Astro 305: Problem Set 4
Due October 26

1 Cosmic Recombination

The universe has a well measured baryon-to-photon number density ratio: $n_b/n_\gamma \approx 2.75 \times 10^{-8}\Omega_b h^2$. The WMAP cosmic microwave background experiment has recently determined $\Omega_b h^2 = 0.024$ to better than 10%.

Calculate (numerically) in eV the $k_B T$ at which the universe recombines by defining that to be an ionization fraction $x_e = n_e/n_{\rm tot} = 0.5$ under the Saha equation. What is the temperature in K? How does that compare with the WMAP determination of 2970K? Give the qualitative reason for the shift.

2 Coordinate Dependence of Stokes Parameters

In polarized radiative transfer it is often necessary to rotate into and out of preferred frames. Here you will derive the rotational properties of the Stokes parameters. I've phrased it here as a review of relativity and tensor mechanics. The analogy here is to R.L. eqn 4.21 and 4.47 for the Lorentz transformation as the Jacobian matrix between x and x' and the way a tensor transforms under a coordinate transformation. Rotations are a subset of the Lorentz transformations.

- (a) Write down the Jacobian matrix associated with a counterclockwise rotation of the 2D $\mathbf{x}$, $\mathbf{y} \rightarrow \mathbf{x}'$, $\mathbf{y}'$ axes by an angle ψ.
- (b) Consider the polarization matrix

$$\begin{pmatrix} I+Q & U+iV \\ U-iV & I-Q \end{pmatrix} \tag{1}$$

  Under rotations, the polarization matrix transforms as a (contravariant) tensor. Write down the transformed polarization matrix and extract its components (I', Q', U', V').
- (c) Construct the quantity $Q \pm iU$. How does it transform under rotations?

3 R&L

Problems 2.2, 2.4