

Growth Function

The linear growth rate equation is given by

$$\begin{aligned} \frac{d^2 G}{d \ln a^2} + \left[\frac{5}{2} - \frac{3}{2} w(a) \Omega_{DE}(a) \right] \frac{dG}{d \ln a} \\ + \frac{3}{2} [1 - w(a)] \Omega_{DE}(a) G = 0, \end{aligned} \quad (1)$$

in a flat universe. Here $\Omega_{DE}(a) = \rho_{DE}(a)/\rho_{\text{crit}}(a)$ and $w(a) = p_{DE}(a)/\rho_{DE}(a)$. Write a ODE solver code that computes $G(a)$ given an input function for $w(a)$ and the value of $\Omega_{DE}(a = 1)$. Take the initial conditions to be $G = 1$, $dG/d \ln a = 0$.

Evaluate $G(a = 1)$ for $\Omega_{DE} = 0.73$ and $w(a) = -1$ and check it against the result given in the previous problem set. Form a lookup table for $G(a)$ and incorporate it into your power spectrum, $\sigma(M, z)$ and mass function codes.

[some helper references on ODE's and spline lookup tables are given in the Boltzmann group problem sets. at the expense of generality you may omit this exercise in favor of using the Λ CDM fitting function from the ARAA by Carroll, Press & Turner]