COSMOLOGICAL SIMULATION 101

PARTICLE-MESH CODE

Huanqing Chen
CosmologyII 2017

N-BODY SIMULATION, PP METHOD

$$\vec{F}_i = -\sum_{j\neq i} \frac{G m_i m_j (\vec{r_i} - \vec{r_j})}{(|\vec{r_i} - \vec{r_j}|^2 + \epsilon^2)^{3/2}}$$

- Direct integration has computation complexity scaling as N^2

  ~10 float point operations per pair of particles

  2.7 GHz*2fpu*2flops/cycle*2cores=20 Gflops , N=10^7 means 1day/step

- Direct integration only used in small N, e.g., planetary systems, star clusters …

MEAN FIELD APPROACH

- BOLTZMANN EQUATION:

$$\frac{\partial f_i}{\partial t} + \dot{x}\frac{\partial f_i}{\partial x} + \ddot{x}\frac{\partial f_i}{\partial \dot{x}} = C[f_i, f_j]$$

- COLLISIONLESS SYSTEM (C=0):

$$\frac{dx}{dt} = u, \frac{du}{dt} = -\nabla\phi$$

$$\nabla^2\phi = 4\pi G\rho$$

PARTICLE-MESH CODE

- Use a mesh for the density and potential

Why using it?

- Resolution under certain scale (~1kpc) not important
- It is FAST! (N+NlogN)
- Large number of particles
- No artificial binaries

EQUATIONS & VARIABLES

- Comoving coordinates x; particle momenta p=av;
- Use scale factor a as time variable

$$\frac{dp}{da} = -\frac{\nabla\phi}{\dot{a}} \qquad \frac{dx}{da} = \frac{p}{\dot{a}a^2}$$

$$\nabla^2\phi = 4\pi G_m a^2 \rho_{cr}\delta$$

EQUATIONS & VARIABLES

- CHANGE PHYSICAL UNIT TO CODE UNIT:

$$x = x_0 \tilde{x} \qquad t = \tilde{t}/H_0$$

$$p = x_0 H_0 \tilde{p} \qquad v_{pec} = (x_0 H_0)\tilde{p}/a$$

$$\phi = \tilde{\phi}(x_0 H_0)^2 \qquad \rho = \frac{\tilde{\rho}}{a^3}\frac{3H_0^2}{8\pi G}\Omega_0$$

SCHEME OF INTEGRATION

$$\frac{d\tilde{p}}{da} = -F(a)\tilde{\nabla}\tilde{\phi} \qquad \frac{d\tilde{x}}{da} = F(a)\frac{\tilde{p}}{a^2}$$

$$\tilde{\nabla}^2\tilde{\phi} = \frac{3\Omega_0}{2a}(\tilde{\rho} - 1) = \frac{3\Omega_0}{2a}\tilde{\delta}$$

$$F(a) = [a^{-1}(\Omega_0 + \Omega_{k0}a + \Omega_{\Lambda 0}a^3)]^{-1/2}$$

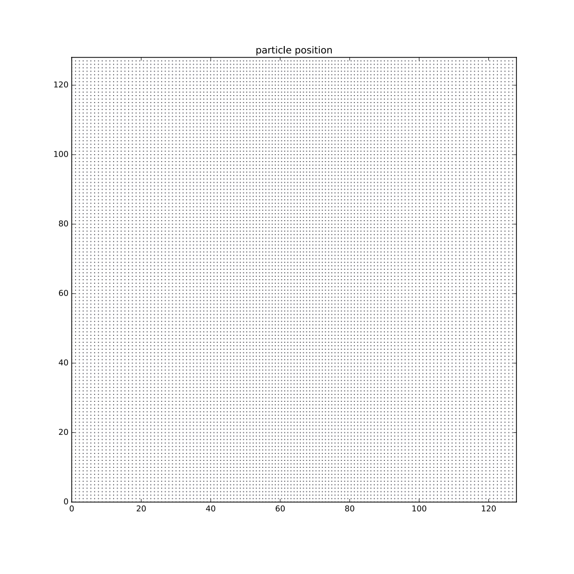
- DENSITY ASSIGNMENT (NGP, CIC, TSC) ~ O(N)
- FFT TO CALCULATE GRADIENT ~ O(NlogN)
- LEAP-FROG (SECOND ORDER)

