Astro 321: Problem Set 4

Due April 28

This problem set develops a piece of the standard toolkit for structure formation calculations. The motivation of some pieces may seem mysterious for now, but save the codes you build here because you will need them again later in the course.

1 Problem 1: Power Spectra

- Define the linear power spectrum as

$$\begin{aligned}\Delta^2(k,a) &\equiv \frac{k^3}{2\pi^2}P(k) \\ &= \frac{4}{25}\delta_\zeta^2\left(\frac{G(a)a}{\Omega_m}\right)^2\left(\frac{k}{H_0}\right)^4\left(\frac{k}{k_{\rm norm}}\right)^{n-1}T^2(k)\end{aligned} \tag{1}$$

  For reference, δ_ζ is the amplitude of the initial curvature fluctuations at a normalization scale of $k_{\rm norm}$. The current best normalization comes from WMAP at a scale of $k_{\rm norm} = 0.05\ {\rm Mpc}^{-1}$ and a value of $\delta_\zeta = 5.07\times10^{-5}$ [WMAP's normalization parameter $A = (1.84\delta_\zeta \times 10^4)^2$ and is chosen to make the observed number of order unity] $G(a)$ gives the suppression of the linear growth due to dark energy such that $G(a \ll 1) = 1$.

  Here n is the spectral index of the *initial* density fluctuations and $T(k)$ is the *transfer function* which defines the linear response (through gravitational perturbation theory) to the initial perturbations. We will see later where that comes from; for now take it to be defined as

$$\begin{aligned}T(k(q)) &= \frac{L(q)}{L(q)+C(q)q^2} \\ L(q) &= \ln(e+1.84q) \\ C(q) &= 14.4 + \frac{325}{1+60.5q^{1.11}}\end{aligned} \tag{2}$$

  q scales k to the horizon size at matter radiation equality and is given for historical reasons by

$$q = \frac{k}{\Omega_m h^2 {\rm Mpc}^{-1}}(T_{\rm CMB}/2.7K)^2 \qquad T_{\rm CMB} = 2.728K \tag{3}$$

  For the experts, this is the transfer function without baryonic or neutrino effects. More accurate results can be obtained by using a numerical transfer function.

- Calculate $\eta(a_{\rm eq})$ and show that $k\eta(a_{\rm eq})$ has the above scaling, $q \propto k\eta(a_{\rm eq})$ in its scalings with Ω_m, h, $T_{\rm CMB}$. For what value of q is $k\eta(a_{\rm eq}) = 1$? At this value of q, what is the value of the transfer function T?

- Write a code in the language of your choice to generate $\Delta^2(k,a)$ with k in units of h ${\rm Mpc}^{-1}$. Leave $G(a)$, Ω_m, n, h as adjustable parameters. Notice that value of q with k in h ${\rm Mpc}^{-1}$ depends on $\Omega_m h$, which is usually called the shape parameter "Γ" in the literature.

- For an $\Omega_m = 0.27$, $h = 0.72$, $n = 1$, $G_0 = G(a=1) = 0.76$ cosmology, plot your result of $\Delta^2(k, a=1)$. For what k in h ${\rm Mpc}^{-1}$ is $\Delta^2 = 1$.