Cosmology with Gravitational Wave Detectors

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Part I: Cosmography

Compact Binary Coalescenses are ***Standard Sirens***

- The amplitude* of a GW from a CBC is

$$\langle h \rangle = 1 \times 10^{-23} \frac{M}{M_\odot}^{2/3} \frac{\mu}{M_\odot} \frac{f}{100\mathrm{Hz}}^{2/3} \frac{100\mathrm{Mpc}}{r}$$

- The timescale is

$$\tau = f/\dot{f} = 7.8 \frac{M}{M_\odot}^{-2/3} \frac{\mu}{M_\odot}^{-1} \frac{f}{100\mathrm{Hz}}^{-8/3} \mathrm{s}$$

- Measuring amplitude, frequency, and chirp rate gives the distance to the CBC

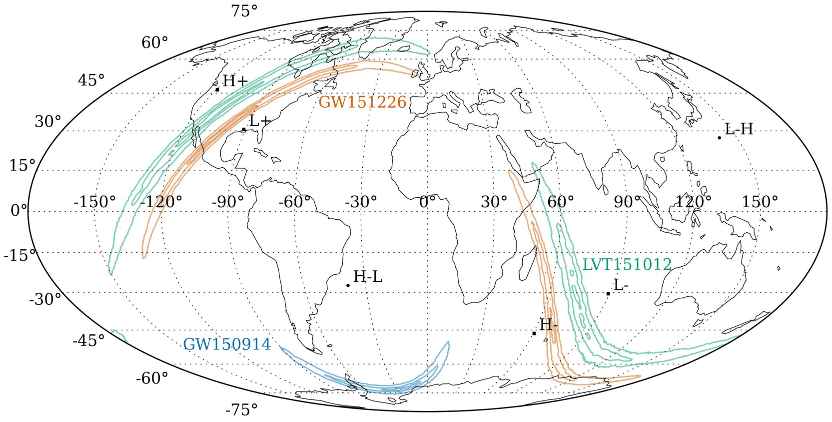
$$r = 780 \frac{f}{100\mathrm{Hz}}^{-2} (\langle h \rangle \times 10^{23} \tau)^{-1} \mathrm{Mpc}$$

- To correct for cosmology, multiply all masses by (1+*z*) and replace physical distance *r* by D_L

Schutz, 1986

Distance Uncertainties from GW signal

- Main uncertainty is measuring the intrinsic amplitude of the signal
- Uncertainty in amplitude of detector response ~ 1/SNR
- Inferring intrinsic amplitude from detector response depends on localization error
- Microlensing



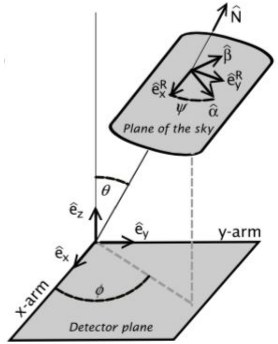
See Holz & Hughes, 2005

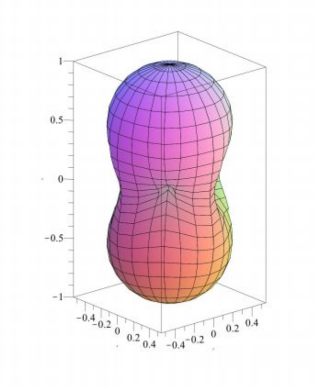
From Abbott et al., Phys. Rev. X 6, 041015 (2016)

Compact Binary Coalescenses are ***Standard Sirens***

- *In reality, the GW amplitude also depends on its *inclination* and *sky position*

$$\frac{\delta L(t)}{L} = F_+(\theta, \phi, \psi)h_+(t) + F_\times(\theta, \phi, \psi)h_\times(t)$$