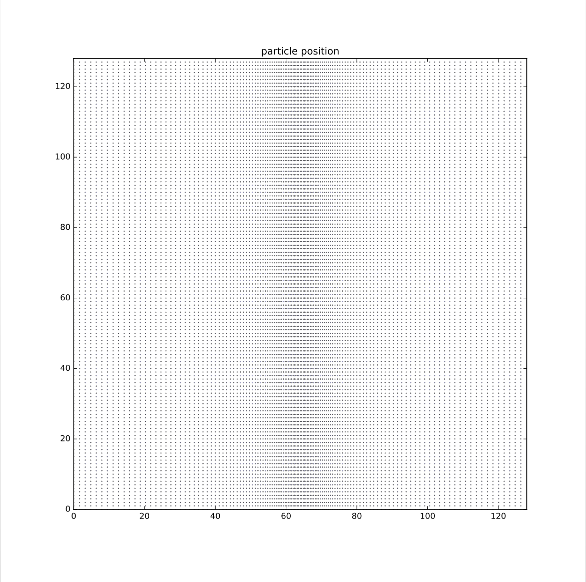
TEST: A SIMPLE SINE WAVE

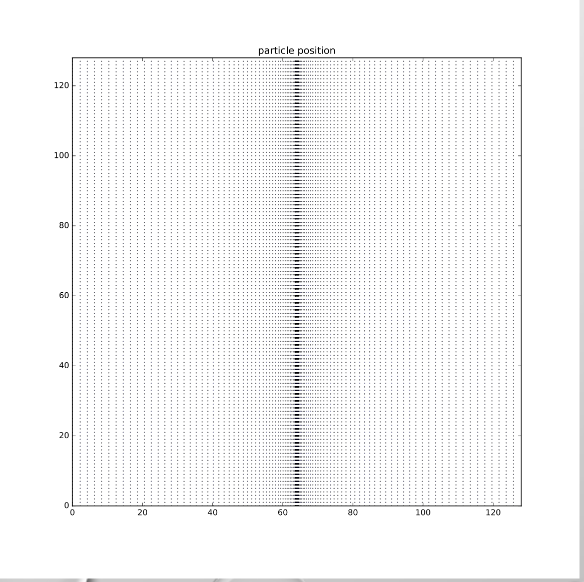
Zeldovich approximation (First-order Lagrangian collapse model):

Expressed by: $x(t) = q + D_{+}(t)S(q)$

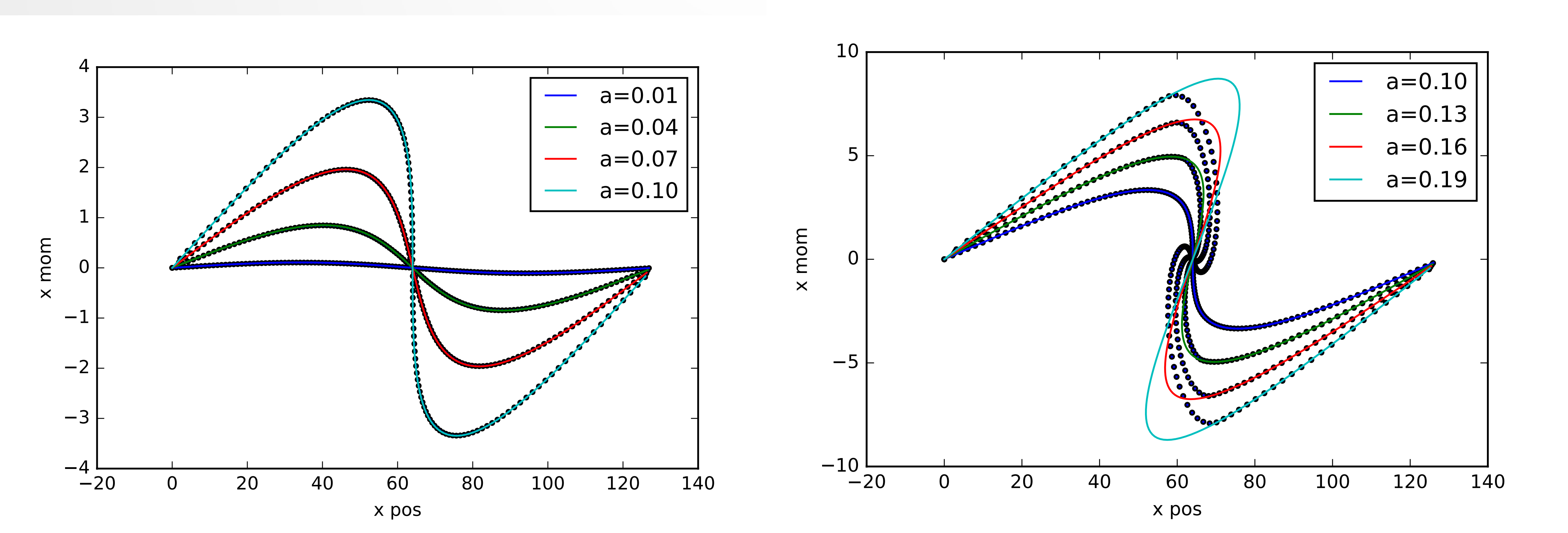
ZA is exact in 1D until the first crossing of particle trajectories occurs.







