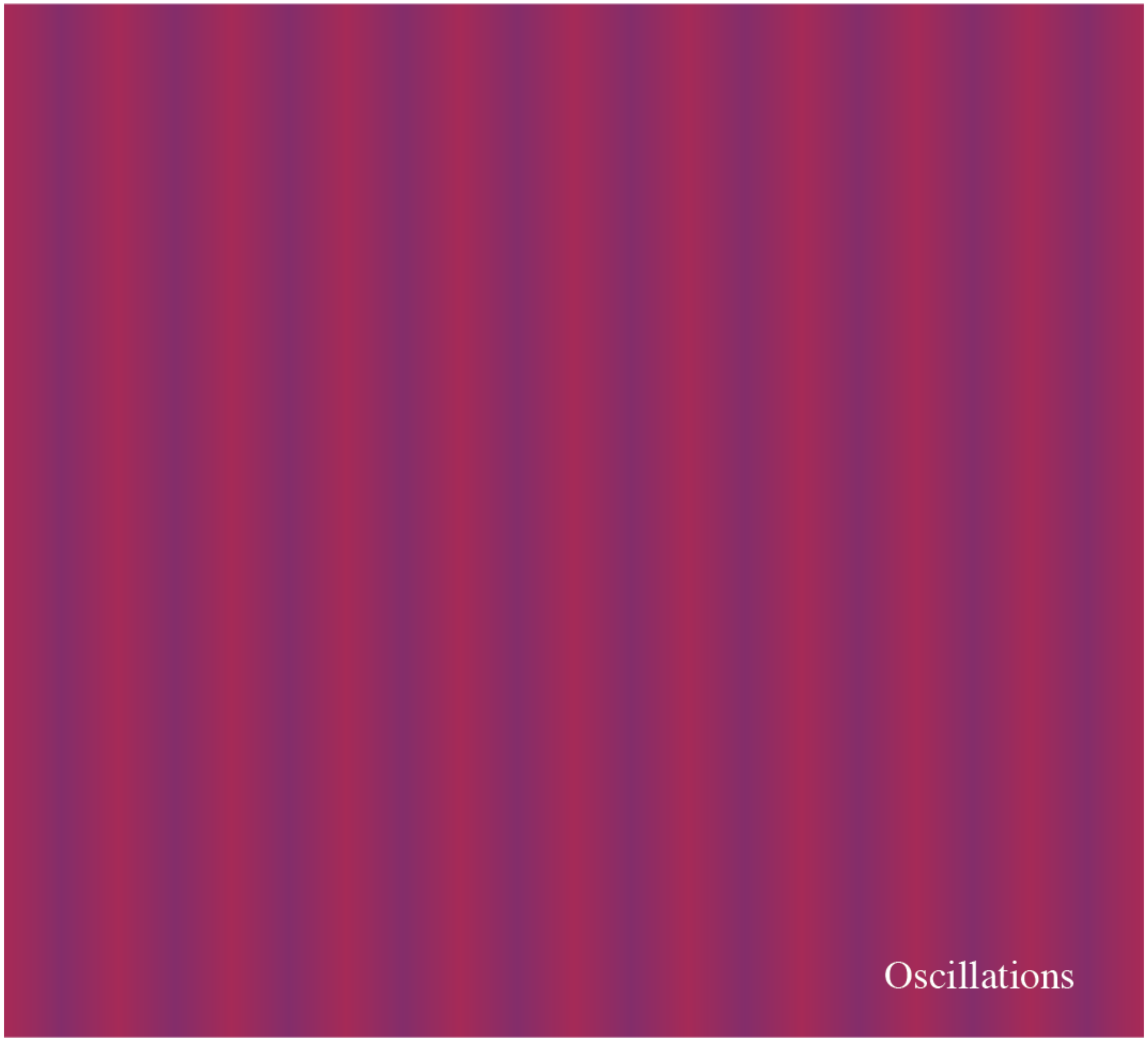
yielding a fundamental scale or frequency, related to the inverse sound horizon

$$k_A = \pi / s_*$$

and a harmonic relationship to the other extrema as $1 : 2 : 3...$

Temperature Anisotropy

- Spatial oscillations frozen at recombination; photons then stream
- Viewed at distance D_* as angular anisotropy $L \approx kD_*$



Peak Location

- The fundmental physical scale is translated into a fundamental angular scale by simple projection according to the angular diameter distance D_A

$$\begin{aligned} \theta_A &= \lambda_A / D_A \\ \ell_A &= k_A D_A \end{aligned}$$

- In a flat universe, the distance is simply $D_A = D \equiv \eta_0 - \eta_* \approx \eta_0$, the horizon distance, and $k_A = \pi / s_* = \sqrt{3}\pi / \eta_*$ so

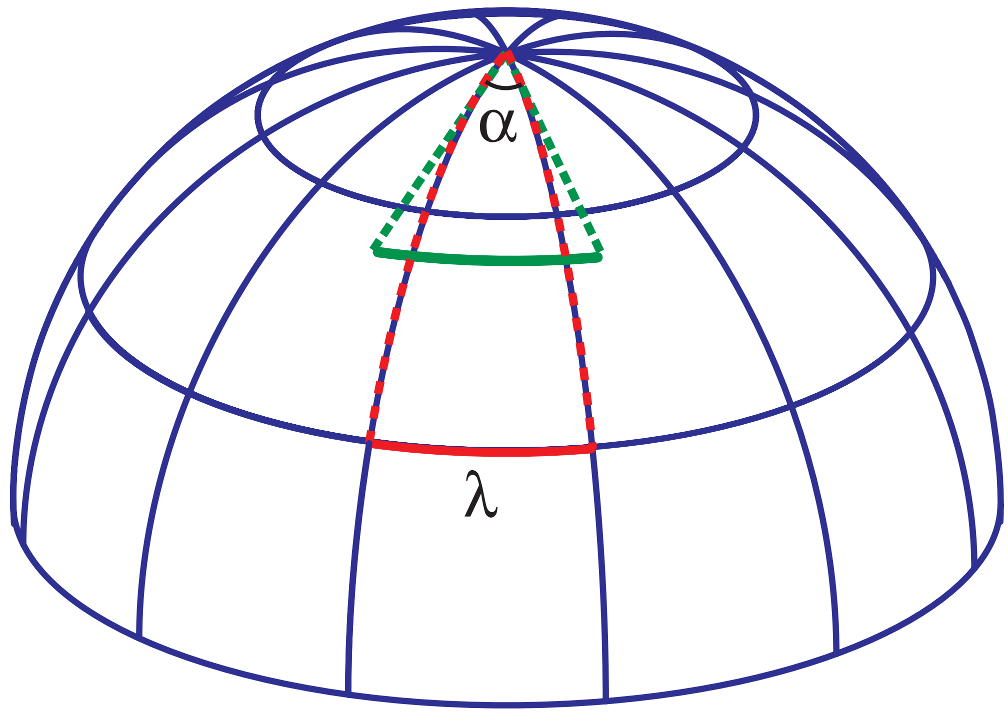
$$\theta_A \approx \frac{\eta_*}{\eta_0}$$

- In a matter-dominated universe $\eta \propto a^{1/2}$ so $\theta_A \approx 1/30 \approx 2^\circ$ or

$$\ell_A \approx 200$$

Curvature

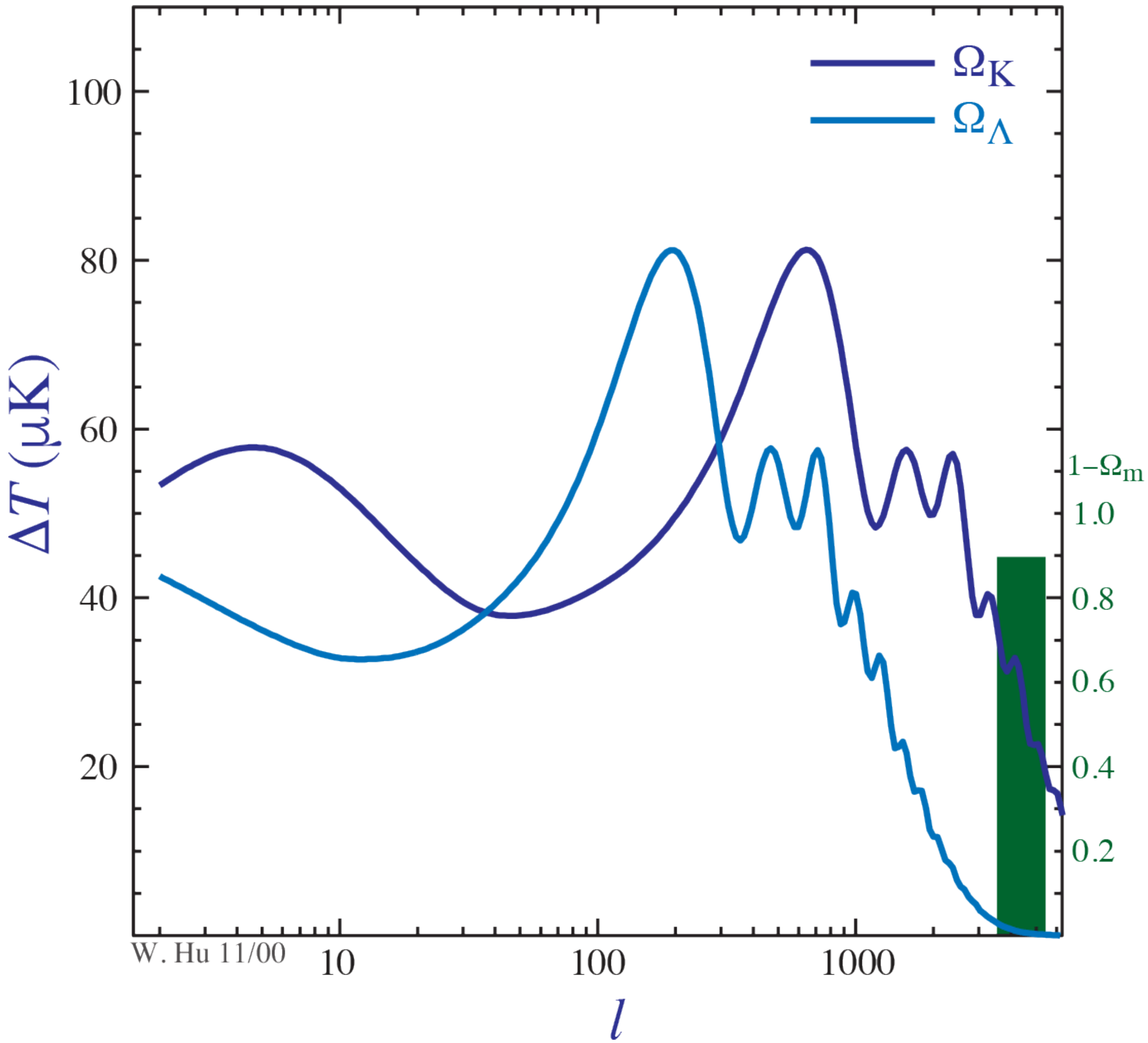
- In a curved universe, the apparent or angular diameter distance is no longer the conformal distance $D_A = R\sin(D/R) \neq D$



