Astro 305: Problem Set 2

Due October 11

1 Scattering Damping

One of the most astrophysically intriguing results from the WMAP CMB satellite is the measurement of the cosmological optical depth of photons to Thomson scattering due to ionized gas at the reionization epoch. Preliminary indications of a high τ were in tension with normal star formation and structure formation. The current best value is $\tau \approx 0.1$ which is still slightly high for normal star formation.

There is therefore a nearly uniform scattering "cloud" sitting in front of cosmological radiation such as the CMB. Work out the implications for the angular distribution of the specific intensity of the CMB.

1. Write down the general radiative transfer equation with the optical depth as the τ independent variable. Specialize it to the case of coherent, isotropic scattering only.

2. Assume an Eddington-like approximation where the specific intensity can be broken up into an isotropic term and its anisotropy

$$I_\nu = J_\nu + \delta I_\nu \tag{1}$$

$$J_\nu \equiv \int \frac{d\Omega}{4\pi} I_\nu \tag{2}$$

Assume that $\delta I_\nu \ll J_\nu$. Keeping only the zeroth order J_ν term, write down the solution $J_\nu(\tau)$ in terms of τ and the initial $J_\nu(0)$. What happens to the average intensity due to the obscuring cloud and why?

3. Solve for the anisotropic term $\delta I_\nu(\tau)$ in terms of τ and the initial anisotropy $\delta I_\nu(0)$. How suppressed is the anisotropy for $\tau = 0.1$?

2 Ideal Gas Law

Using the Maxwell Boltzmann distribution

$$f = e^{-(E-\mu)/kT} \tag{3}$$

derive the ideal gas law using the general relations for pressure and number density

$$n = g \int \frac{d^3q}{(2\pi\hbar)^3} f \tag{4}$$

$$p = g \int \frac{d^3q}{(2\pi\hbar)^3} \frac{|q|^2 c^2}{3E(q)} f \tag{5}$$

assuming non relativistic particles. What is the equation of state $p(u,T)$ and how do you interpret it in terms of the thermal velocity $v = \sqrt{3kT/m}$ of the particles? compare that with the equation of state of radiation and argue that the pressure for a given energy density u is much less for non-relativistic particles.

3 R&L

Problems 1.5, 1.7, 1.9