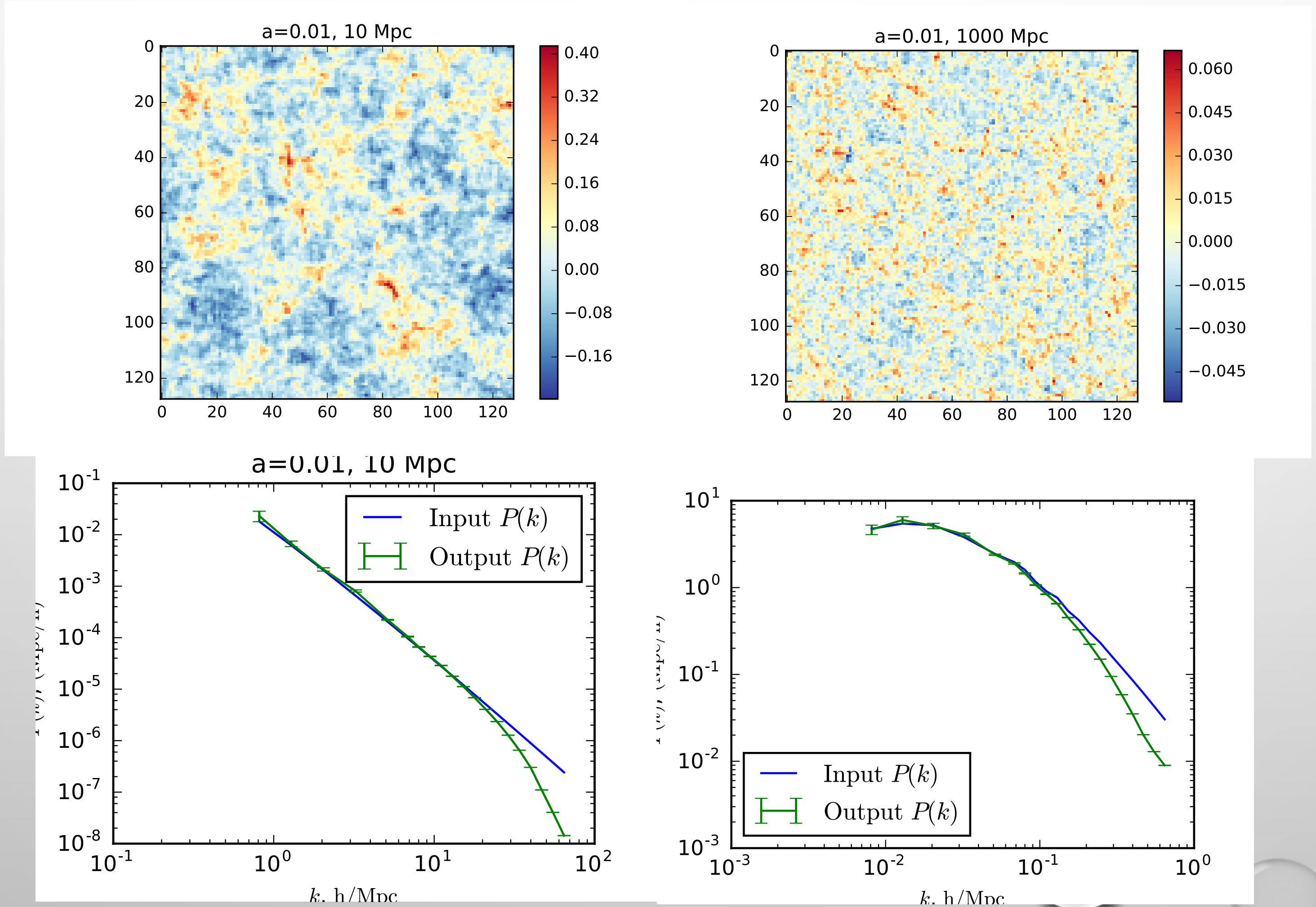
TEST: ZELDOVICH APPROXIMATION



COSMOLOGICAL INITIAL CONDITION

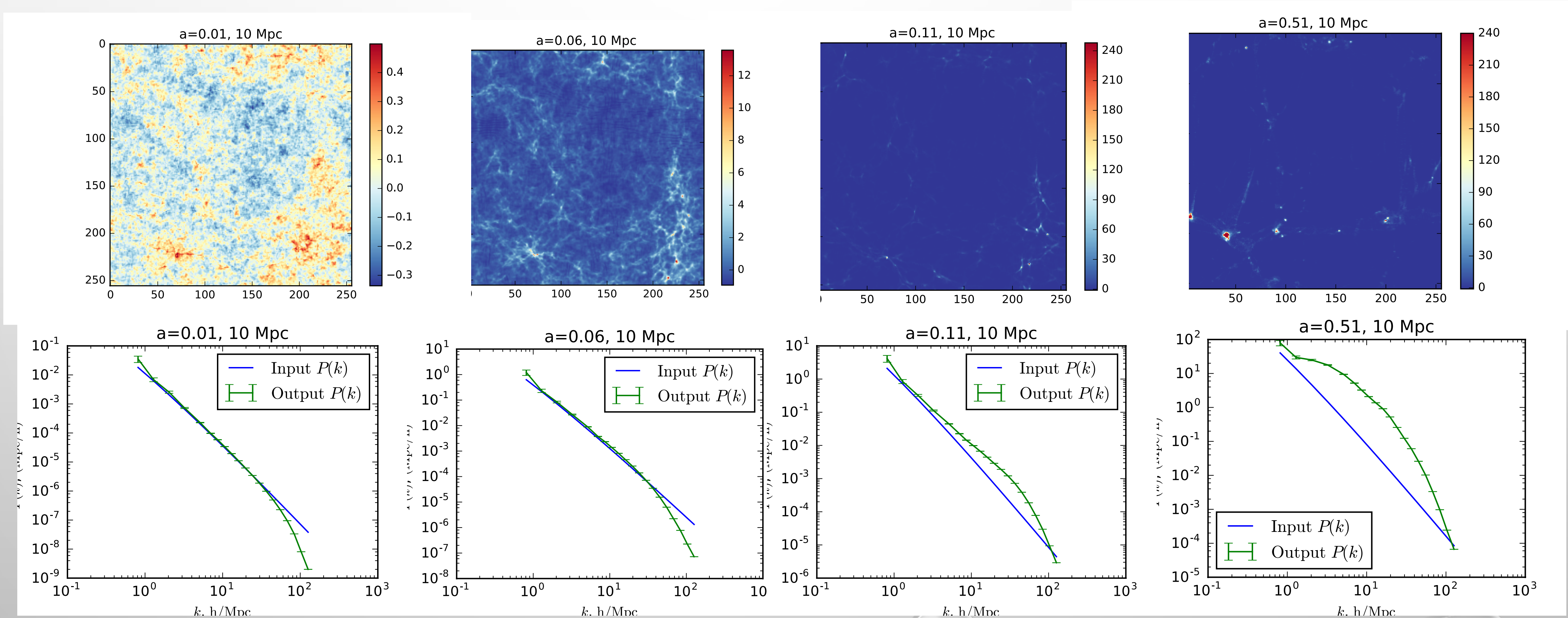
- Generate a random Gaussian field
- Modulate it by power spectrum in Fourier space
- Compute displacement vector field in Fourier space
- Transform displacement vector back to real space
- Move particles

