Astro 321: Problem Set 5

Due Feb. 12

1 Problem 1: Transfer function

Derive the basic features of the transfer function, or linear response to an initial perturbation.

- Consider the Poisson equation
$$k^2\Phi = 4\pi G a^2 \rho \Delta \tag{1}$$
where ρ is the total density and Δ is the density fluctuation. Use the Friedmann equation assuming a constant equation of state w to eliminate $4\pi G a^2 \rho$ in favor of conformal time η. What is the relationship between Φ and Δ? which is larger outside the horizon $k\eta \ll 1$.

- Recall from the last problem set that if stresses are negligible $\zeta =$ const.; if the equation of state w is constant this implies $\Phi =$ const. Given an initial curvature perturbation Φ_0 what is the density fluctuation Δ at horizon corssing $k\eta = 1$?

- Inside the horizon during radiation domination, pressure fluctuations $\Delta p/p = \Delta\rho/\rho = \Delta$ and hence dominate over gravity in the Euler equation. Argue that this implies the density fluctuation Δ oscillates around zero with a constant amplitude given by its value near horizon crossing (ignore viscosity). The dark matter does not participate in the oscillations but also has its density fluctuation $\Delta_m = \Delta$ frozen in at horizon crossing. Use the Poisson equation to express Φ in terms of Φ_0 and $k\eta$ in this regime neglecting the oscillations (these become the acoustic peaks in the CMB but are not relevant for the dark matter). What happens in the opposite regime of $k\eta \ll 1$?

- When the universe becomes matter dominated, pressure fluctuations become negligible compared with density fluctuations and the curvature Φ stays constant on all scales. Define the transfer function as $T(k) = \Phi(\eta_0)/\Phi_0$. What is the behavior as $k \to 0$ and $k \to \infty$. What is the critical scale? Evaluate this scale in an $\Omega_m = 1/3$, $h = 1/\sqrt{2}$ cosmology (expressed in h^{-1}Mpc). Hint: consider the horizon scale at matter-radiation equality.