Astro 305: Problem Set 6
Due November 22

1 Compton-y distortions

Recall that the Kompaneets equation is given by

$$\frac{\partial f}{\partial t} = \frac{d\tau}{dt}\frac{k_B T_e}{m_e c^2}\frac{1}{x^2}\frac{\partial}{\partial x}\left[x^4\left(\frac{\partial f}{\partial x} + f + f^2\right)\right], \tag{1}$$

where $x = h\nu/kT_e$.

Consider small deviations of the spectrum from the blackbody form

$$f = \frac{1}{e^{h\nu/k_B T} - 1}. \tag{2}$$

- Show
$$f + f^2 \approx -\frac{T}{T_e}\frac{\partial f}{\partial x}, \tag{3}$$
- Transform variables from time t to the Compton-y parameter
$$y = \int dt \frac{d\tau}{dt}\frac{k_B(T_e - T)}{m_e c^2}, \tag{4}$$
and write the Kompaneets equation in the form of a diffusion equation $\frac{\partial f}{\partial y} = \ldots$ The Kompaneets equation describes an upwards diffusion in energy of the photons via scattering off hotter electrons.
- Again assuming small deviations, insert the blackbody form eqn. (**??**) into the right hand side of the Kompaneets/diffusion equation. Trivially integrate the equation to show that the change in the distribution function is given by
$$\frac{\Delta f}{f} = -y x_\nu e^{x_\nu} f\left(4 - x_\nu \coth\frac{x_\nu}{2}\right), \tag{5}$$
where $x_\nu = h\nu/k_B T$.
- Define the thermodynamic brightness temperature as the temperature of a blackbody that has the same f at a given frequency as the perturbed spectrum. Convert $\Delta f/f$ to $\Delta T/T$. What happens as $x_\nu \to 0$? What happens at $x_\nu \to \infty$. Argue that there must be a frequency (independent of y) at which $\Delta T/T = 0$. Numerically find this value of x_ν. Convert your answer to frequency (in GHz) and wavelength (cm) assuming $T = 2.725$K. This is known as the null in the thermal Sunyaev Zeldovich effect.

2 RL: 7.4, 5.1, 5.2