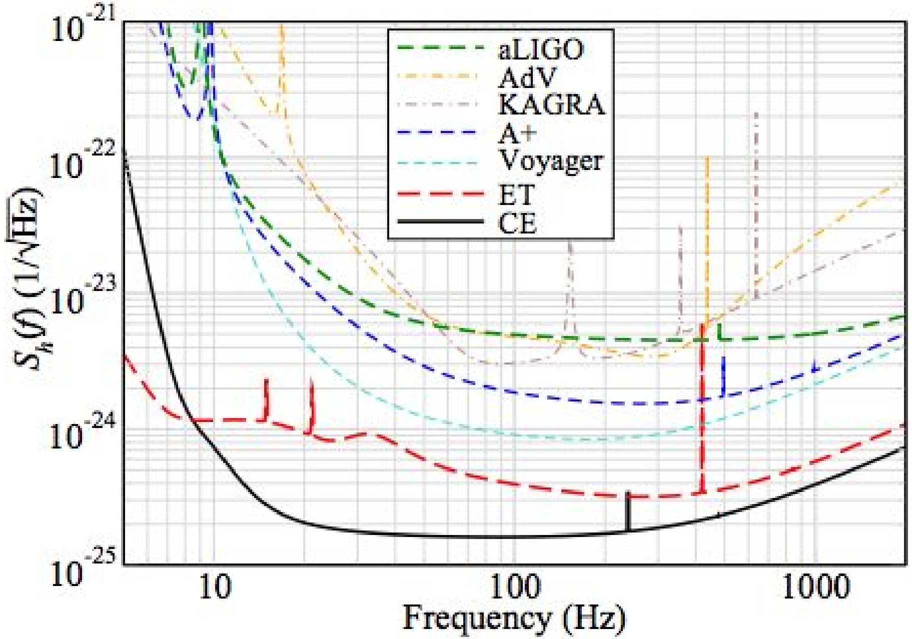


- The degree of elliptic polarization gives the binary inclination
- The differences in arrival times between multiple detectors gives sky position

GW signal gives D_L but not the redshift

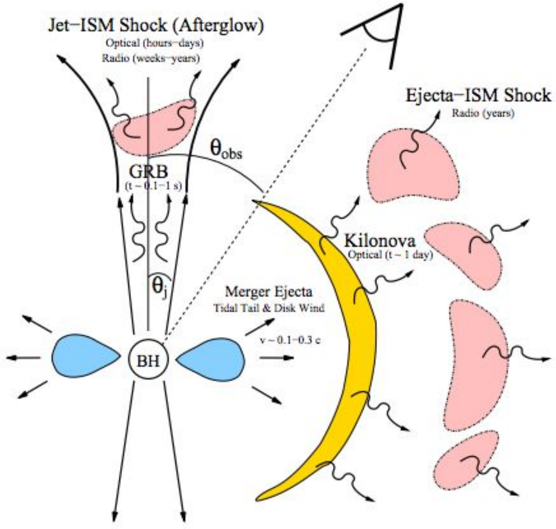
- Scale-invariance of CBC solution means that a signal with z=2 and chirp mass M looks like signal with $z=0$ and chirp mass $M/3$
- Solutions:
  - Identify EM counterpart
  - Identify the host galaxy
  - GW signal of binary neutron star systems with known EOS

Future Ground-Based Detector Sensitivities



Identifying the Source Redshift: EM Counterpart

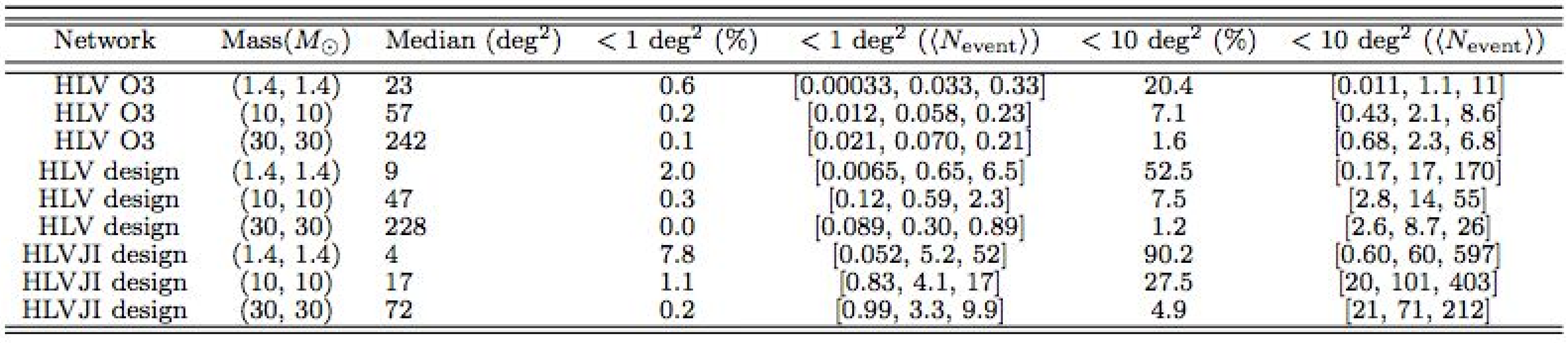
- Associated sGRB, kilonova, afterglow for neutron star mergers
- Detection rates unclear, most mergers may not have detectable sGRB because of beaming angle



From Metzger & Berger, 2011, arxiv: 1108.6056

Identifying the Source Redshift: Host Galaxy

- Cross correlation with galaxy surveys, take into account incompleteness
- Targeted follow up of localization region for well-localized events

