Astro 305: Problem Set 6

Due Nov 15

1 Lyman α Forest Power Spectrum

The IGM is nearly fully ionized but retains a small neutral component due to ionization balance which fluctuates with the density to form a forest of Lyα absorption ($1s \to 2p$) in the spectra of distant quasars. One use of this forest is to measure the (cosmological) density fluctuations in the IGM. Derive the scaling between optical depth and density. Hint: keeping track of dimensions will help you sort out the physical form of the relations below.

- Using the Einstein relations, write down the optical depth to (true) absorption by the Lyα line in terms of the transition wavelength λ [cm], the transition rate $\Gamma = A_{21}$ [s^{-1}], the line profile $\phi(\nu)$ [s], the neutral hydrogen fraction $x_{\rm HI}$, and the baryon number density n_b [cm^{-3}].
- To find $x_{\rm HI}$ in the presence of ionizing flux, write down the ionization balance equation for $dx_{\rm HI}/dt =$ [sources-sinks] given photoionization and recombination. Express your answer in terms of the isotropic ionizing specific intensity J_ν [ergs s^{-1} cm^{-2} Hz^{-1} sr^{-1}], recombination coefficient R [cm^3 s^{-1}], photoionization cross section as a function of frequency σ_ν [cm^2] and baryon number density n_b [cm^{-3}].
- The photoionization cross section σ_ν gives the strength of a bound-free transition and has a strong frequency dependence. Where in ν do you expect the ionizing specific intensity to be most important. Call this value $J_{\rm HI}$.
- Assuming ionization equilibrium show how the Lyα optical depth as a scales with baryon mass density ρ_b, $J_{\rm HI}$ and temperature T. You may assume that the recombination coefficient $R \propto T^{-0.7}$ and the medium is nearly completely ionized. Take a barotropic equation of state of $p_b \propto \rho_b^\gamma$ and reduce this scaling to one of ρ_b and $J_{\rm HI}$.
- Explain how optical depth (absorption) measurements in the spectrum of quasars at high redshift can measure density fluctuations in the baryons and hence the power spectrum of large-scale structure.

2 R&L

Problems 10.7