- Objects in a closed universe are further than they appear! gravitational lensing of the background...
- Curvature scale of the universe must be substantially larger than current horizon

Curvature in the CMB

- Curvature and Λ – consistent with flat ΛCDM



Restoring Gravity

- Take a simple photon dominated system with gravity
- Continuity altered since a gravitational potential represents a stretching of the spatial fabric that dilutes number densities – formally a spatial curvature perturbation
- Think of this as a perturbation to the scale factor $a \to a(1 + \Phi)$ so that the cosmogical redshift is generalized to

$$\frac{\dot{a}}{a} \to \frac{\dot{a}}{a} + \dot{\Phi}$$

so that the continuity equation becomes

$$\dot{\Theta} = -\frac{1}{3} k v_\gamma - \dot{\Phi}$$

Restoring Gravity

- Gravitational force in momentum conservation $\mathbf{F} = -m\nabla\Psi$ generalized to momentum density modifies the Euler equation to

$$\dot{v}_\gamma = k(\Theta + \Psi)$$

- General relativity says that Φ and Ψ are the relativistic analogues of the Newtonian potential and that $\Phi \approx -\Psi$.
- In our matter-dominated approximation, Φ represents matter density fluctuations through the cosmological Poisson equation

$$k^2\Phi = 4\pi G a^2 \rho_m \Delta_m$$

where the difference comes from the use of comoving coordinates for k (a^2 factor), the removal of the background density into the background expansion $(\rho\Delta_m)$ and finally a coordinate subtlety that enters into the definition of Δ_m

Constant Potentials

- In the matter dominated epoch potentials are constant because infall generates velocities as $v_m \sim k\eta\Psi$
- Velocity divergence generates density perturbations as $\Delta_m \sim -k\eta v_m \sim -(k\eta)^2\Psi$
- And density perturbations generate potential fluctuations

$$\Phi = \frac{4\pi G a^2 \rho \Delta}{k^2} \approx \frac{3}{2}\frac{H^2 a^2}{k^2}\Delta \sim \frac{\Delta}{(k\eta)^2} \sim -\Psi$$

keeping them constant. Note that because of the expansion, density perturbations must grow to keep potentials constant.

Constant Potentials

- More generally, if stress perturbations are negligible compared with density perturbations ($\delta p \ll \delta\rho$) then potential will remain roughly constant
- More specifically a variant called the Bardeen or comoving curvature is strictly constant

$$\mathcal{R} = \mathrm{const} \approx \frac{5+3w}{3+3w}\Phi$$

where the approximation holds when $w \approx$const.

Oscillator: Take Two

- Combine these to form the simple harmonic oscillator equation

$$\ddot{\Theta} + c_s^2 k^2 \Theta = -\frac{k^2}{3}\Psi - \ddot{\Phi}$$

- In a CDM dominated expansion $\dot{\Phi} = \dot{\Psi} = 0$. Also for photon domination $c_s^2 = 1/3$ so the oscillator equation becomes

$$\ddot{\Theta} + \ddot{\Psi} + c_s^2 k^2 (\Theta + \Psi) = 0$$

- Solution is just an offset version of the original

$$[\Theta + \Psi](\eta) = [\Theta + \Psi](0)\,\cos(ks)$$

- $\Theta + \Psi$ is also the observed temperature fluctuation since photons lose energy climbing out of gravitational potentials at recombination

Effective Temperature

- Photons climb out of potential wells at last scattering
- Lose energy to gravitational redshifts
- Observed or effective temperature

$$\Theta + \Psi$$

- Effective temperature oscillates around zero with amplitude given by the initial conditions
- Note: initial conditions are set when the perturbation is outside of horizon, need inflation or other modification to matter-radiation FRW universe.
- GR says that initial temperature is given by initial potential

Sachs-Wolfe Effect and the Magic 1/3

- A gravitational potential is a perturbation to the temporal coordinate [formally a gauge transformation]

$$\frac{\delta t}{t} = \Psi$$

- Convert this to a perturbation in the scale factor, in a matter dominated expansion $a \propto t^{2/3}$ so

$$\frac{\delta a}{a} = \frac{2}{3}\frac{\delta t}{t}$$

- CMB temperature is cooling as $T \propto a^{-1}$ so

$$\Theta + \Psi \equiv \frac{\delta T}{T} + \Psi = -\frac{\delta a}{a} + \Psi = \frac{1}{3}\Psi$$

Sachs-Wolfe Normalization

- Use measurements of $\Delta T/T \approx 10^{-5}$ in the Sachs-Wolfe effect to infer $\Delta^2_{\mathcal{R}}$
- Recall in matter domination $\Psi = -3\mathcal{R}/5$ and so $\Delta T/T = -\mathcal{R}/5$
- So that the amplitude of initial curvature fluctuations is $\Delta_R \approx 5 \times 10^{-5}$
- This then determines the amplitude of the inflationary power spectrum $A_S = \Delta^2_{\mathcal{R}}$ in the previous lecture set

Baryon Loading

- Baryons add extra mass to the photon-baryon fluid
- Controlling parameter is the momentum density ratio:

$$R \equiv \frac{p_b + \rho_b}{p_\gamma + \rho_\gamma} \approx 30\Omega_b h^2 \left(\frac{a}{10^{-3}}\right)$$

of order unity at recombination

- Momentum density of the joint system is conserved

$$(\rho_\gamma + p_\gamma)v_\gamma + (\rho_b + p_b)v_b \approx (p_\gamma + p_\gamma + \rho_b + \rho_\gamma)v_\gamma = (1 + R)(\rho_\gamma + p_\gamma)v_\gamma$$

- Momentum density ratio enters as

$$[(1 + R)v_\gamma]^{\cdot} = k\Theta + (1 + R)k\Psi$$

- Oscillations around hydrostatic equilibrium point:
  $\Theta + (1 + R)\Psi = 0$ – like clusters, measurement of dark matter

New Euler Equation

- Photon continuity remains the same

$$\dot{\Theta} = -\frac{k}{3} v_\gamma - \dot{\Phi}$$

- Modification of oscillator equation

$$\frac{d}{d\eta}[(1+R)\dot{\Theta}] + \frac{1}{3}k^2\Theta = -\frac{1}{3}k^2(1+R)\Psi - \frac{d}{d\eta}[(1+R)\dot{\Phi}]$$