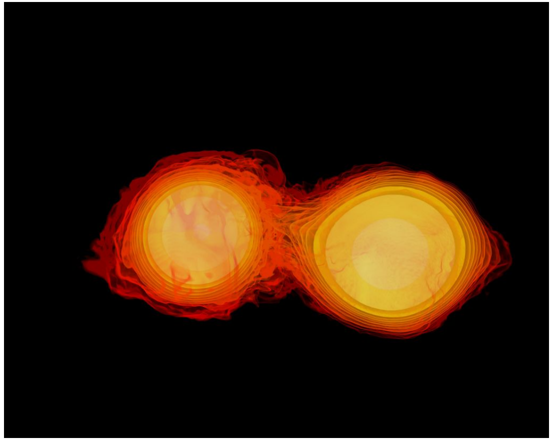


Network	Mass($M_\odot$)	Median (deg^2)	< 1 deg^2 (%)	< 1 deg^2 ($\langle N_{\mathrm{event}} \rangle$)	< 10 deg^2 (%)	< 10 deg^2 ($\langle N_{\mathrm{event}} \rangle$)
HLV O3	(1.4, 1.4)	23	0.6	[0.00033, 0.033, 0.33]	20.4	[0.011, 1.1, 11]
HLV O3	(10, 10)	57	0.2	[0.012, 0.058, 0.23]	7.1	[0.43, 2.1, 8.6]
HLV O3	(30, 30)	242	0.1	[0.021, 0.070, 0.21]	1.6	[0.68, 2.3, 6.8]
HLV design	(1.4, 1.4)	9	2.0	[0.0065, 0.65, 6.5]	52.5	[0.17, 17, 170]
HLV design	(10, 10)	47	0.3	[0.12, 0.59, 2.3]	7.5	[2.8, 14, 55]
HLV design	(30, 30)	228	0.0	[0.089, 0.30, 0.89]	1.2	[2.6, 8.7, 26]
HLVJI design	(1.4, 1.4)	4	7.8	[0.052, 5.2, 52]	90.2	[0.60, 60, 597]
HLVJI design	(10, 10)	17	1.1	[0.83, 4.1, 17]	27.5	[20, 101, 403]
HLVJI design	(30, 30)	72	0.2	[0.99, 3.3, 9.9]	4.9	[21, 71, 212]

From Chen & Holz 2016, arxiv:1612.01471

Identifying the Source Redshift: Neutron Star EOS

- Expect to detect 10^4- 10^7 BNS systems per year with Einstein Telescope
- GW signal is modified for neutron stars with tidal deformability
- Tidal deformability is a function of EOS and rest mass
- It is expected that the EOS will be well-constrained after *O*(30) BNS detections (del Pozzo et al. 2013, arxiv:1307.8338)



Part II: Stochastic Gravitational Wave Background

Cosmological versus Astrophysical GW Background

- Cosmological background:
  - Emitted ~10^{-24} seconds after the Big Bang
  - Test inflationary models
  - Topological phase transitions and cosmic strings
  - String theory
- Astrophysical background:
  - Compact binaries
  - Pulsars
  - Core-collapse supernovae

The GW Energy Density Spectrum

- Energy density spectrum defined as:

$$\Omega_{\mathrm{gw}} := \frac{d\rho_{\mathrm{gw}}/\rho_c}{d\ln f}$$

- The resulting strain spectral noise density in the detector:

$$S_{\mathrm{gw}}(f) = \frac{3H_0^2}{10\pi^2} f^{-3}\,\Omega_{\mathrm{gw}}(f)$$

- If there are enough sources, this will look like Gaussian noise
  - Need to understand detector noise very well
  - Cross-correlation between detectors, for co-located detectors:

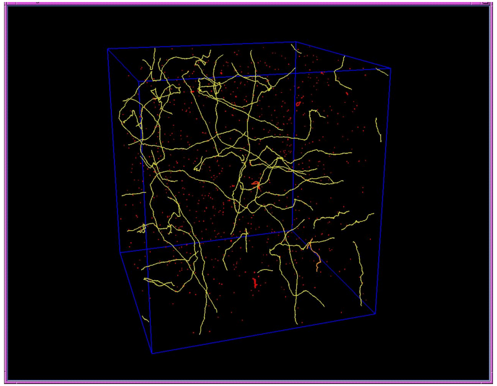
$$\Omega_{\mathrm{gw}}^{1/2} h_{100} = \left(\frac{S_h^{1/2}}{3.1\times 10^{-18}\ \mathrm{Hz}^{-1/2}}\right)\left(\frac{f}{10\ \mathrm{Hz}}\right)^{5/4}\left(\frac{T}{3\ \mathrm{yrs}}\right)^{-1/4}$$

