Explorations of Planck-scale Noise in Noncommutative Holographic Spacetime[1]

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[1] C. J. Hogan, "Interferometers as Holograpic Clocks," arXiv:1002.4880 [gr-qc]

Motivation

- Events in spacetime are not pointlike. In quantum mechanics, the position of an event arises from an operator that acts on a wavefunction state.

- Experimentally, physically realizable measurement systems collapse the probabilities to eigenfunctions and results in limited precision.

- Standard theories of gravitation place a fundamental minimum on time intervals at the Planck scale:

$$t_P \equiv \sqrt{\frac{\hbar G_N}{c^5}} = 5.39 \times 10^{-44}\text{sec}$$

- It is reasonable to expect that a new type of uncertainty in defining specetime positions would emerge from this Planckian behavior, but the exact characteristics are unknown and have not been observed.

Holographic Geometries

- Black holes are objects of maximum entropy; they are formed when relativistic gases with too much energy collapse gravitationally. The entropy of a black hole is proportional to the area of the event horizon.[2]
- This inspired interpretations of quantum gravity and string theory that theorized that the information contained within a volume can be encoded on its boundary, especially in cases of light-like gravitational horizons.[3] [4] [5]
- There is speculation that the observed three dimensions are an effective description of a 2D information structure, but mathematical precision is difficult to obtain because the cosmological horizon is finite and growing.

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[2] J. D. Bekenstein, "Universal upper bound on the entropy-to-energy ratio for bounded systems". *Phys. Rev. D* **23** (215): 287–298 (Jan 1981)

[3] G. 't Hooft, "Dimensional reduction in quantum gravity," in "Conference on Particle and Condensed Matter Physics (Salamfest)", edited by A. Ali, J. Ellis, and S. Randjbar-Daemi (World Scientic, Singapore, 1993), arXiv:gr-qc/9310026

[4] L. Susskind, "The World As A Hologram," J. Math. Phys. 36, 6377 (1995)

[5] R. Bousso, "The holographic principle," Rev. Mod. Phys. 74, 825 (2002)

Noncommutative Geometries

- Noncommutative geometries involving spacetime coordinates that do not commute have been around for a while, and in particular have been manifested in string theory.[6]

- However, the typical treatment (that involves deformations by Moyal algebras in 3D) leads to a modified field theory similar to a Planckian filter in 3+1 dimensions, which has more degrees of freedom than the upper bound posited by the holographic principle.

- Craig Hogan has suggested, as an effective model, hypothetical position operators with a 2D commutator and the interpretation of spacetime position as being defined by interactions of null fields with matter.

- The theory is holographic, with one degree of freedom per 2D Planck area.

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[6] N. Seiberg and E. Witten, "String theory and noncommutative geometry," JHEP 9909, 032 (1999)

Operator Description

- Consider only matter and radiation. An operational definition of position for a surface can be formed by the boundary condition and how an electromagnetic field interacts with it, e.g. using the reflected phase.

- So far classical – the surface is an average of atoms, and the field is a sum of plane modes propagating in a flat metric; both remain unquantized. Different from gravitational waves or theories with quantized field modes.

- Now for a given body, choose any direction in its rest frame, and within the plane thus defined, postulate the quantum commutation relation:

$$[\hat{x}_i, \hat{x}_j] = \frac{i}{2\pi}(ct_P)^2\epsilon_{ij}$$

- These operators do not choose a particular direction in space, but depend on the rest frame of the body whose position is being measured in a specific direction.

Holographic Nature

- Given a spacelike surface, this imposes a Planckian limit on the degree of freedom in transverse spacelike directions. Can draw an analogy between $\hbar$ as a quantum of action in phase space and this newly defined quanta of a 2D planck surface area.

- Coefficient chosen to match information flux with the entropy surface density of black hole event horizons.

- The position state of matter affects the configuration of a reflected radiation field. Such an observation of position in a particular direction collapses the system into an eigenstate of that direction, so the relative transverse position is not determined until the radiation is measured a macroscopic distance away.

- Holographic nonlocality causes new uncertainty in position that is shared coherently by unconnected bodies.

Interferometers

- Measuring spacetime intervals with quantum operators: let us imagine looking at clocks as we observe the phase of a wave traveling between two events. Such operators are spacially oriented.

- So position measurements in different directions, compared over macroscopic time intervals, have limited accuracy.

- Consider a Michelson interferometer with arm length L. The signal at the dark port measures arm length difference via reflections off the beam splitter. These events are at times separated by $2L$, in two directions.

- In operator language, we are measuring:

$$\hat{X}(t) = \widehat{x_2}(t) - \widehat{x_1}\left(t - \frac{2L}{c}\right)$$

- What if we compare phases of null fields in two directions every $2L/c$?

Statistical Properties of Noise

- Conjecture: Repeated interactions of null waves with matter in two directions constitute a series of discrete measurements, for when the wavefunction collapses to an eigenstate of one direction it is undetermined in the other direction.