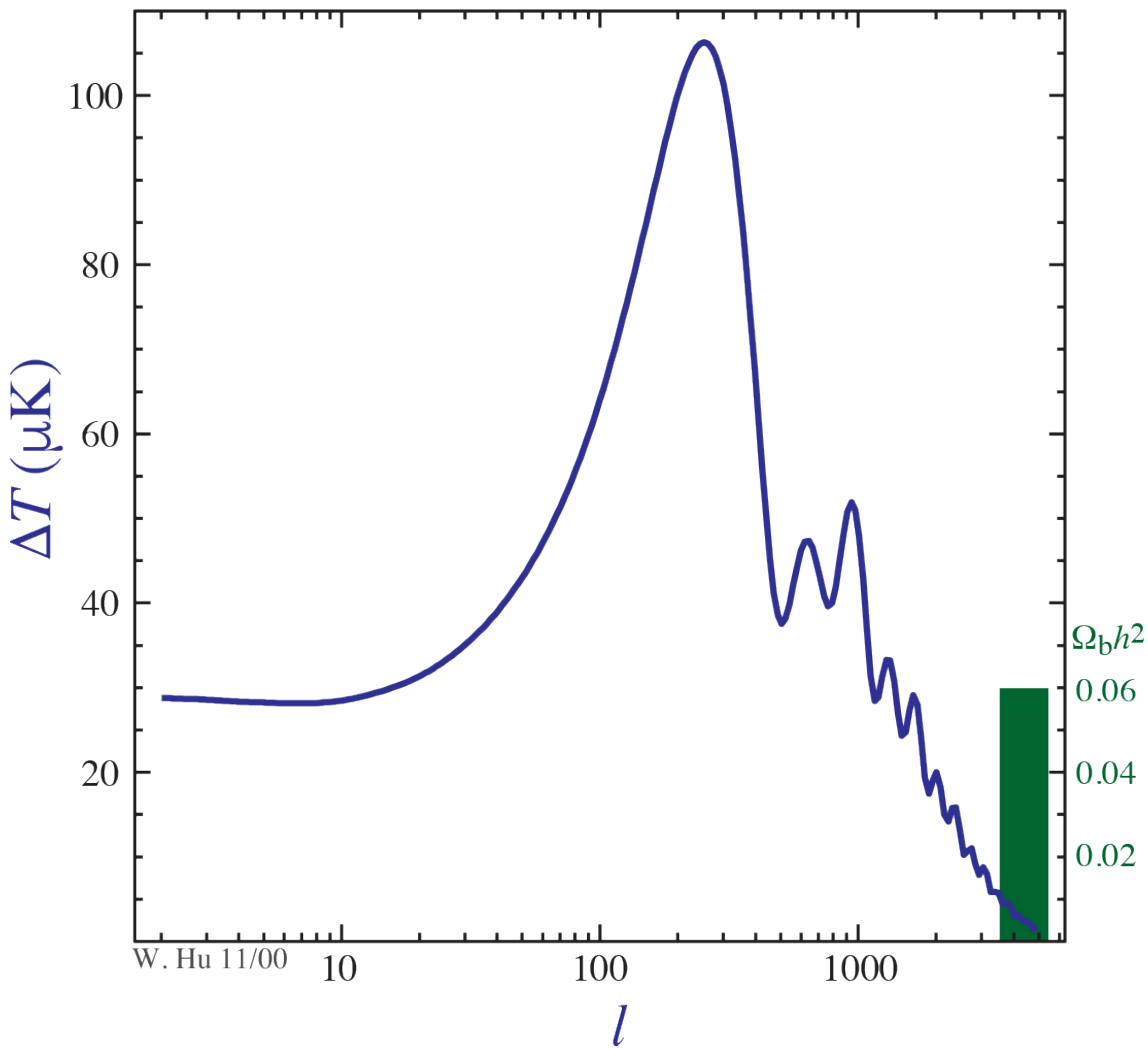
- In a CDM dominated expansion $\dot{\Phi} = \dot{\Psi} = 0$ and the adiabatic approximation where the sound speed evolves slowly

$$c_s = \sqrt{\frac{1}{3}\frac{1}{1+R}}$$

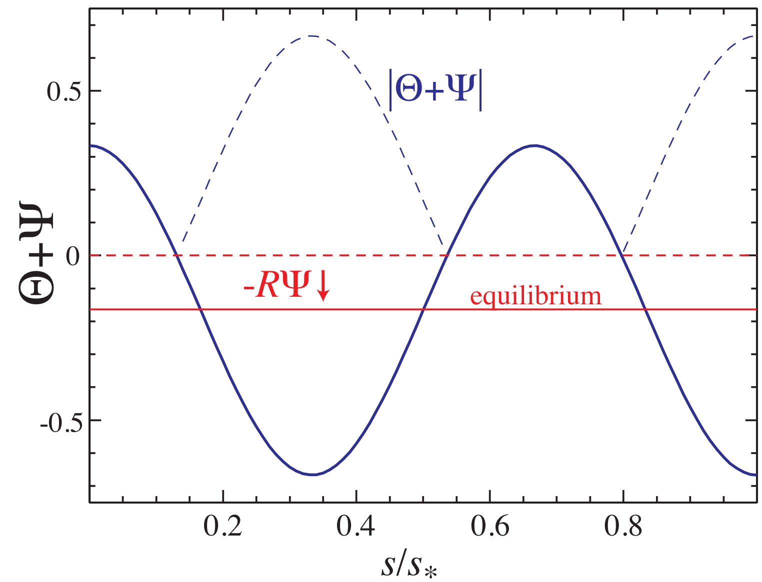
$$[\Theta + (1+R)\Psi](\eta) = [\Theta + (1+R)\Psi](0)\cos(ks)$$

Baryons in the CMB

- Modulation, amplitude, sound horizon scale



Baryon Peak Phenomenology



- Photon-baryon ratio enters in three ways
- Overall larger amplitude:

$$[\Theta + (1+R)\Psi](0) = \frac{1}{3}(1+3R)\Psi(0)$$

- Even-odd peak modulation of effective temperature

$$[\Theta + \Psi]_{\rm peaks} = [\pm(1+3R) - 3R]\,\frac{1}{3}\Psi(0)$$

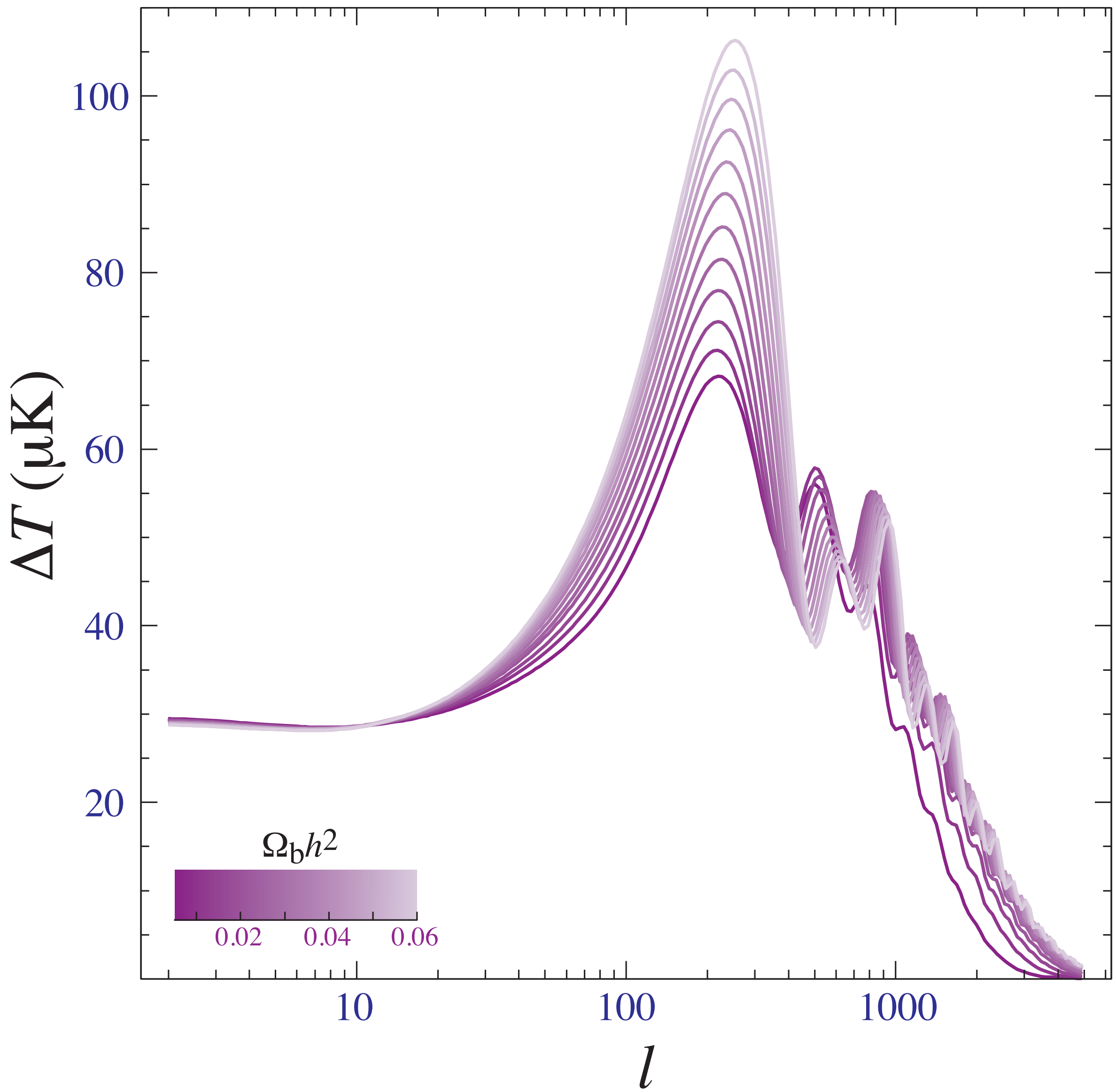
$$[\Theta + \Psi]_1 - [\Theta + \Psi]_2 = [-6R]\frac{1}{3}\Psi(0)$$

- Shifting of the sound horizon down or ℓ_A up

$$\ell_A \propto \sqrt{1+R}$$

Baryons in the Power Spectrum

- Relative heights of peaks



Oscillator: Take Three and a Half

- The not-quite-so simple harmonic oscillator equation is a forced harmonic oscillator

$$(c_s^{-2}\dot{\Theta})^{\cdot} + k^2\Theta = -\frac{k^2}{3}c_s^{-2}\Psi - (c_s^{-2}\dot{\Phi})^{\cdot}$$

  changes in the gravitational potentials alter the form of the acoustic oscillations

