# Primordial Non-Gaussianity

**Sam Passaglia**[1]

[1]University of Chicago KICP

# In This Discussion

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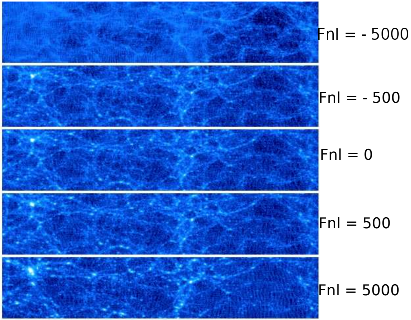
# Non-Gaussianity in Single-Field Slow-Roll

# Gaussian and Non-Gaussian Curvature Perturbations

- Inflation produces curvature perturbations $\zeta$. Are they Gaussian or non-Gaussian?
- Local (Quadratic) Ansatz: $\zeta = \zeta_G + 3/5 f_{\mathsf{NL}}(\zeta_G^2 - \langle\zeta_G^2\rangle)$
- 'For canonical single field slow roll inflation $f_{\mathsf{NL}}$ is of order $n_s - 1$, simply understood from separate universe perspective' - Wayne Hu




(Dalal et al 2008)

# Single Field Consistency Relation (following de Putter et al. 2015)

- $\mathrm{d}s^2 = a^2(\tau)[-\mathrm{d}\tau^2 + e^{2\zeta(\tau,\mathbf{x})}\,\mathrm{d}\mathbf{x}^2]$
  For superhorizon modes in Unitary Gauge (Equifield Surfaces set a 'clock')

- $\zeta = \zeta_{\text{short}} + \zeta_{\text{long}}$

- Locally, can't know about long-wavelength mode.

- In appropriate coordinate system:
  $\mathrm{d}\tilde{s}^2 = a^2(\tau)\Big[-\mathrm{d}\tau^2 + e^{2\tilde{\zeta}_s(\tau,\tilde{\mathbf{x}})}\,\mathrm{d}\tilde{\mathbf{x}}^2\Big]$

- $\mathrm{d}s^2 = \mathrm{d}\tilde{s}^2$

# Single Field Consistency Relation

- Apply a spatial dilation with $\zeta_l$: $\mathbf{x} \equiv \tilde{\mathbf{x}}(1-\zeta_l)$
  (We can move freely from $\zeta_l$ / $\zeta_l(\mathbf{x})$ / $\zeta_l(\tilde{\mathbf{x}})$)

- $\mathrm{d}\tilde{s}^2 = a^2(\tau) e^{2\tilde{\zeta}_s(\tau,\tilde{\mathbf{x}})}\,\mathrm{d}\tilde{\mathbf{x}}^2 = a^2(\tau) e^{2\tilde{\zeta}_s(\tau,\tilde{\mathbf{x}})+2\zeta_l}\,\mathrm{d}\mathbf{x}^2$
  So spatial dilations generate large-scale curvature.

- $\zeta_s(\mathbf{x}) = \tilde{\zeta}_s(\tilde{\mathbf{x}}) = \tilde{\zeta}_s(\mathbf{x}(1+\zeta_l)) = (1+\zeta_l(\mathbf{x}\cdot\nabla))\tilde{\zeta}_s$

- In terms of long/short split, ansatz is $\zeta_s = (1 + \frac{6}{5}\zeta_l f_{\mathsf{NL}})\zeta_s^G$

$$\implies f_{\mathsf{NL}} = \frac{5}{6}\,\mathbf{x}\cdot\nabla \sim -\frac{5}{6}\frac{\mathrm{d}\ln\sqrt{\Delta_\zeta^2(k)}}{\mathrm{d}\ln k} = \boxed{-\frac{5}{12}(n_s-1)}$$

# Shapes of Non-Gaussianity

- This was 'squeezed' limit.
- $\langle\zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle$ characterizes non-gaussianity
- Function of 3*3 - 3 (momentum conservation) - 3 (rotational invariance) - 1 (scaling invariance) = 2 variables = leg ratios
- We just computed $k_2/k_3 << k_1/k_2$: i.e long modes interacting with short mode
- $f_{\text{NL}}^{\text{Equil.}} \sim \mathcal{O}(1)$

# Non-Gaussianity in the EFT of Inflation

# EFT Idea (Following Cheung et al. 2008 + Wayne's Slides)

- Write down most general $\mathcal{L}$ consistent with symmetries of the problem
- Inflation: Want $\mathcal{L}$ with broken time diffs describing perturbations around flat FRW
- $\mathcal{L}(R_{\mu\nu\sigma\rho}, g^{00}, K_{\mu\nu}, \nabla_\mu, t)$
- Lowest Order Operators should be most important

# Unitary Gauge Action

$$S = \int \mathrm{d}^4x\sqrt{-g}\Big[\frac{1}{2}M_{\mathsf{Pl}}^2 R + \sum_{n=0}^{\infty}\frac{1}{n!}M_n^4(t)(g^{00}+1)^n$$
$$-\frac{1}{2}\bar{M}_1(t)^3(g^{00}+1)\delta K_\mu^\mu$$
$$-\frac{1}{2}\bar{M}_2(t)^2(\delta K_\mu^\mu)^2 - \frac{1}{2}\bar{M}_3(t)^2\delta K_\nu^\mu \delta K_\mu^\nu + ...\Big]$$

