

1 Coordinate Dependence of Stokes Parameters

In polarized radiative transfer it is often necessary to rotate into and out of preferred frames. Here you will derive the rotational properties of the Stokes parameters. I've phrased it here as a review of relativity and tensor mechanics. The analogy here is to R.L. eqn 4.21 and 4.47 for the Lorentz transformation as the Jacobian matrix between x and x' and the way a tensor transforms under a coordinate transformation. Rotations are a subset of the Lorentz transformations.

- (a) Write down the Jacobian matrix associated with a counterclockwise rotation of the 2D $\mathbf{x}, \mathbf{y} \rightarrow \mathbf{x}', \mathbf{y}'$ axes by an angle ψ .
- (b) Consider the polarization matrix

$$\begin{pmatrix} I + Q & U + iV \\ U - iV & I - Q \end{pmatrix} \quad (1)$$

Under rotations, the polarization matrix transforms as a (contravariant) tensor. Write down the transformed polarization matrix and extract its components (I', Q', U', V').

- (c) Construct the quantity $Q \pm iU$. How does it transform under rotations?

2 Plasma Effects

Radiation pulses from a pulsar are measured at two frequencies $\nu_1 = 2\text{GHz}$ and $\nu_2 = 1\text{GHz}$. Their pulses are separated by $\Delta t_{21} = 1\text{s}$ and their linear polarization is rotated by $\Delta\Theta_{21} = \pi$ rad. Assume that the intervening medium has a density of $n = 0.03$ electrons cm^{-3} . [For the numerical evaluation: the classical electron radius $r_0 = (e^2/mc^2) = 2.82 \times 10^{-13}$ cm, the electron rest mass $mc^2 = 8.182 \times 10^{-7}$ ergs and $(1G)^2 = 1$ erg cm^{-3} .]

- 1. Express the distance to the pulsar in terms of the observables and fundamental constants. Evaluate the distance in cm for the given parameters.
- 2. Express the line of sight average magnetic field

$$\langle B_{\parallel} \rangle = \frac{\int n B_{\parallel} ds}{\int n ds} \quad (2)$$

in terms of the observables and fundamental constants. Evaluate the magnetic field in μG for the given parameters.

3 R&L

Problems 2.2, 2.4