N-BODY SIMULATION, PP METHOD

\[ \vec{F}_i = - \sum_{j \neq i} \frac{G m_i m_j (\vec{r}_i - \vec{r}_j)}{(\|\vec{r}_i - \vec{r}_j\|^2 + \epsilon^2)^{3/2}} \]

• Direct integration has computation complexity scaling as \( N^2 \)
  
  \( \sim 10 \) float point operations per pair of particles

  \( 2.7 \text{ GHz} \times 2 \text{fpu} \times 2 \text{flops/cycle} \times 2 \text{cores} = 20 \text{ Gflops} \), \( N=10^7 \) means 1 day/step

• Direct integration only used in small \( N \), e.g., planetary systems, star clusters ...
MEAN FIELD APPROACH

• BOLTZMANN EQUATION:

\[
\frac{\partial f_i}{\partial t} + \dot{x} \frac{\partial f_i}{\partial x} + \ddot{x} \frac{\partial f_i}{\partial x} = C[f_i, f_j].
\]

• COLLISIONLESS SYSTEM (C=0):

\[
\frac{dx}{dt} = u, \quad \frac{du}{dt} = -\nabla \phi
\]

\[
\nabla^2 \phi = 4\pi G \rho
\]
PARTICLE-MESH CODE

• Use a mesh for the density and potential

Why using it?
• Resolution under certain scale (~1kpc) not important
• It is FAST! (N+NlogN)
• Large number of particles
• No artificial binaries
EQUATIONS & VARIABLES

• Comoving coordinates $x$; particle momenta $p=av$;
• Use scale factor $a$ as time variable

\[
\frac{dp}{da} = -\frac{\nabla \phi}{\dot{a}} \quad \frac{dx}{da} = \frac{p}{\dot{a}a^2}
\]

\[
\nabla^2 \phi = 4\pi G_m a^2 \rho_{cr} \delta
\]
EQUATIONS & VARIABLES

• CHANGE PHYSICAL UNIT TO CODE UNIT:

\[ x = x_0 \tilde{x} \quad t = \tilde{t}/H_0 \]

\[ p = x_0 H_0 \tilde{p} \quad \nu_{pec} = (x_0 H_0) \tilde{p}/a \]

\[ \phi = \tilde{\phi}(x_0 H_0)^2 \quad \rho = \frac{\tilde{\rho}}{a^3} \frac{3H_0^2}{8\pi G} \Omega_0 \]
SCHEME OF INTEGRATION

\[ \frac{d\tilde{p}}{da} = -F(a)\tilde{\nabla} \tilde{\phi} \quad \frac{d\tilde{x}}{da} = F(a)\frac{\tilde{p}}{a^2} \]

\[ \tilde{\nabla}^2 \tilde{\phi} = \frac{3\Omega_0}{2a}(\tilde{\rho} - 1) = \frac{3\Omega_0}{2a} \tilde{\delta} \]

\[ F(a) = [a^{-1}(\Omega_0 + \Omega_{k0}a + \Omega_{\Lambda0}a^3)]^{-1/2} \]

- DENSITY ASSIGNMENT (NGP, CIC, TSC) \(\sim O(N)\)
- FFT TO CALCULATE GRADIENT \(\sim O(N\log N)\)
- LEAP-FROG (SECOND ORDER)
TEST: A SIMPLE SINE WAVE

Zeldovich approximation (First-order Lagrangian collapse model):

Expressed by: \[ x(t) = q + D_+(t)S(q) \]

ZA is exact in 1D until the first crossing of particle trajectories occurs.
TEST: ZELDOVICH APPROXIMATION
COSMOLOGICAL INITIAL CONDITION

- Generate a random Gaussian field
- Modulate it by power spectrum in Fourier space
- Compute displacement vector field in Fourier space
- Transform displacement vector back to real space
- Move particles
Deviation from the true power spectrum is due to mass assignment onto finite grids using an FFT. Jing 2005 provided a way to recover it.
POWER SPECTRA
HALO PROFILE

63 Mpc

10 Mpc
HALO PROFILE

$slope \approx -4 \sim -3$

1 Mpc
HALO MASS FUNCTION

Halos are detected using ROCKSTAR (Behroozi et. al. 2013)
HALO MASS FUNCTION
OTHER SCHEMES

• Tree Code:
  Gridless, use multipole expansion to calculation gravitational force

• Adaptive Mesh Refinement Method (AMR, e.g., ENZO):
  Grid elements are concentrated where a higher resolution is needed

• P$^3$M CODE:
  PM+ direct summation over neighboring particles

• Adaptive Meshes P$^3$M (AP$^3$M)

• Hybrid PM-Tree Scheme (GADGET2)
REFERENCES

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• HTTP://ASTRO.UCHICAGO.EDU/~ANDREY/TALKS/PM/PM.PDF
• JING 2005
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