- If the forcing term has a temporal structure that is related to the frequency of the oscillation, this becomes a driven harmonic oscillator
- Term involving Ψ is the ordinary gravitational force
- Term involving Φ involves the $\dot{\Phi}$ term in the continuity equation as a (curvature) perturbation to the scale factor

Potential Decay

- Matter-to-radiation ratio

$$\frac{\rho_m}{\rho_r} \approx 24\Omega_m h^2 \left(\frac{a}{10^{-3}}\right)$$

  of order unity at recombination in a low Ω_m universe

- Radiation is not stress free and so impedes the growth of structure

$$k^2\Phi = 4\pi G a^2 \rho_r \Delta_r$$

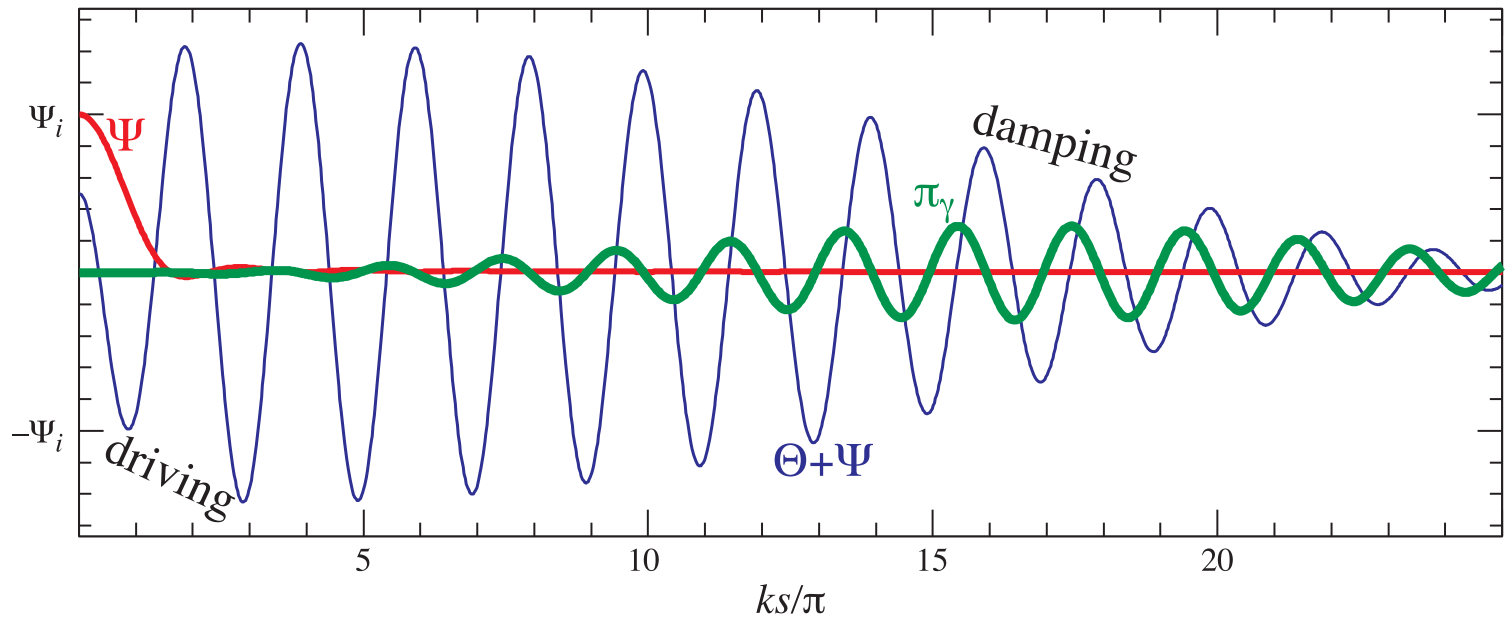
  $\Delta_r \sim 4\Theta$ oscillates around a constant value, $\rho_r \propto a^{-4}$ so the Netwonian curvature decays.

- General rule: potential decays if the dominant energy component has substantial stress fluctuations, i.e. below the generalized sound horizon or Jeans scale

Radiation Driving

- Decay is timed precisely to drive the oscillator - close to fully coherent

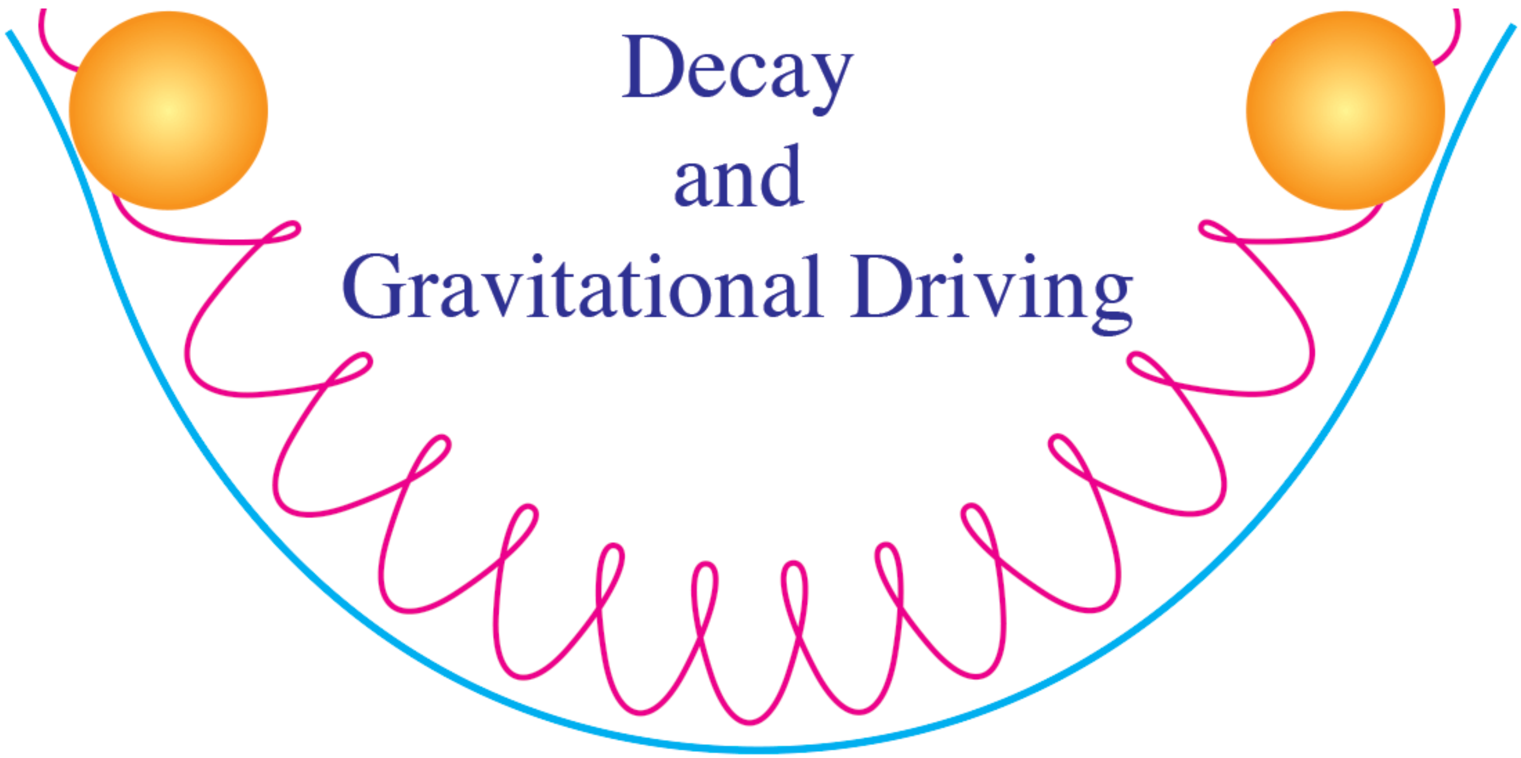
$$|[\Theta + \Psi](\eta)| = |[\Theta + \Psi](0) + \Delta\Psi - \Delta\Phi|$$
$$= |\frac{1}{3}\Psi(0) - 2\Psi(0)| = |\frac{5}{3}\Psi(0)|$$



- $5\times$ the amplitude of the Sachs-Wolfe effect!

