Astro 305: Problem Set 3
Due October 18

1 Liouville Equation

Consider the propagation of radiation with a distribution function $f(\mathbf{x}, \mathbf{q}, t)$ in the absence of collisions and processes that change the photon momenta. Note that $d\mathbf{x}/dt = c\hat{\mathbf{n}}$ where $\mathbf{q} = q\hat{\mathbf{n}}$.

(a) Write down the Liouville equation for f in terms of the independent variables $\mathbf{x}$, $\mathbf{n}$ at a fixed q.

(b) Take the Fourier transform of the Liouville equation

$$f(\mathbf{k}, \mathbf{n}, t) = \int d^3x e^{-i\mathbf{k}\cdot\mathbf{x}} f(\mathbf{x}, \mathbf{n}, t) \tag{1}$$

A single plane wave or Fourier moment that begins in an isotropic state satisfies the plane parallel approximation by definition for all time and so the angular dependence can be decomposed into Legendre polynomials:

$$f(\mathbf{k}, \mathbf{n}, t) = \sum (-i)^\ell f_\ell(\mathbf{k}, t) P_\ell(\hat{\mathbf{k}} \cdot \hat{\mathbf{n}}) \tag{2}$$

where $\mathbf{k} = k\hat{\mathbf{k}}$. This is the generalization of the Eddington approximation.

(c) Write the Liouville equation as an infinite series of moment equations in ℓ. These are the Boltzmann hierarchy equations. What happens to the initially isotropic radiation as time progresses? Draw a simple ray tracing diagram and explain why.

2 R&L

Problems 9.4, 9.5