Mass Functions and Bias

Consider the Jenkins et al (2001) mass function:

\[
\frac{dn}{d\ln M} = 0.315 \rho_m \frac{d\ln \sigma^{-1}}{M} \frac{d\ln M}{d\ln M} \exp[-|\ln \sigma^{-1} + 0.61|^{3.8}].
\]  (1)

and dig out your code for computing \(\sigma(M)\) from the previous problem set.

- Modify your code to also calculate \(d\ln \sigma^{-1}/d\ln M\). Hint: again start with the tophat in \(R\) and compute \(d\sigma^2_R/d\ln R\) by differentiating the window under the integral; the rest is just chain-ruling \(M(R)\).

- Integrate the mass function above \(3 \times 10^{14} h^{-1} M_\odot\). What is the number density of such (cluster sized) objects in \(h^3\) Mpc\(^{-3}\) in the same cosmology as the previous problem sets?

- An alternate form of the mass function by Sheth & Torman is more accurate at low masses and the consideration of halo bias.

\[
\frac{dn}{d\ln M} = \frac{\rho_m}{M} f(\nu) \frac{d\nu}{d\ln M}
\]  (2)

\[
\nu f(\nu) \propto \sqrt{\frac{2}{\pi} a \nu^2 [1 + (a \nu^2)^p]} \exp[-a \nu^2/2]
\]  (3)

where \(a = 0.75\), \(p = 0.3\), \(\nu = 1.69/\sigma\) and the proportionality is chosen such that \(\int d\nu f(\nu) = \int d(\ln \nu) \nu f(\nu)\). Show that the two mass functions differ significantly only at low masses.

- The bias as a function of mass is given in Press-Schechter theory as

\[
b(M) = 1 + \frac{\nu^2 - 1}{\delta_c} + \frac{2p}{\delta_c [1 + (a \nu^2)^p]}
\]  (4)

Take \(\delta_c\) the threshold for spherical collapse to be \(\delta_c = 1.68\). Plot \(b(M)\) from \(10^{11} M_\odot\) to \(10^{16} M_\odot\). By integrating over the Sheth-Torman mass function, find the average bias of objects \(> 3 \times 10^{14} h^{-1} M_\odot\).