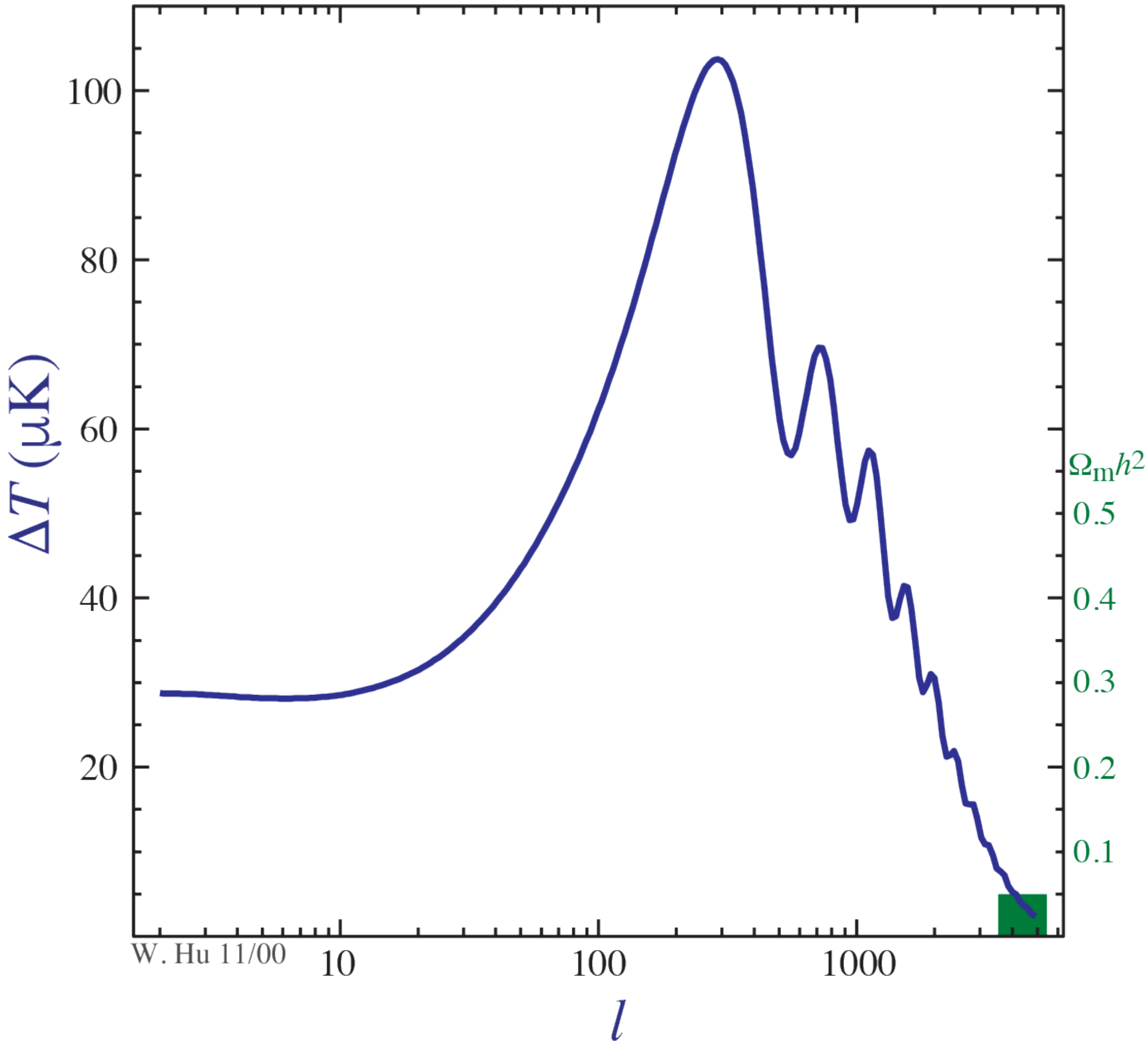
Radiation Driving

- Cartoon version (doubled by local scale factor Φ effect):



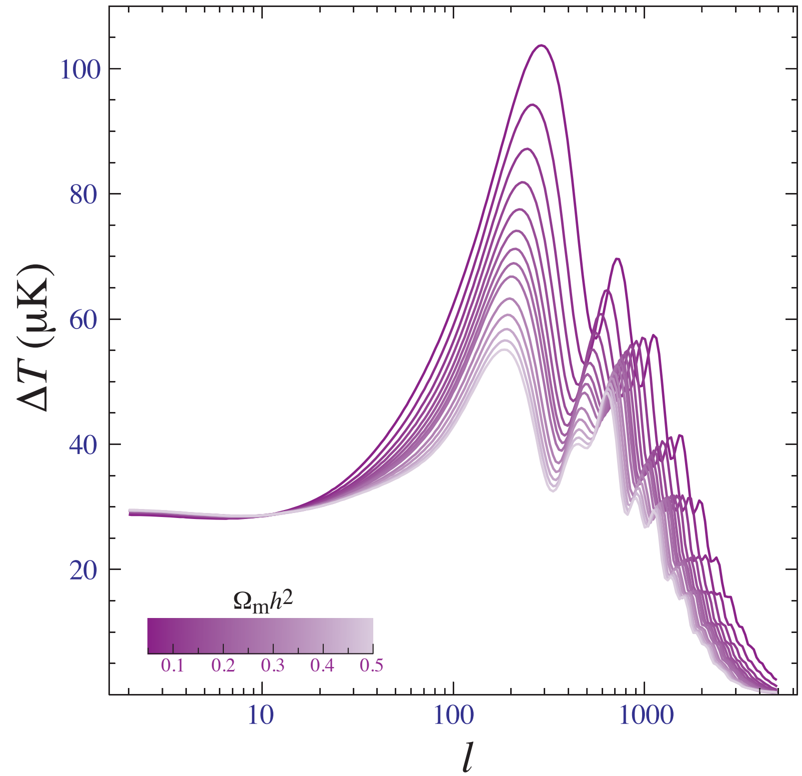
Cold Dark Matter in the CMB

- Hydrostatic equilibrium, oscillation forcing, damping



Matter-Radiation in the Power Spectrum

- Coherent approximation is exact for a photon-baryon fluid but reality is reduced to $\sim 4\times$ because neutrino contribution is free streaming not fluid like
- Neutrinos drive the oscillator less efficiently and also slightly change the phase of the oscillation



- Actual initial conditions are $\Theta + \Psi = \Psi/2$ for radiation domination but comparison to matter dominated SW correct
- With 3 peaks, it is possible to solve for both the baryons and dark matter densities, providing a calibration for the sound horizon
- Higher peaks check consistency with assumptions: e.g. extra relativistic d.o.f.s

Damping

- Tight coupling equations assume a perfect fluid: no viscosity, no heat conduction
- Fluid imperfections are related to the mean free path of the photons in the baryons

$$\lambda_C = \dot{\tau}^{-1} \quad \text{where} \quad \dot{\tau} = n_e \sigma_T a$$

is the conformal opacity to Thomson scattering

- Dissipation is related to the diffusion length: random walk approximation

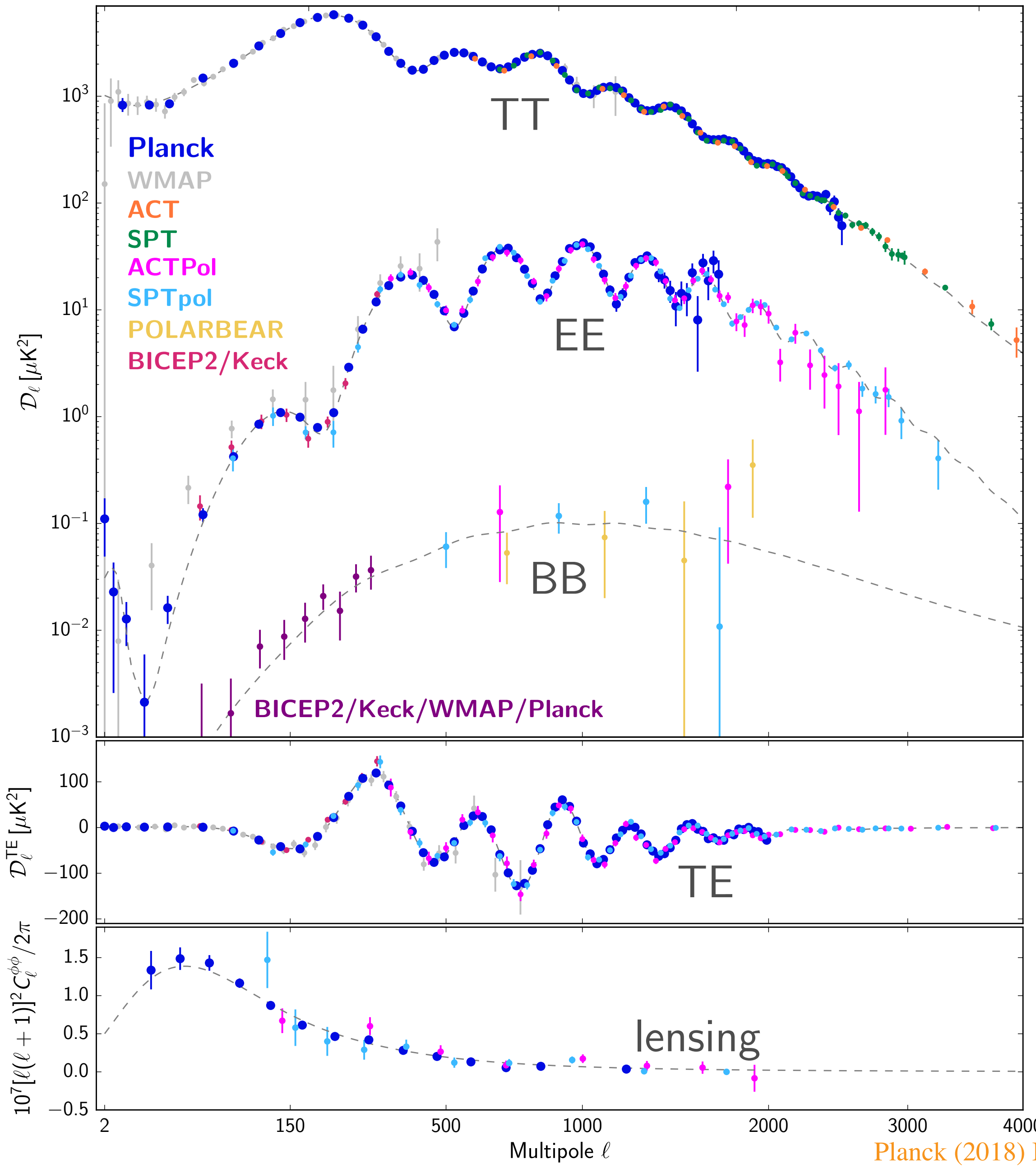
$$\lambda_D = \sqrt{N} \lambda_C = \sqrt{\eta / \lambda_C}\, \lambda_C = \sqrt{\eta \lambda_C}$$

the geometric mean between the horizon and mean free path

- $\lambda_D / \eta_* \sim$ few %, so expect the peaks $:> 3$ to be affected by dissipation