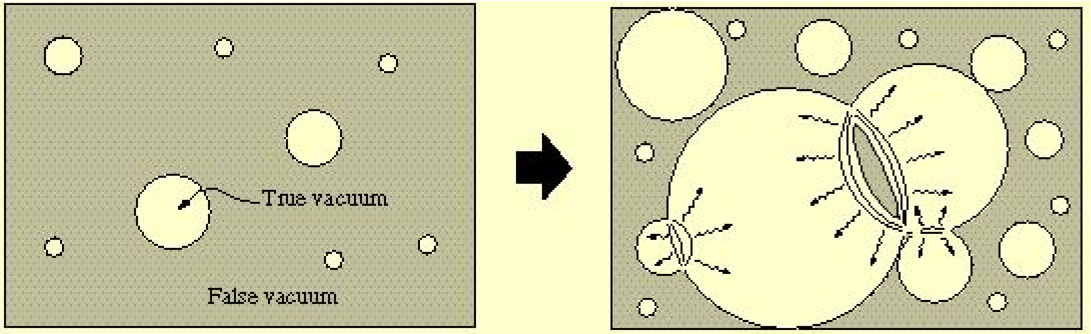
The Shape of the GW Energy Spectrum

- It is *not* a blackbody spectrum like the CMB unless gravity was in thermal equilibrium with other matter and radiation (but if it were, the temperature would be 0.9K)
- CBC background would have power law dependence with index ⅔
- Higher frequencies carry information about earlier times

The Shape of the GW Energy Spectrum

- Inflation would lead to a flat spectrum assuming nearly constant *H(t)*
- Networks of cosmic strings scale with the Hubble length, produce flat spectrum in frequencies accessible to ground-based detectors
- Bubble collisions from first-order phase transitions produce a strongly peaked spectrum at frequency = 1/time of phase transition

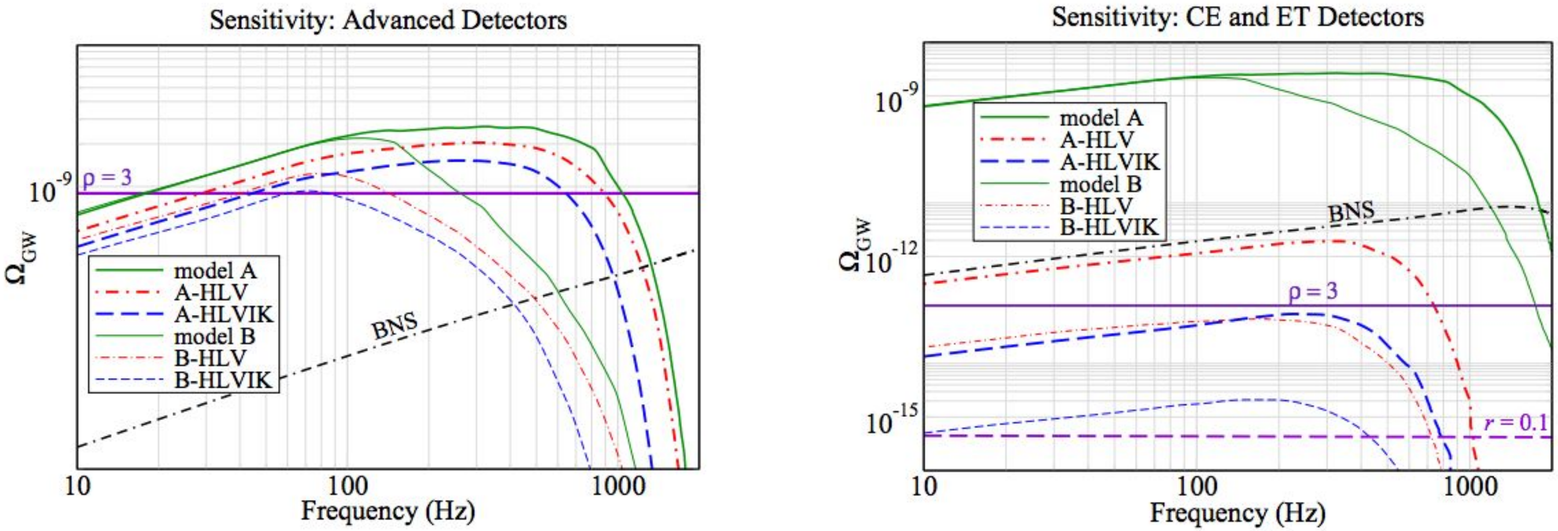




The Astrophysical Background is Likely to Dominate

- LIGO’s first detection suggests a higher coalescence rate of massive BBHs than expected
- LIGO is likely to detect a stochastic astrophysical background, but this will dominate the primordial background
- Third generation detectors will be able to detect & resolve most of these sources, enabling a reduction in the astrophysical background (Regimbau et al., arxiv:1611.08943)

Sensitivity to Stochastic Background



From radiation dominated era, most optimistic $\Omega_{Gw} \sim 10^{-10}$, usually $O(10^{-12})$ -- see Giblin & Thrane, arxiv:1410.4779

Summary

- Gravitational-wave standard sirens are a promising tool for cosmography
- We may or may not detect a stochastic gravitational wave background of primordial origin