Astro 321

Lecture Notes Set 8
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Halo Bias

- If halos are formed without regard to the underlying density fluctuation and move under the gravitational field then their number density is an unbiased tracer of the dark matter density fluctuation

$$\left( \frac{\delta n}{n} \right)_{\text{halo}} = \left( \frac{\delta \rho}{\rho} \right)$$

- However, spherical collapse says the probability of forming a halo depends on the initial density field

- Large scale density field acts as “background” enhancement of probability of forming a halo or “peak”

- Peak-Background Split (Efstathiou 1998; Cole & Kaiser 1989; Mo & White 1997)
Peak-Background Split

- Schematic Picture:
Perturbed Mass Function

- Density fluctuation split

\[ \delta = \delta_b + \delta_p \]

- Lowers the threshold for collapse

\[ \delta_{cp} = \delta_c - \delta_b \]

so that \( \nu = \delta_{cp}/\sigma \)

- Taylor expand number density \( n_M \equiv dn/d \ln M \)

\[ n_M + \frac{dn_M}{d\nu} \frac{d\nu}{d\delta_b} \delta_b \ldots = n_M \left[ 1 + \frac{(\nu^2 - 1)}{\sigma \nu} \right] \]

if mass function is given by Press-Schechter

\[ n_M \propto \nu \exp(-\nu^2/2) \]
Halo Bias

• Halos are biased tracers of the “background” dark matter field with a bias $b(M)$ that is given by spherical collapse and the form of the mass function

$$\frac{\delta n_M}{n_M} = [1 + b(M)] \delta$$

• For Press-Schechter

$$b(M) = 1 + \frac{\nu^2 - 1}{\delta_c}$$

• Improved by the Sheth-Torman mass function

$$b(M) = 1 + \frac{a\nu^2 - 1}{\delta_c} + \frac{2p}{\delta_c[1 + (a\nu^2)^p]}$$

with $a = 0.75$ and $p = 0.3$ to match simulations.
Numerical Bias

- Example of halo bias from a simulation (from Hu & Kravstov 2002)
What is a Halo?

- Mass function and halo bias depend on the definition of mass of a halo.
- Agreement with simulations depend on how halos are identified.
- Other observables (associated galaxies, X-ray, SZ) depend on the details of the density profile.
- Fortunately, simulations have shown that halos take on a near universal form in their density profile at least on large scales.
**NFW Halo**

- **Density profile** well-described by *(Navarro, Frenk & White 1997)*

\[
\rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}
\]

![Graph](image)
Transforming the Masses

- NFW profile gives a way of transforming different mass definitions

$$M \ (h^{-1} M_{\odot})$$

$$n (>M) \ (h^3 \text{Mpc}^{-3})$$

$$\Delta_{180} \ (\text{wrt mean})$$

$$\Delta_{666}$$

Jenkins et al.

Rescaled

$$W_m = G = 0.15; \text{ flat; } h = 0.65; s_8 = 1.07$$
Lack of Concentration?

- NFW parameters may be recast into $M_v$, the mass of a halo out to the virial radius $r_v$ where the overdensity wrt mean reaches $\Delta_v = 180$.

- Concentration parameter

  $$c \equiv \frac{r_v}{r_s}$$

- CDM predicts $c \sim 10$ for $M_*$ halos. Too centrally concentrated for galactic rotation curves?

- Possible discrepancy has lead to the exploration of dark matter alternatives: warm ($m \sim$keV) dark matter, self-interacting dark-matter, annihilating dark matter, ultra-light “fuzzy” dark matter, . . .
The Halo Model

- NFW halos, of abundance $n_M$, given by mass function, clustered according to the halo bias $b(M)$ and the linear theory $P(k)$

- Power spectrum example:

![Graph showing power spectra](image)
Non-Linear Power Spectrum

- Non-linear power spectrum is composed of dark matter halos that are clustered according to the halo bias + the clustering due to the halo density profile

\[ P_{nl}(k, z) = I_2^2(k, z)P(k, z) + I_1(k, z) \]

where

\[ I_2(k, z) = \int d\ln M \left( \frac{M}{\rho_m(z = 0)} \right) \frac{dn}{d\ln M} b(M) y(k, M) \]

\[ I_1(k, z) = \int d\ln M \left( \frac{M}{\rho_m(z = 0)} \right)^2 \frac{dn}{d\ln M} y^2(k, M) \]

and \( y \) is the Fourier transform of the halo profile with \( y(0, M) = 1 \)

\[ y(k, M) = \frac{1}{M} \int_0^{r_h} dr 4\pi r^2 \rho(r, M) \frac{\sin(kr)}{kr} \]
Galaxy Power Spectrum

- For galaxies, one defines a halo occupation distribution which determines the number of galaxies (satisfying a certain observational criteria) that can occupy a halo of mass $M$

- Take a simple example of a mass selection on the galaxies, then
  \[ N(M) = 0 \text{ for } M < M_{th} \text{ and above threshold} \]
  \[ N(M) = C + S(M) \text{ where } C = 1 \text{ accounts for the central galaxy and satellite galaxies follow a poisson distribution with mean} \]
  \[ S(M) \approx \frac{M}{30M_{th}} \]
Galaxy Power Spectrum

• Then assuming that satellites are distributed according to the mass profile

\[ P_{\text{gal}}(k, z) = I_2^2(k, z) P(k, z) + I_1(k, z) \]

where

\[ I_2(k, z) = \frac{1}{n_{\text{gal}}} \int d\ln M \frac{dn}{d\ln M} b(M) [C + y(k, M)S(M)] \]

\[ I_1(k, z) = \frac{1}{n_{\text{gal}}^2} \int d\ln M \frac{dn}{d\ln M} [S^2(M)y^2(k, M) + 2CS(M)y(k, M)] \]

• Break between the one and two halo regime first seen by SDSS
Galaxy Power Spectrum

- Example (Seljak 2001)

- An explanation of the nearly power law galaxy spectrum
Incredible, Extensible Halo Model

- An industry developed to build semi-analytic models for wide variety of cosmological observables based on the halo model
- Idea: associate an observable (galaxies, gas, ...) with dark matter halos
- Let the halo model describe the statistics of the observable
- The overextended halo model?
Halo Temperature

- Motivate with isothermal distribution, correct from simulations

\[ \rho(r) = \frac{\sigma^2}{2\pi G r^2} \]

- Express in terms of virial mass \( M_v \) enclosed at virial radius \( r_v \)

\[ M_v = \frac{4\pi}{3} r_v \rho_m \Delta_v = \frac{2}{G} r_v \sigma^2 \]

- Eliminate \( r_v \), temperature \( T \propto \sigma^2 \) velocity dispersion

- Then \( T \propto M_v^{2/3} (\rho_m \Delta_v)^{1/3} \) or

\[
\left( \frac{M_v}{10^{15} h^{-1} M_\odot} \right) = \left[ \frac{f}{(1 + z)(\Omega_m \Delta_v)^{1/3}} \frac{T}{1\text{keV}} \right]^{3/2}
\]

- Theory (X-ray weighted): \( f \sim 0.75 \); observations \( f \sim 0.54 \).

Difference is crucial in determining cosmology from cluster counts!