INITIAL POWER SPECTRA

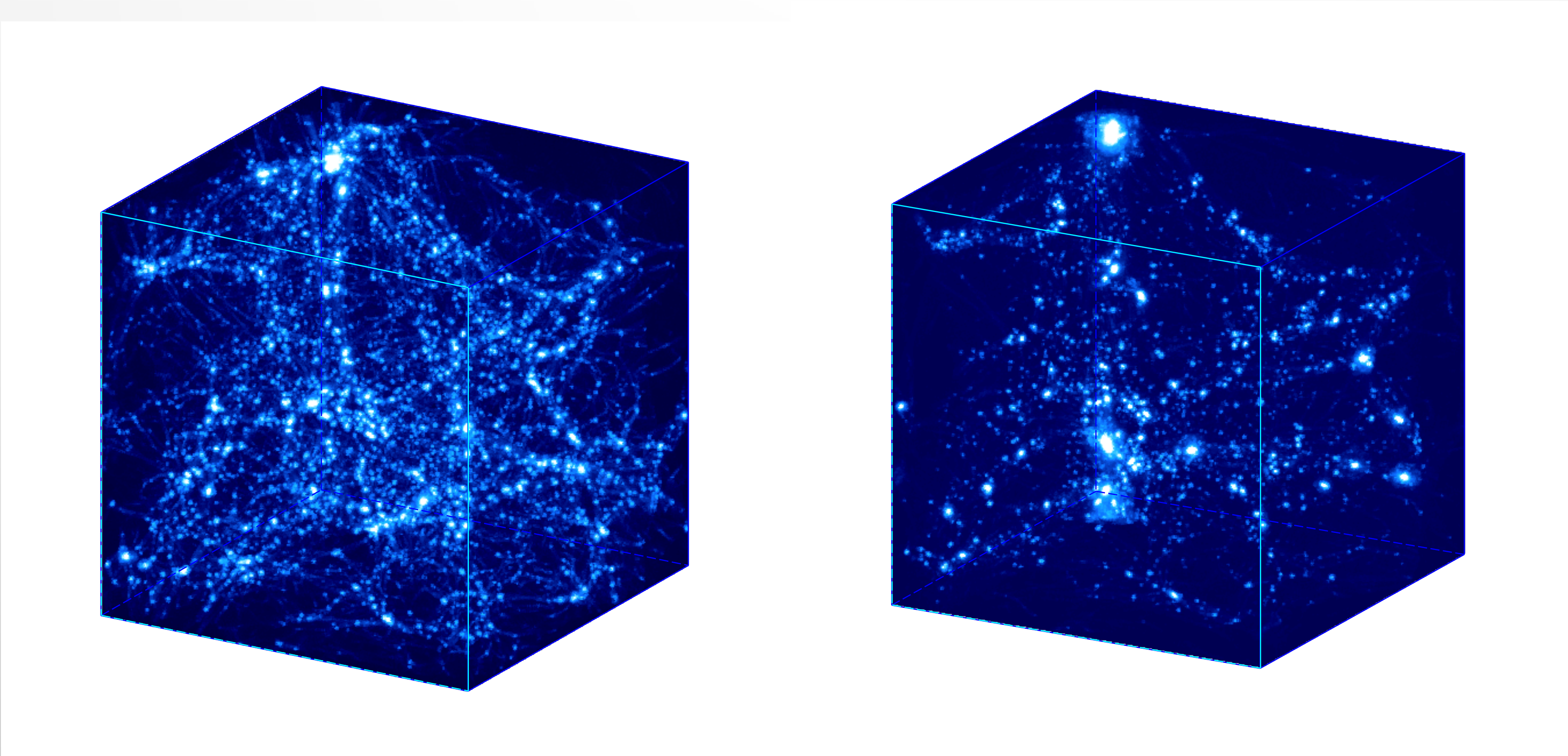


Deviation from the true power spectrum is due to mass assignment onto finite grids using an FFT.
Jing 2005 provided a way to recover it.

