Non-Linear Matter Power Spectrum

Put together the various pieces of the last few problem sets to form the two and one halo terms of the non-linear power spectrum $P_{nl}(k, z)$ from the linear power spectrum P(k, z):

$$P_{\rm nl}(k,z) = I_2^2(k,z)P(k,z) + I_1(k,z)$$
(1)

where

$$I_2(k,z) = \int d\ln M \left(\frac{M}{\rho_m(z=0)}\right) \frac{dn}{d\ln M} b(M) y(k,M)$$
(2)

$$I_1(k,z) = \int d\ln M \left(\frac{M}{\rho_m(z=0)}\right)^2 \frac{dn}{d\ln M} y^2(k,M)$$
(3)

Show that $P_{nl}(k, z) \to P(k, z)$ for $k \to 0$ for the Sheth-Torman mass function (what happens with if you take the Jenkins mass function). Plot the logarithmic power spectra $\Delta_m^2(k)$ at z = 0 and z = 1 for the cosmology of the previous problem sets. Separate the total contribution into the two and one halo terms.

Extra credit: add in galaxy occupation to form the galaxy power spectrum. See the Physics Reports review by Sheth & Cooray for details. For definiteness, take N(M) = C + S(M) where C = 1, $S(M) = M/30M_{\text{th}}$, if $M > M_{\text{th}} = 10^{12}h^{-1}M_{\odot}$ (else C = S = 0).