Astro 305: Problem Set 6
Due Nov 15

## 1 Lyman $\alpha$ Forest Power Spectrum

The IGM is nearly fully ionized but retains a small neutral component due to ionization balance which fluctuates with the density to form a forest of Ly $\alpha$  absorption (1s  $\rightarrow$  2p) in the spectra of distant quasars. One use of this forest is to measure the (cosmological) density fluctuations in the IGM. Derive the scaling between optical depth and density. Hint: keeping track of dimensions will help you sort out the physical form of the relations below.

- Using the Einstein relations, write down the optical depth to (true) absorption by the Ly $\alpha$  line in terms of the transition wavelength  $\lambda$  [cm], the transition rate  $\Gamma = A_{21}$  [s<sup>-1</sup>], the line profile  $\phi(\nu)$  [s], the neutral hydrogen fraction  $x_{\rm HI}$ , and the baryon number density  $n_b$  [cm<sup>-3</sup>].
- To find  $x_{\rm HI}$  in the presence of ionizing flux, write down the ionization balance equation for  $dx_{\rm HI}/dt = [{\rm sources-sinks}]$  given photoionization and recombination. Express your answer in terms of the isotropic ionizing specific intensity  $J_{\nu}$  [ergs s<sup>-1</sup> cm<sup>-2</sup> Hz<sup>-1</sup> sr<sup>-1</sup>], recombination coefficient R [cm<sup>3</sup> s<sup>-1</sup>], photoionization cross section as a function of frequency  $\sigma_{\nu}$  [cm<sup>2</sup>] and baryon number density  $n_b$  [cm<sup>-3</sup>].
- The photoionization cross section  $\sigma_{\nu}$  gives the strength of a bound-free transition and has a strong frequency dependence. Where in  $\nu$  do you expect the ionizing specific intensity to be most important. Call this value  $J_{\rm HI}$ .
- Assuming ionization equilibrium show how the Ly $\alpha$  optical depth as a scales with baryon mass density  $\rho_b$ ,  $J_{\rm HI}$  and temperature T. You may assume that the recombination coefficient  $R \propto T^{-0.7}$  and the medium is nearly completely ionized. Take a barotropic equation of state of  $p_b \propto \rho_b^{\gamma}$  and reduce this scaling to one of  $\rho_b$  and  $J_{\rm HI}$ .
- Explain how optical depth (absorption) measurements in the spectrum of quasars at high redshift can measure density fluctuations in the baryons and hence the power spectrum of large-scale structure.

## 2 R&L

Problems 10.7