- The accumulated uncertainty after N measurements:

$$N\Delta x_1 \Delta x_2 = \frac{N}{2\pi}(ct_P)^2$$

- Over a macroscopic time interval $\tau \equiv N t_P$ the growth is linear in time. We can draw an analogy with quantum random walk errors (one Planck time per Planck time) observed in ideal atomic clocks. The variance shows the above linearity, while the fractional error decreases like τ^{-1}.

- At $\tau = 2L/c$, the beamsplitter has a definite position at every time that fixes the relative x_1 and x_2 phases. So we need $\tau \ll 2L/c$ for the above to hold.

Wavefunction Description[7 8]

- Think about position-space wavepackets at two times ($\psi_1(t)$, $\psi_2(t+2L/c)$) and directions. Their widths are given by $\Delta x_1(t)$ and $\Delta x_2(t+2L/c)$.

- Fourier transform to wavenumber space. After the first reflection, the wavepacket has spread to width $\Delta k_1 \approx 1/\Delta x_1$.

- This leads to a new measurement uncertainty in the definition of the rest frame: the effective transverse velocity at which the wavepacket interacts with matter, with the corresponding Planckian uncertainty, is:

$$v_2 \pm \Delta v_2 = c l_P k_1 \pm c l_P \Delta k_1/2 \approx c l_P k_1 \pm \frac{c l_P}{2\Delta x_1}$$

- After time $2L/c$, the phase shift due to this uncertainty grows with the propagation distance:

$$\Delta x_2 \approx 2L\Delta v_2/c \approx L l_P/\Delta x_1$$

Wavefunction Description[7] [8]

- So the phase difference $X = x_1 - x_2$ has a variance of:

$$\Delta X^2 = \Delta x_1^2 + \Delta x_2^2 \approx \Delta x_1^2 + (\frac{Ll_P}{\Delta x_1})^2 \geq 2Ll_P$$

- Over shorter time intervals $\tau < 2L/c$, the variance becomes:

$$\Delta X \approx \sqrt{c\tau l_P}.$$

- This picture makes it clear that the phenomenon predicted is not a simple spatial random walk. The limited bandwidth in the frequency space causes an uncertainty in the effective transverse velocity in the rest frame, and this complementary effect causes two clocks oriented along two axes to deviate by a Planck time per Planck time.

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[7] C. J. Hogan, "Measurement of Quantum Fluctuations in Geometry" Phys. Rev. D 77, 104031 (2008)
[8] C. J. Hogan, "Indeterminacy of Quantum Geometry" Phys Rev D.78.087501 (2008)

Correlated Interferometers

- How can we experimentally verify characteristics unique to the predicted phenomenon? After all, the point of an effective theory is precisely that.
- The holographic displacement calculated above is same for positions of unconnected bodies given a rest frame and direction that are shared.
- Classically, all bodies share zero displacement. The change added into the theory was a transverse jitter, but this jitter is spatially and directionally coherent. This means if we have two approximately parallel paths close together such that the transverse separation is much shorter than the paths themselves, the classical coherency is preserved.
- The nonlocal effect can be used for diagnosis: Two interferometers occupying the same volume and aligned along the same axes can be cross-correlated. The wavefunctions of the spacetime measured must collapse into the same state; holographic fluctuations share a common mode.

Experimental Possibilities

- Atomic clocks currently demonstrate errors around[9]:

$$\Delta\nu(\tau)/\nu = 2.8 \times 10^{-15}/\sqrt{\tau/sec}$$

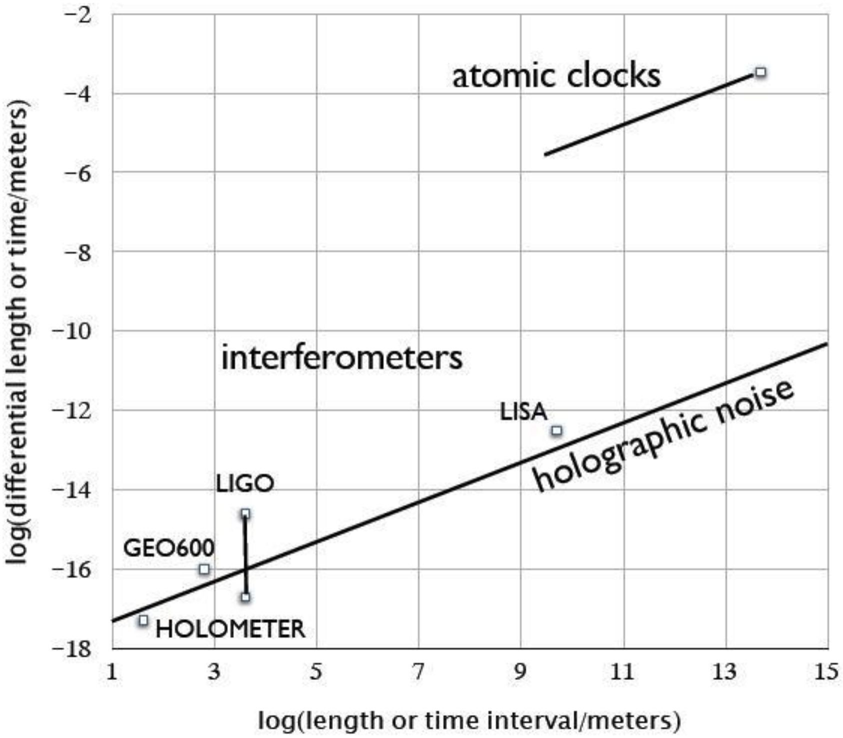
- Does not get close to the predicted Planckian directional position error:

$$\frac{\Delta\nu(\tau)}{\nu} \approx \frac{\Delta t(\tau)}{\tau} = \sqrt{\frac{5.39 \times 10^{-44}\text{sec}}{2\pi\tau}}$$

- But over short but macroscopic time intervals, interferometers can be used as precision clocks. LIGO, VIRGO, and GEO-600 nearly reach the phase accuracy needed if operated at their respective most sensitive frequencies.

- LIGO seems to be precise enough at first glance, but would not be able to detect metric and holographic fluctuations to the same precision because 1D Fabry-Perot cavities in the arms used to amplify phase displacements from gravitational waves have no effect on holographic noise.

Gravitational Interferometer Sensitivity[9 10 11]



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[9] C.-W. Chou, D.B. Hume, J.C.J. Koelemeij, D.J. Wineland, and T. Rosenband, "Frequency Comparison of Two High-Accuracy Al+ Optical Clocks", Physical Review Letters, 104, 070802 (2010)

[10] B. P. Abbott et al. [LIGO Scientic Collaboration and VIRGO Collaboration], "An Upper Limit on the Stochastic Gravitational-Wave Background of Cosmological Origin," Nature 460, 990 (2009)

[11] H. L□uck et al., "The upgrade of GEO600," J. Phys. Conf. Ser. 228, 012012 (2010) [arXiv:1004.0339 [gr-qc]]

Proposed Holometer

- We would like to isolate the holographic portion of noise from other sources of noise. We start with an optimal frequency 2~3 orders of magnitude higher than gravitational wave detectors, around $c/2L$.
- At such high frequencies, photon shot noise is dominant. But by using two cross-correlated interferometers, we can average over time and reduce the relative magnitude of the uncorrelated photon shot noise, whereas the correlated holographic noise would integrate up.
- The GEO600 experiment achieves sufficient photon shot noise level. LIGO uses correlated interferometers but not ones that are co-located. The challenge is to record correlated signals around 3.74MHz(40m/L).
- Will be able to observe an effective behavior that preserves classical coherence and Lorentz invariance in a single direction but violates them in different noncommuting directions.

Statistical Properties of Noise

- Time-domain autocorrelation function for a single interferometer:

$$\Xi(\tau) \equiv \lim_{T\to\infty} (2T)^{-1} \int_{-T}^{T} X(t)X(t+\tau)dt \equiv \langle X(t)X(t+\tau)\rangle$$

- The mean square displacement demonstrates a Planckian random walk:

$$\langle [X(t) - X(t+\tau)]^2 \rangle = 2\langle X^2 \rangle - 2\Xi(\tau) = c^2 t_P \tau/\pi$$

- The total variance is $\langle \mathrm{X}^2 \rangle = \Xi(0) = ct_P L/\pi$.

- Again, around $\tau = 2L/c$, the radiation along the two axes are no longer independent due to the beamsplitter. That is the time it takes for light to make a single round trip, and after that causal boundary the relative phases no longer experience differential random walks. Therefore:

$$\Xi(\tau) = (ct_P/2\pi)(2L - c\tau) \text{ for } 0 < \mathrm{c}\tau < 2\mathrm{L} \text{ and 0 otherwise}$$

Statistical Properties of Noise

- For two correlated interferometers A and B, the cross correlation is:

$$\Xi(\tau)_\times \equiv \lim_{T\to\infty}(2T)^{-1}\int_{-T}^{T} X_A(t)X_B(t+\tau)dt \equiv \langle X_A(t)X_B(t+\tau)\rangle$$

- Assume that each null plane wave shares the same holographic displacements in the transverse direction. So when two wavefronts are traveling parallel to each other at the same time, the two interferometers share differential phase perturbations. But obviously we cannot have two interferometers at the exact same location, so assume a displacement ΔL.

- $\Xi_\times(\tau) = (ct_P/2\pi)(2L - 2\Delta L - c\tau)$ for $0 < c\tau < 2L - 2\Delta L$

- This correlated behavior is counterintuitive, but we are hoping for an experimental breakthrough in the spirit of the Bohr atom model.

- Have a good spring break everyone!