Astro 321: Problem Set 7

You may submit this as a group.

Runge-Kutta Integration

- Go to `http://www.nr.com`, download chapter 16 and read the section on 4th order Runge-Kutta.
- Familiarize yourself with the use of their "rk4" routine, find another standard package, or write your own integrator.
- Test your code against a simple harmonic oscillator system for which you can find analytic results:

$$\dot{\Theta} = -kv \qquad (1)$$

$$\dot{v} = k\Theta \qquad (2)$$

  What criteria should you use to choose the time step - think about the time scales in the oscillator problem. Check convergence in the time step and the final result against the analytic solution for $\Theta(0) = 1$, $v(0) = 0$, $k = 0.1$ (Mpc^{-1}). The typical unit of choice for a Boltzmann code is k in Mpc^{-1} and conformal time in Mpc (without the h's just to make life confusing when comparing to large scale structure conventions). You are free to choose your own units but make sure you are consistent!

- Use the rk4 method to solve for $\eta(a)$ for a universe with matter and radiation only. Take "log" time steps: i.e. each step should be a small fraction of the current a. For simplicity drop the neutrino contribution to the radiation, i.e. take $\Omega_m = 1 - \Omega_\gamma$. Compare your solution with the analytic one for a reasonable value of h.

For oscillating linear perturbations in the expanding universe, the appropriate time step is the mimimum of the linear and logarithmic criteria above. This is OK until you want to deal with the transition from a tightly coupled system to freely streaming radiation – then the collision time scale enters. In this case (not required in the course) you may want to punt and use a package, e.g. "dverk" that chooses the time step for you. This approach is generally less efficient.