Astro 282: Problem Set 4

Due April 28

1 Problem 1: Thomson scattering

Refer to problem set # 1 for the Thomson cross section σ_T and the solutions to # 2 for the conformal time in a matter-radiation universe.

- (a) Scattering rate. Given the Thomson cross section σ_T and a density of free electrons $n_e = n_b$ (fully ionized hydrogen case), calculate the time in seconds it takes a typical photon to scatter at an epoch a (hint: essentially dimensional analysis since photons travel at the speed of light $c = 1$). Compare this time with the Hubble or expansion time $H^{-1}(a)$. Express the ratio in terms of $\Omega_m h^2$ and $\Omega_b h^2$ and recall that you may neglect dark energy (or curvature) for $a \ll 1$. What is the redshift at which this ratio is unity? Why is this number different from the last scattering epoch claimed in class $z = 1000$.

- (b) Mean free path. Convert your answer in (a) to physical length units. This is the mean free path that a photon travels before scattering off an electron. What is the mean free path λ_c in comoving coordinates as a function of a and $\Omega_b h^2$?

- (c) Diffusion. Photons random walk in the baryons due to the mean free path and so they can only travel a distance $\lambda_D = \sqrt{\lambda_c \eta}$ where η is the conformal time. Calculate λ_D at $z = 1000$ for $\Omega_b h^2 = 0.02$, $h = 0.7$, $\Omega_m = 1/3$ and compare it to η at the same epoch. Since the horizon scale corresponds to $\ell \sim 200$, where in multipole do you expect diffusion effects to become important?