Astro 243: Problem Set 6

1 Lyth Bound

Recall from class that the tensor to scalar ratio r is related to the slow roll parameter ϵ_H as $r = 16\epsilon_H$ and

$$\epsilon_H = 4\pi G \frac{(d\phi/dt)^2}{H^2} \tag{1}$$

(1) Estimate $\Delta\phi$, i.e. how far ϕ must have rolled in $N = \Delta \ln a$ efolds of the expansion as a function of r and express your answer in units of the reduced Planck mass $M_{\rm pl} = 1/\sqrt{8\pi G}$. (2) Recall that inflation is supposed to last for $N \sim 60$ efolds. If $r \sim 10^{-1}$, near the current upper limits, does the roll exceed a Planck mass range $\Delta\phi > M_{\rm pl}$ and therefore violate the so called Lyth bound?

2 $m^2\phi^2$ Inflation

Suppose inflation was described by a simple mass term m so that the potential $V = m^2\phi^2/2$. Recall that in a friction dominated roll $3H d\phi/dt = -V'$ and $H^2 \approx 8\pi G V/3$. (1) Give the expression for $\epsilon_H(\phi)$. Where is the field in $M_{\rm pl}$ units when inflation ends ($\epsilon_H = 1$)? (2) In a friction dominated roll

$$\delta_1 = \epsilon_H - \frac{1}{8\pi G}\frac{V''}{V} \tag{2}$$

Give the expression for $\delta_1(\phi)$. (3) Give the scalar tilt $n_s(\phi) - 1$ (see notes for the slow-roll formula). The observed value of the tilt in the CMB is $n_s - 1 \approx -0.04$. What is the field value that corresponds to CMB scales? Is the Lyth bound violated given that the inflaton must roll between these two field points in (1) and (3)?