Astro 243: Problem Set 2

1 Ryden 3.2, 3.3 + followup questions

(1) Suppose you are a two-dimensional being, living on the surface of a sphere with radius R. An object of width $dl \ll R$ is at a distance r (we called this D in class and $dl = d\Sigma$) from you (remember, all distances are measured on the surface of the sphere). What angular width $d\theta$ will you measure for the object? Explain the behavior of $d\theta$ as $r \to 0$ and $r \to \pi R$. What happens if I go even further toward $r \to 2\pi R$?

(2) Now consider a large finite angular width of $\Delta\theta = \pi/2$ and put the large object that subtends this angle at $r = \pi R/2$ (the equator). The object and you form a "triangle" on the sphere. What is the sum of the angles of this triangle and how does it differ from a flat triangle?

(3) Suppose you are still a two-dimensional being, living on the same sphere of radius R. Show that if you draw a circle of radius r, the circles circumference will be

$$C = 2\pi R \sin(r/R) \qquad (1)$$

Explain the relationship between C and the angular diameter distance $R\sin(r/R)$.

(4) Idealize the Earth as a perfect sphere of radius $R = 6371$km. If you could measure circumferences with an error of ± 1 meter, how large a circle would you have to draw on the Earths surface to convince yourself that the Earth is spherical rather than flat? Comment on whether it is possible to ever empirically prove that the universe is exactly spatially flat?