Astro 243: Problem Set 4

1 Dark Energy (cf. Ryden 5.5)

One speculation in cosmology is that the dark energy may take the form of phantom energy with an equation-of-state parameter $w < -1$. Suppose that the universe is spatially flat and contains matter with a density parameter Ω_m, and phantom energy with a density parameter $\Omega_{\rm DE} = 1 - \Omega_m$ and equation-of-state parameter $w < -1$. How does the phantom dark energy density change with a? Comment on the compatibility with and implications for energy conservation from the general standpoint including pressure. At what scale factor $a_{\rm DE}$ are the energy density of phantom energy and matter equal? Write down the Friedmann equation for this universe in the limit that $a \gg a_{\rm DE}$. Assuming that $a_{\rm DE} < 1$ and using approximation, show that the scale factor a goes to infinity at a finite cosmic time $t_{\rm rip}$ from today t_0, given by the relation

$$H_0(t_{\rm rip} - t_0) \approx \frac{2}{3|1+w|}(1-\Omega_m)^{-1/2}. \tag{1}$$

2 Dark Matter

Suppose that the dark matter in the galaxy is composed of objects with a mass ten times that of Jupiter $M \sim 10^{-2} M_\odot$ and the halo extends with a flat rotation curve of ($v = 220$km/s) out to at least half way to the LMC. (1) Estimate the expected time scale of the brightening of a star in the LMC at a distance of 52kpc by microlensing. Note that the rotation velocity provides a typical velocity for objects in the halo and that lensing is most efficient when the lens is half way to the source. (2) Estimate the number density (in kpc^{-3}) of these objects at this point.