Near Perfection in 6 Numbers

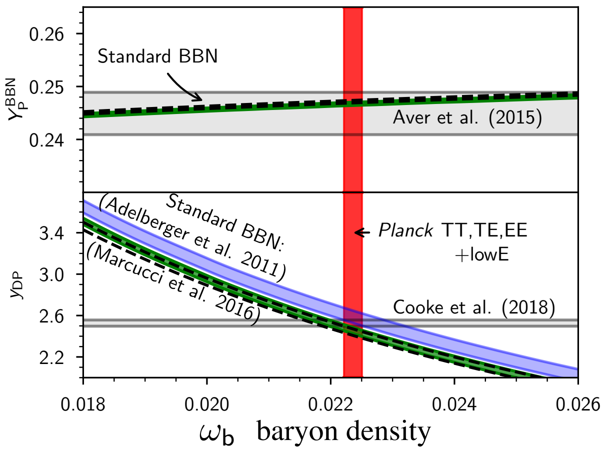
- All this precision data described by 6 ΛCDM parameters
  - – $\Omega_c h^2$: CDM
  - – $\Omega_b h^2$: baryons
  - – θ_s: sound scale
  - – A_s: amplitude
  - – n_s: tilt
  - – τ: reionization
- Measured to sub percent precision (except τ)



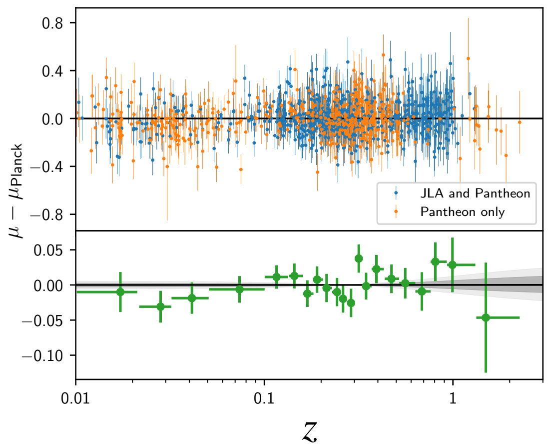
Predictive Power

- Predicts all other observables, which direct measurements test

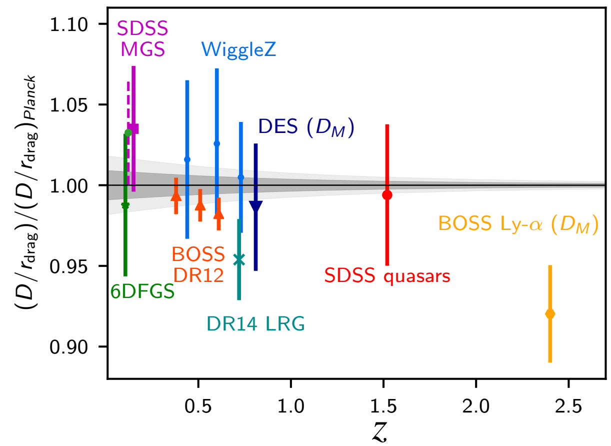
BBN

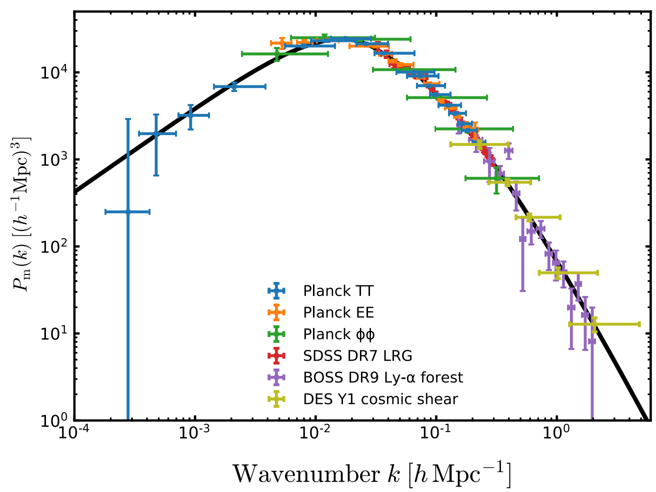


SNIa

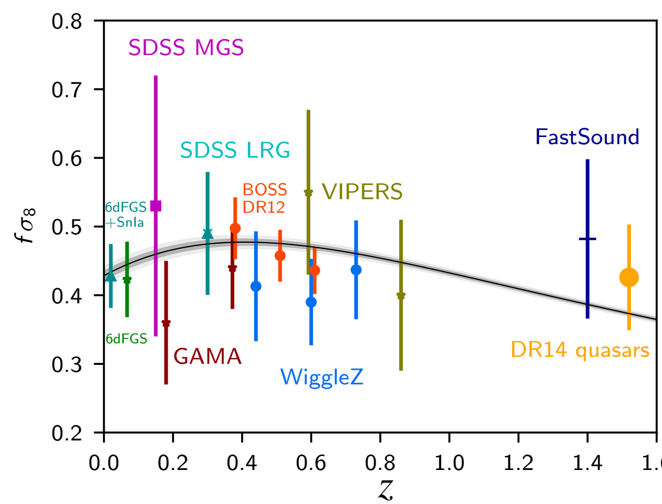


BAO





$P(k)$



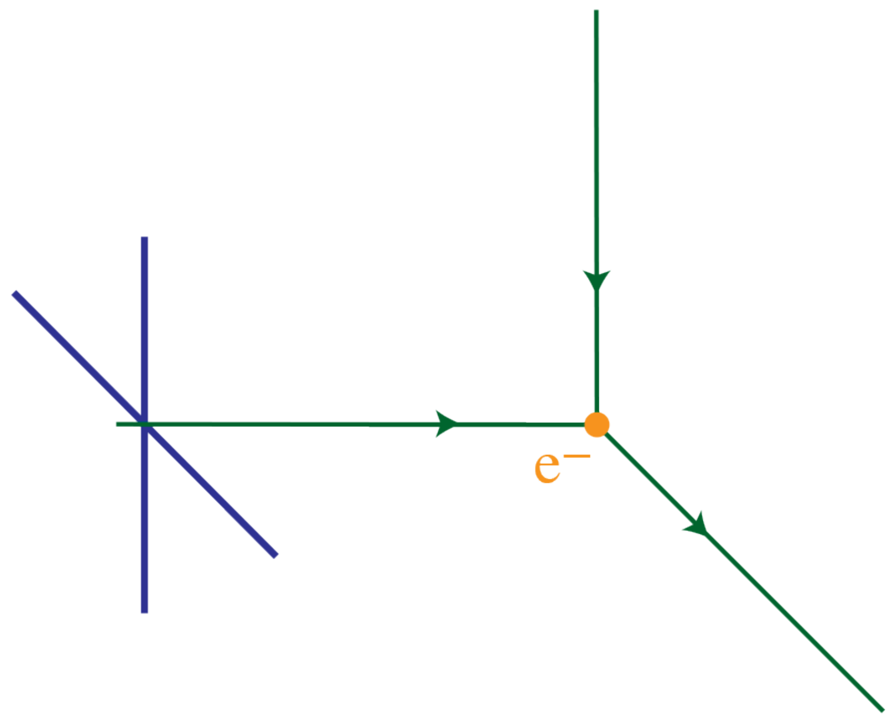
growth rate

Planck (2018) I,VI

- Good agreement, even weak lensing, clusters, and yes H_0 ($< 10\%$)