POWER SPECTRA



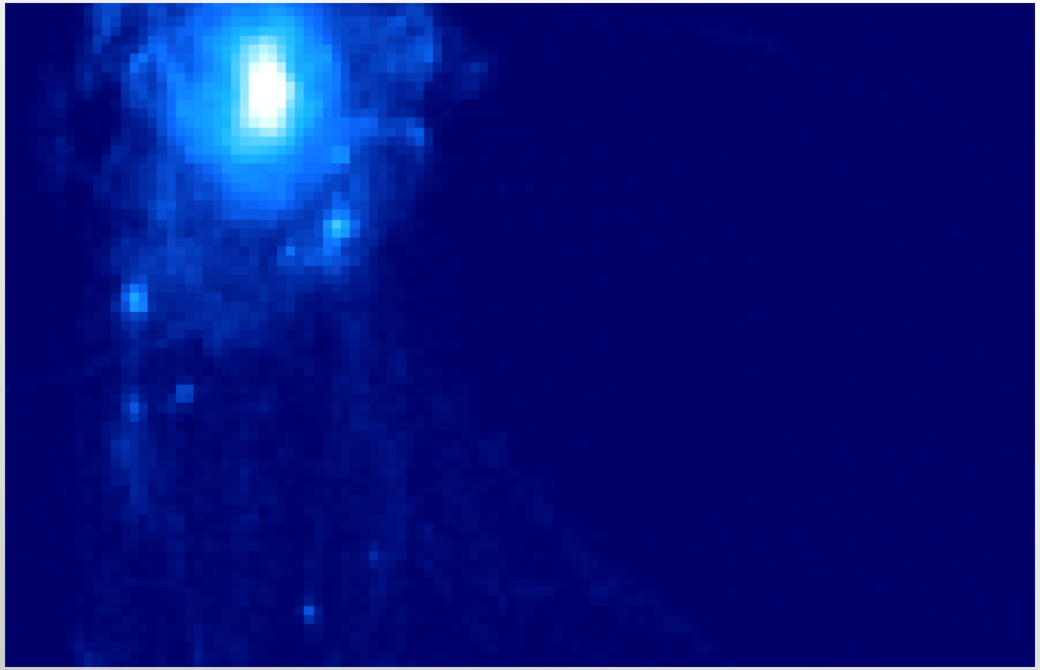
HALO PROFILE



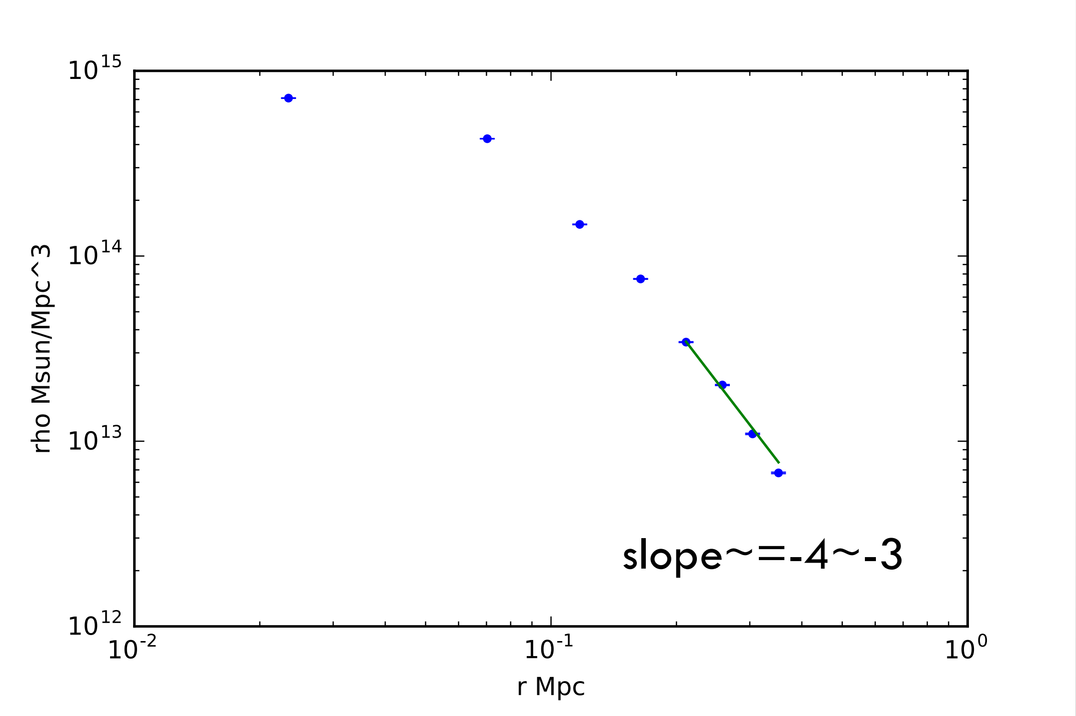
63 Mpc

10 Mpc

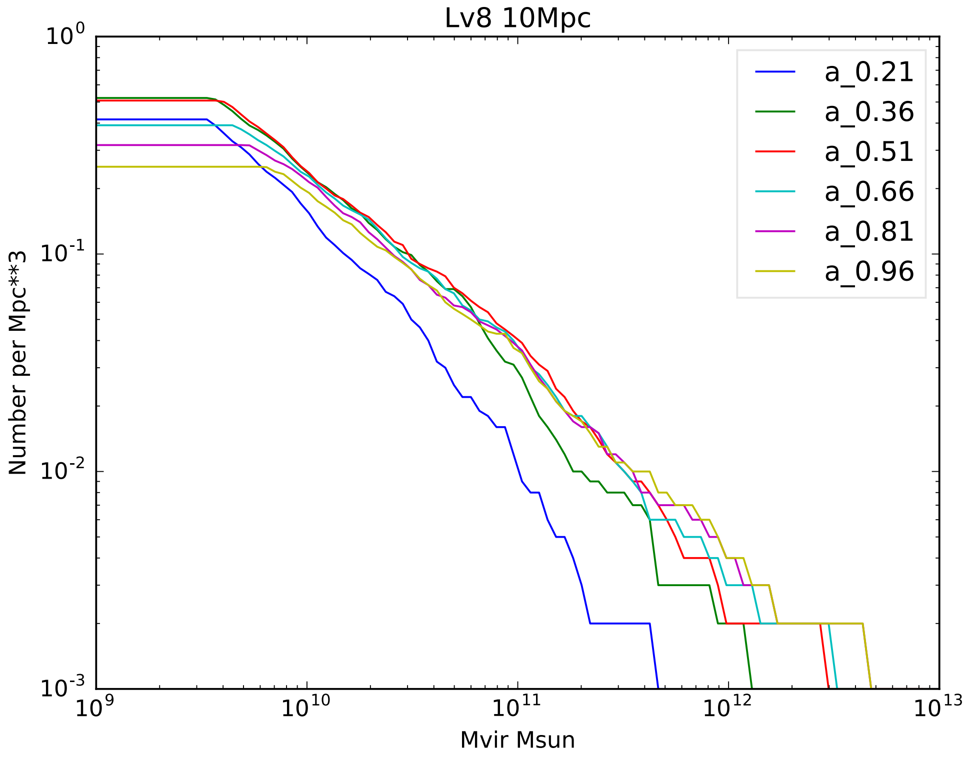
HALO PROFILE

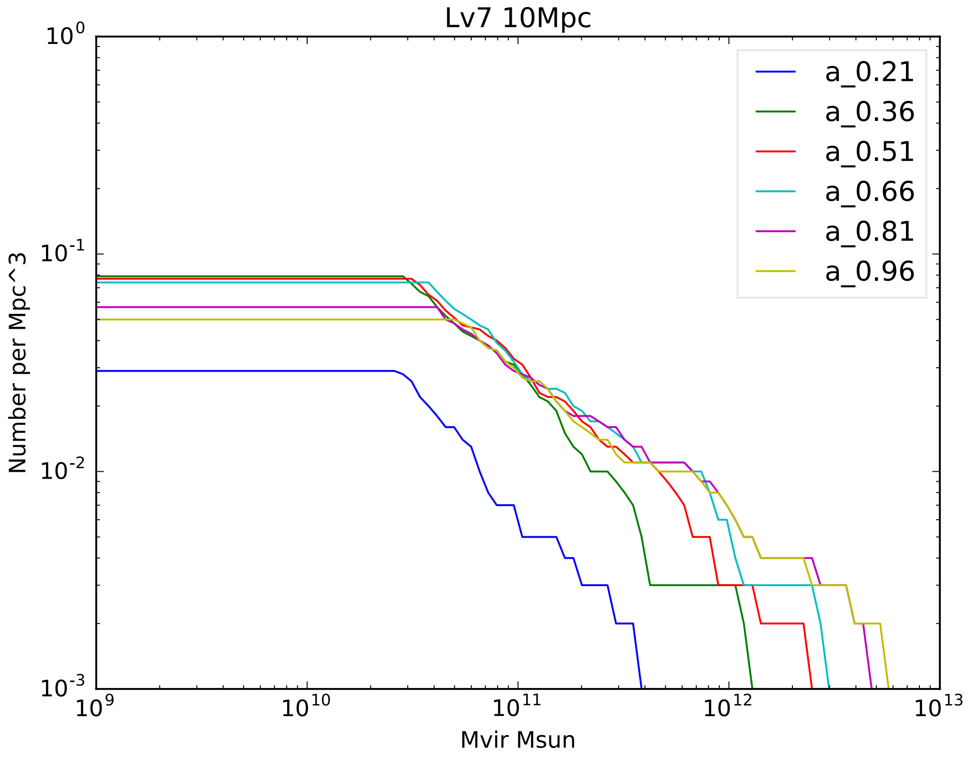


1 Mpc



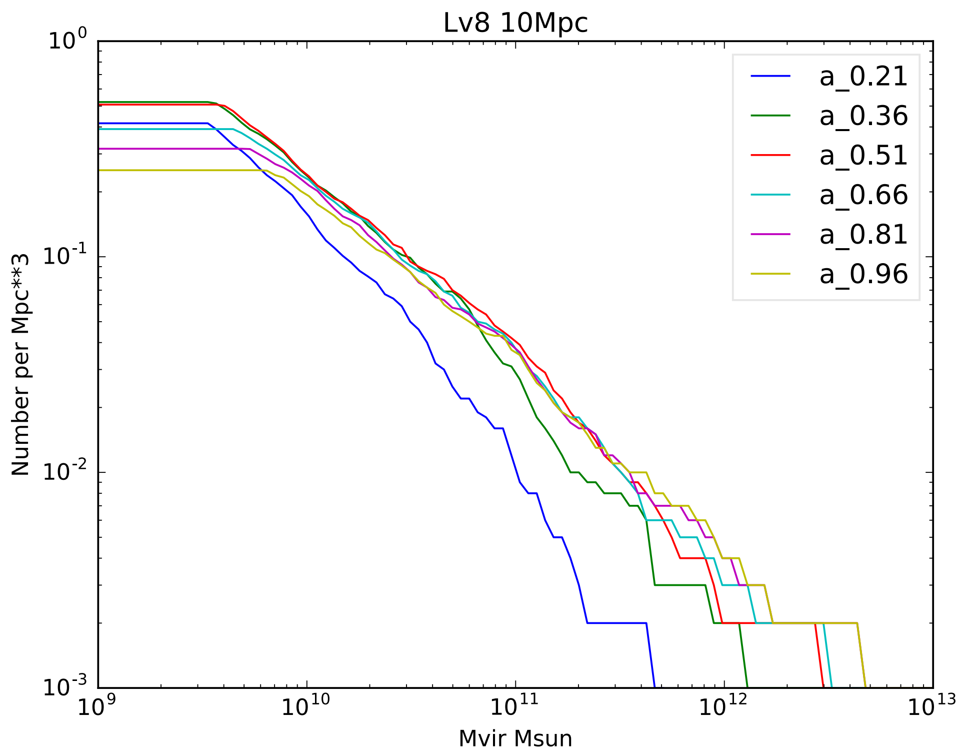
HALO MASS FUNCTION

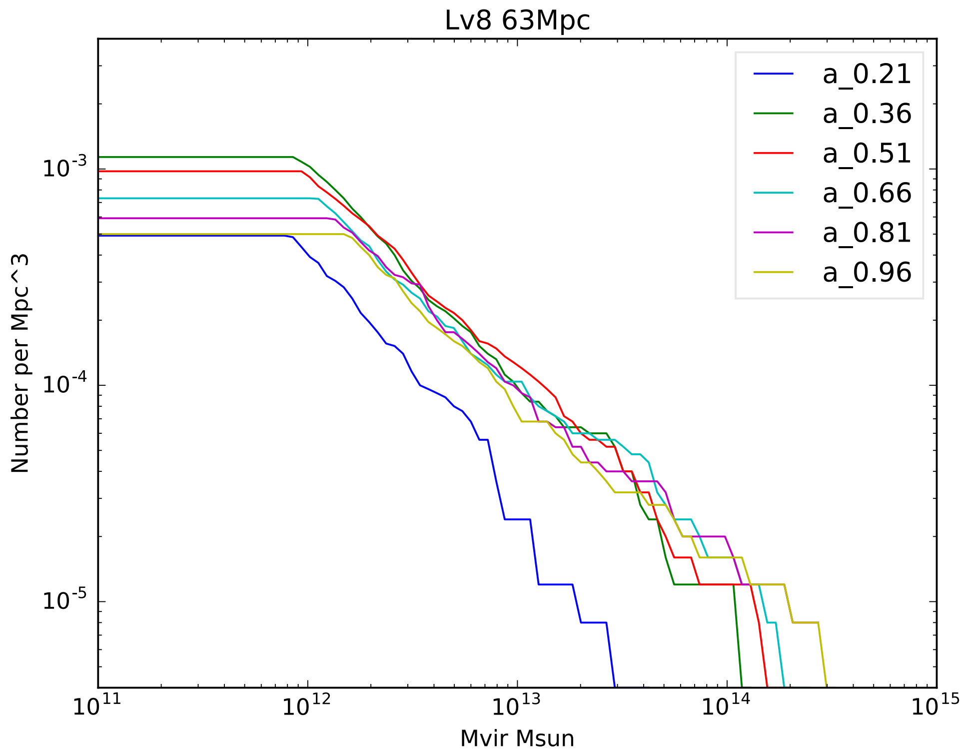




Halos are detected using ROCKSTAR (Behroozi et. al. 2013)

HALO MASS FUNCTION





OTHER SCHEMES

- Tree Code:

  Gridless, use multipole expansion to calculation gravitational force

- Adaptive Mesh Refinement Method (AMR, e.g., ENZO):

  Grid elements are concentrated where a higher resolution is needed

- P^3M CODE:

  PM+ direct summation over neighboring particles

- Adaptive Meshes P^3M (AP^3M)
- Hybrid PM-Tree Scheme (GADGET2)

REFERENCES

- KLYPIN, A & HOLTZMAN 1997
- HTTP://ASTRO.UCHICAGO.EDU/~ANDREY/TALKS/PM/PM.PDF
- JING 2005
- PETER S. BEHROOZI, RISA H. WECHSLER, HAO-YI WU 2013
- DOI:10.4249/SCHOLARPEDIA.3930