- (From now on neglect the extrinsic curvature terms)
- $M_0$ and $M_1$ terms are linear in perturbations: Fixed by requiring the background to be FRW
- Not diff invariant: Time symmetry broken by design
- Theory has 3 d.o.f. : 2 graviton helicities + 1 scalar mode

# Stueckelberg Trick: Make the Scalar d.o.f into a Scalar Field

- Perform a time-diffeomorphism $t \to \tilde{t} = t + \xi(t, x^i)$ (i.e. a change in time-slicing)
- See how the action changes: This is the scalar d.o.f.
- Promote the parameter $\xi(t, x^i)$ to a scalar field $\pi(t, x^i)$ (with a slightly strange transformation rule $\pi(x) \to \pi(x) - \xi^0(x)$).
- We will have a diff invariant theory with the symmetries/background/perturbations we want

# How does the Action change?

- $t \to t+\pi$ ; $g^{00} \to \frac{\delta(t+\pi)}{\delta x^{\mu}} \frac{\delta(t+\pi)}{\delta x^{\nu}} g^{\mu\nu}$; $M_n^4(t) \to M_n^4(t+\pi)$

- In general, many time-space mixing terms in action like: $\delta_i \pi g^{0i}, \dot{\pi} g^{00}, \ldots$

- If we gauge-fix by choosing spatially flat gauge, can neglect all the spatial metric perturbations
  Justifies neglecting extrinsic curvature terms: their contributions will be... fourth order?

- See Wayne's notes for a better explanation!

# Goldstone Action in Spatially Flat Gauge to Cubic Order

$$S_\pi = \int \mathrm{d}^4x\sqrt{-g}\Big[(M_{\mathsf{Pl}}^2\epsilon_H H^2 + 2M_2^4)\dot{\pi}^2 - M_{\mathsf{Pl}}^2\epsilon_H H^2\frac{(\delta_i\pi)^2}{a^2}$$
$$+ 2M_2^4(\dot{\pi}^3 - \dot{\pi}\frac{(\delta_i\pi)^2}{a^2}) - \frac{4}{3}M_3^4\dot{\pi}^3$$
$$+ \text{fourth order}\Big]$$

- Recall $\epsilon_H = -\dot{H}/H^2$
- Removed the background-fixed terms

# Effective Sound Speed of $\pi$

- $c_s^{-2} = 1 + \frac{2M_2^4}{M_{Pl}^2 \epsilon_H H^2}$
  temporal/spacial coefficients at quadratic order

- Because $M_2$ is both in cubic and quadratic terms in Lagrangian expect non-gaussianity if $c_s << 1$

- $\frac{\dot{\pi}(\delta_i \pi)^2}{a^2 \dot{\pi}^2} \sim \frac{k \pi_{\mathsf{rms}}}{c_s a}$

- ($\pi$ related to curvature fluctuations by $\zeta = -H\pi$ in single-field slow-roll)

$$\boxed{f_{\mathsf{NL}}^{\mathsf{equil}} \sim \frac{1}{c_s^2}}$$

# Observational Constraints

# From CMB

- Measure from 3-point functions of T and E
  $f_{\mathsf{NL}}^{\mathsf{local}} = 0.8 \pm 5.0$; $f_{\mathsf{NL}}^{\mathsf{equil}} = 4 \pm 43$; $f_{\mathsf{NL}}^{\mathsf{ortho}} = 26 \pm 21$
- But most interesting parameter range is $f_{\mathsf{NL}} \sim \mathcal{O}(1)$
- How can we reach this? S4?

| Type | *Planck* actual (forecast) | CMB-S4 | CMB-S4 + low-$\ell$ *Planck* | Rel. improvement |
|---|---|---|---|---|
| Local | $\sigma(f_{\rm NL}) = 5\,(4.5)$ | $\sigma(f_{\rm NL}) = 2.6$ | $\sigma(f_{\rm NL}) = 1.8$ | 2.5 |
| Equilateral | $\sigma(f_{\rm NL}) = 43\,(45.2)$ | $\sigma(f_{\rm NL}) = 21.2$ | $\sigma(f_{\rm NL}) = 21.2$ | 2.1 |
| Orthogonal | $\sigma(f_{\rm NL}) = 21\,(21.9)$ | $\sigma(f_{\rm NL}) = 9.2$ | $\sigma(f_{\rm NL}) = 9.1$ | 2.4 |

(S4 science book)

# From Large Scale Structure?

- Multi-field models often give a scale dependent halo bias $b(k) \propto \frac{f_{\mathrm{NL}}^{squeeze}}{k^2}$ (Dalal et al. 2007)

- From 800,000 photometric quasars (Leistedt 2014): $-49 < f_{\mathrm{NL}}^{\text{squeeze}} < 31$

- LSST forecasts:
  Systematics free $\sigma(f_{\mathrm{NL}}) \sim \mathcal{O}(1)$.
  Systematics-full $\sigma(f_{\mathrm{NL}}) \sim \mathcal{O}(30)$.
  Systematics-cleaned? $\sigma(f_{\mathrm{NL}}) \sim \mathcal{O}(5)$.

- Is it worth doing the analysis for DES - test bed for LSST?

# Conclusion

- Non-gaussianity is a powerful probe of inflation
- EFT language makes it clear $f_{NL}$ related to effective sound speed of inflation
- On the verge of an interesting parameter space

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