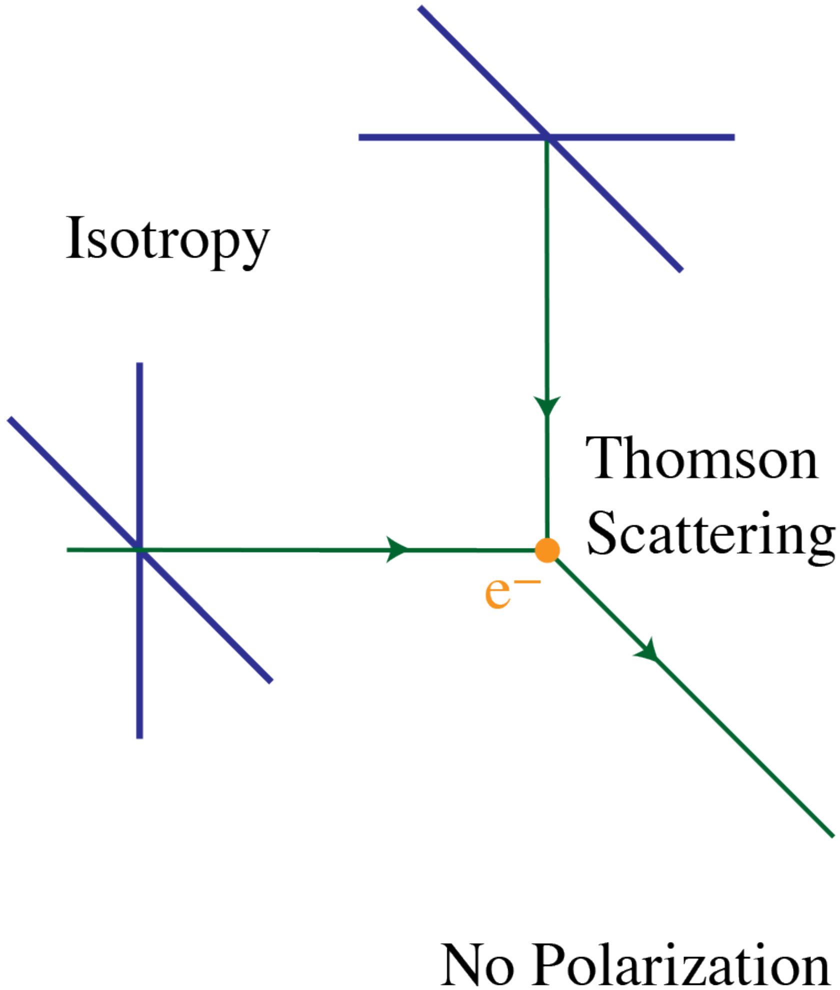
Polarization and Gravitational Waves

- Thomson scattering generates linear polarization



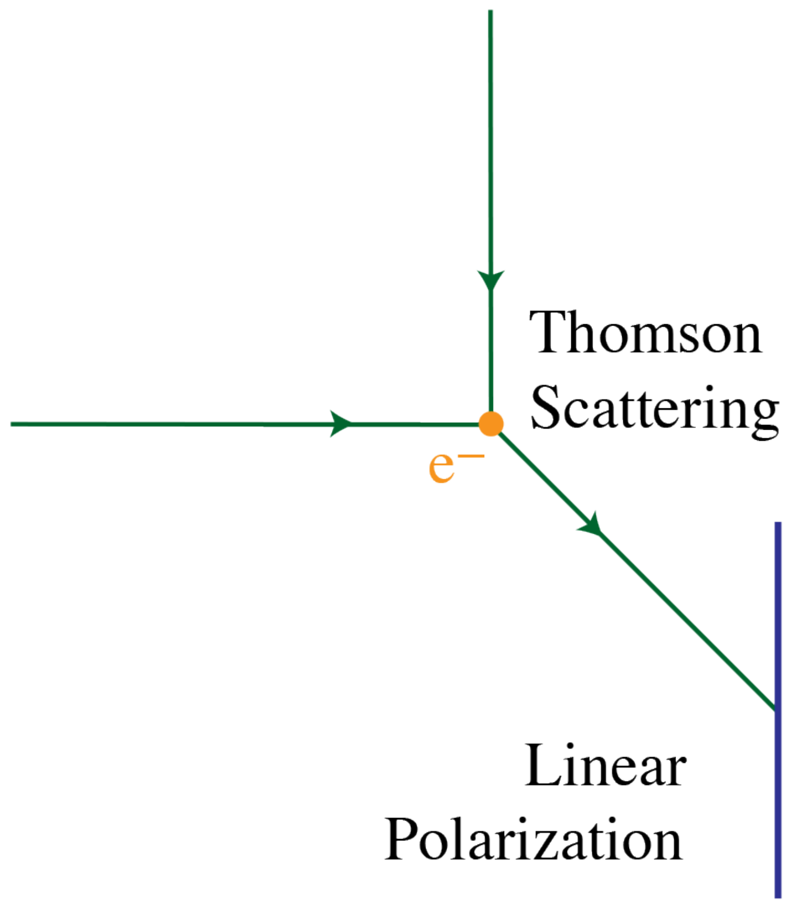
Polarization and Gravitational Waves

- An isotropic medium by symmetry leads to no net polarization

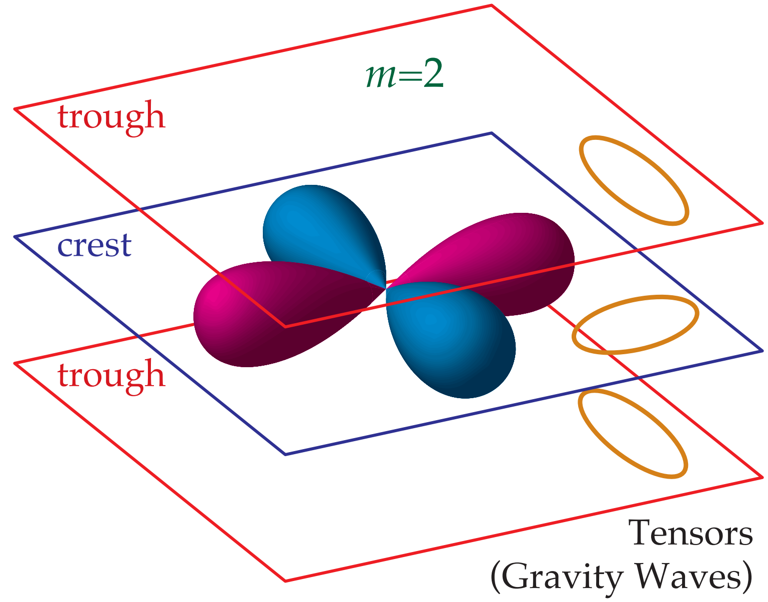


Polarization and Gravitational Waves

- Quadrupole anisotropy provides polarization source



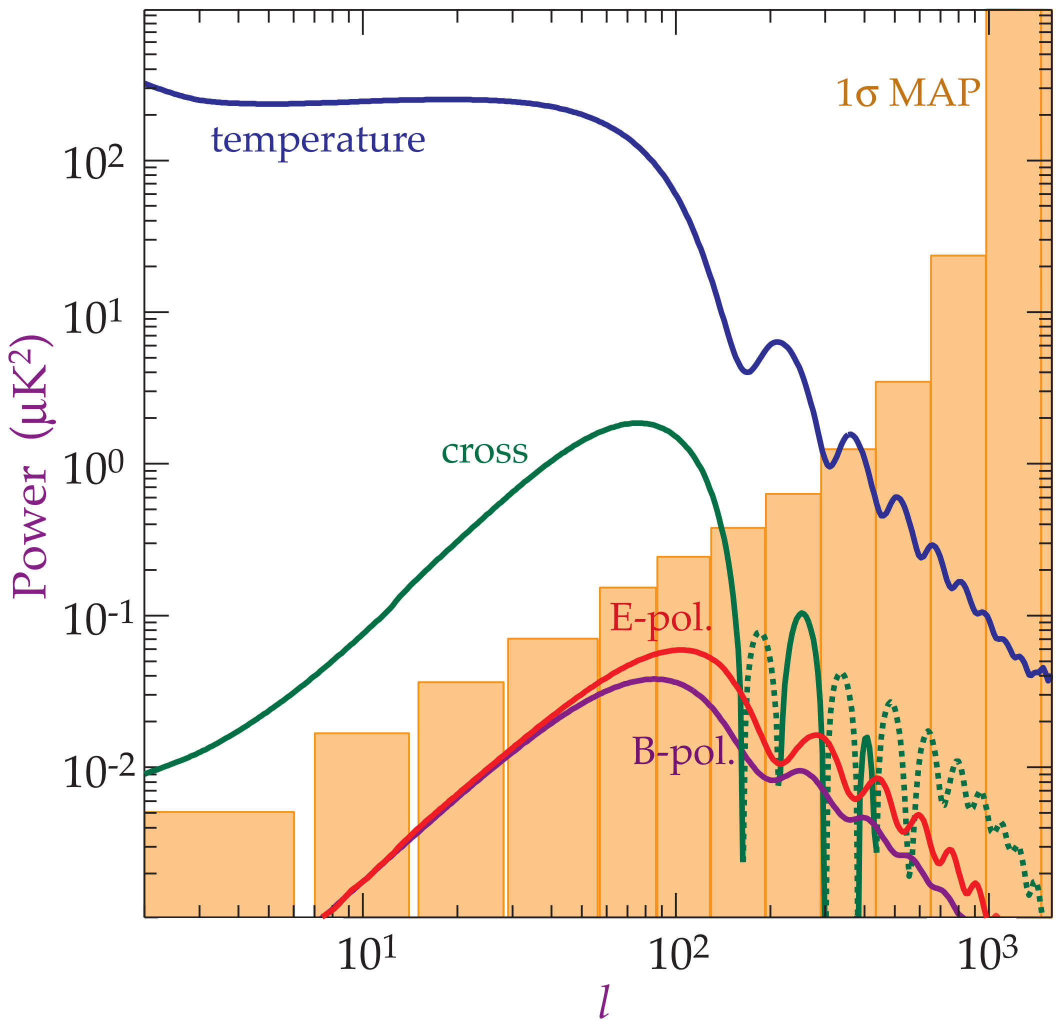
Gravitational Wave Quadrupoles



- Changing transverse-traceless distortion of space creates a quadrupole CMB anisotropy much like the distortion of test ring of particles
- As the tensor mode enters the horizon it imprints a quadrupole temperature $\ell = 2, m = \pm 2$ in plane wave coordinates $\mathbf{k} \parallel \mathbf{z}$
- Modes that cross before recombination: effect erased by rescattering $e^{-\tau}$ due to its isotropizing effect
- Modes that cross after recombination: integrate contributions along the line of sight - tensor ISW effect

Observability

- Gravitational waves from inflation yet to be detected but is the goal of many current and future CMB experiments



Transfer Function

- Transfer function transfers the initial Newtonian curvature to its value today (linear response theory)

$$T(k) = \frac{\Phi(k, a=1)}{\Phi(k, a_{\rm init})} \frac{\Phi(k_{\rm norm}, a_{\rm init})}{\Phi(k_{\rm norm}, a=1)}$$

- Conservation of Bardeen curvature: Newtonian curvature is a constant when stress perturbations are negligible: above the horizon during radiation and dark energy domination, on all scales during matter domination
- When stress fluctuations dominate, perturbations are stabilized by the Jeans mechanism
- Hybrid Poisson equation: Newtonian curvature, comoving density perturbation $\Delta \equiv (\delta\rho/\rho)_{\rm com}$ implies Φ decays

$$(k^2 - 3K)\Phi = 4\pi G a^2 \rho \Delta \sim \eta^{-2} \Delta$$

Transfer Function

- Freezing of Δ stops at $\eta_{\rm eq}$

$$\Phi \sim (k\eta_{\rm eq})^{-2}\Delta_H \sim (k\eta_{\rm eq})^{-2}\Phi_{\rm init}$$

- Transfer function has a k^{-2} fall-off beyond $k_{\rm eq} \sim \eta_{\rm eq}^{-1}$

$$\eta_{\rm eq} = 15.7(\Omega_m h^2)^{-1}\left(\frac{T}{2.7K}\right)^2 \text{Mpc}$$

- Small correction since growth with a smooth radiation component is logarithmic not frozen
- Transfer function is a direct output of an Einstein-Boltzmann code

Fitting Function

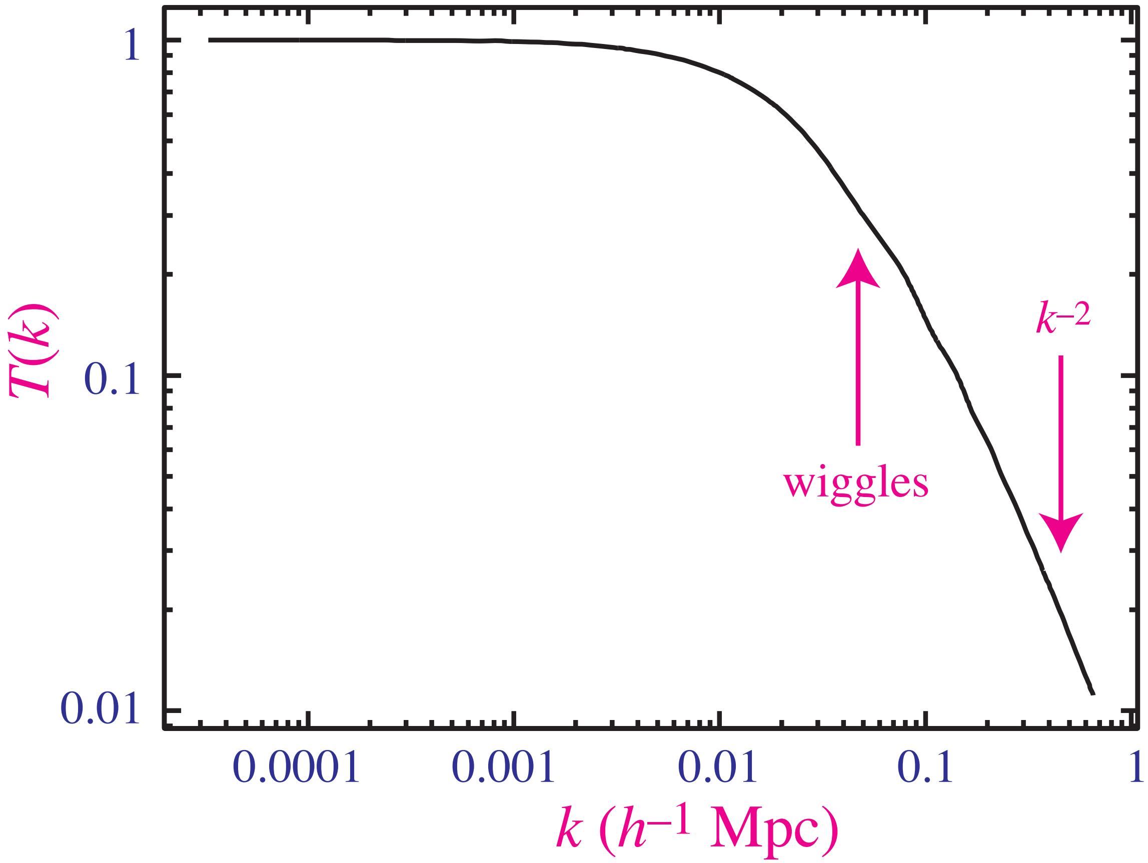
- Alternately accurate fitting formula exist, e.g. pure CDM form:

$$T(k(q)) = \frac{L(q)}{L(q) + C(q)q^2}$$
$$L(q) = \ln(e + 1.84q)$$
$$C(q) = 14.4 + \frac{325}{1 + 60.5q^{1.11}}$$
$$q = k/\Omega_m h^2 \mathrm{Mpc}^{-1} (T_{\rm CMB}/2.7K)^2$$

- In h Mpc^{-1}, the critical scale depends on $\Gamma \equiv \Omega_m h$ also known as the shape parameter

Transfer Function

- Numerical calculation

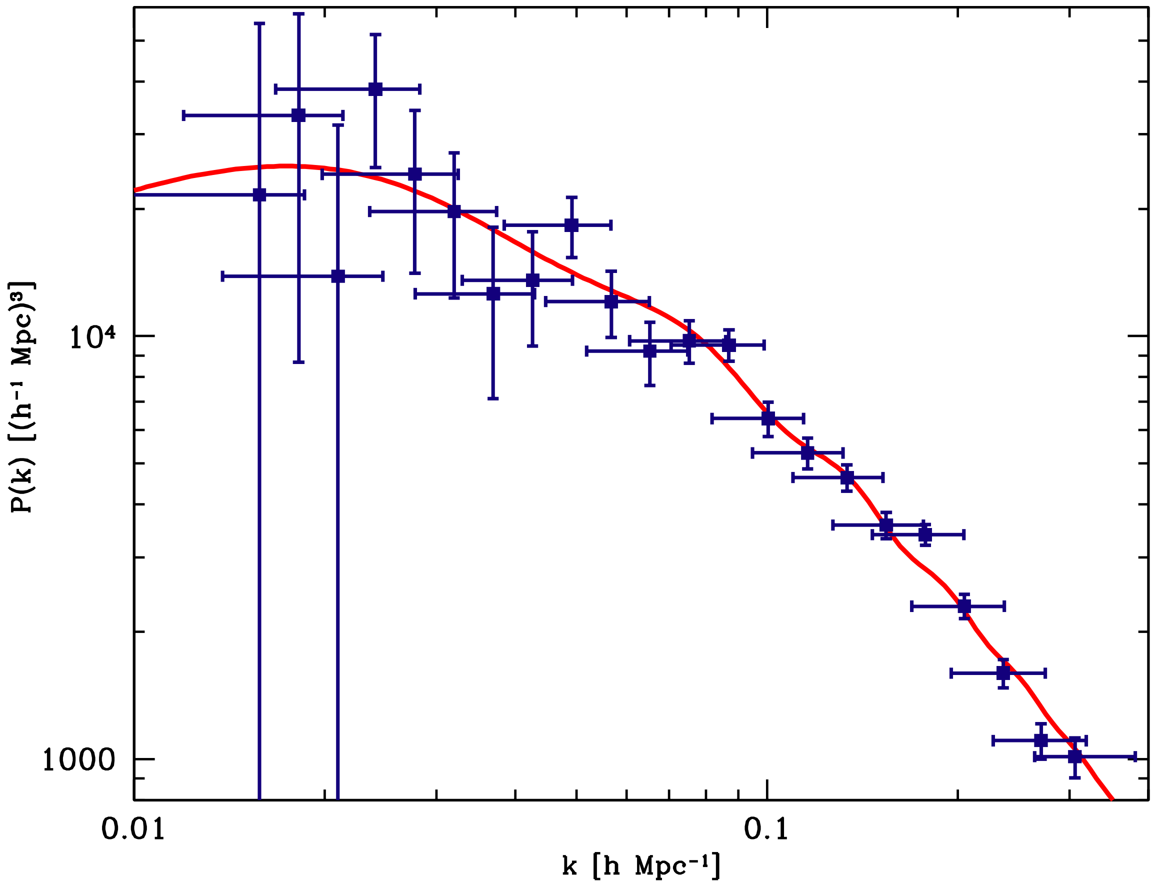


Baryon Acoustic Oscillations

- Baryons caught up in the acoustic oscillations of the CMB and impart acoustic oscillations to the transfer function. Density enhancements are produced kinematically through the continuity equation $\delta_b \sim (k\eta)v_b$ and hence are out of phase with CMB temperature peaks
- Dissipation of the acoustic oscillations eliminates both the CMB and baryon perturbations – known as Silk damping for the baryons. This suppression and the general fact that baryons are caught up with photons was one of the main arguments for CDM
- Detected first in the SDSS LRG survey.
- An excellent standard ruler for angular diameter distance $D_A(z)$ since it does not evolve in redshift in linear theory
- Radial extent of BAO gives $H(z)$

Power Spectrum

- SDSS data



- Power spectrum defines large scale structure observables: galaxy clustering, velocity field, Lyα forest clustering, cosmic shear