Primordial Non-Gaussianity

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In This Discussion

Non-Gaussianity in Single-Field Slow-Roll

Non-Gaussianity in the EFT of Inflation

Observational Constraints
Non-Gaussianity in Single-Field Slow-Roll
Gaussian and Non-Gaussian Curvature Perturbations

- Inflation produces curvature perturbations $\zeta$. Are they Gaussian or non-Gaussian?

- Local (Quadratic) Ansatz:
  $$\zeta = \zeta_G + \frac{3}{5} f_{NL} (\zeta_G^2 - \langle \zeta_G^2 \rangle)$$

- ‘For canonical single field slow roll inflation $f_{NL}$ is of order $n_s - 1$, simply understood from separate universe perspective’ - Wayne Hu

(Dalal et al 2008)
Single Field Consistency Relation (following de Putter et al. 2015)

\[ ds^2 = a^2(\tau)[-d\tau^2 + e^{2\zeta(\tau,x)} \, dx^2] \]

For superhorizon modes in Unitary Gauge (Equifield Surfaces set a ‘clock’)

\[ \zeta = \zeta_{\text{short}} + \zeta_{\text{long}} \]

Locally, can’t know about long-wavelength mode.

In appropriate coordinate system:

\[ d\tilde{s}^2 = a^2(\tau)\left[ -d\tau^2 + e^{2\tilde{\zeta}(\tau,\tilde{x})} \, d\tilde{x}^2 \right] \]

\[ ds^2 = d\tilde{s}^2 \]
Apply a spatial dilation with $\zeta_l$: $x \equiv \tilde{x}(1 - \zeta_l)$
(We can move freely from $\zeta_l / \zeta_l(x) / \zeta_l(\tilde{x})$)

$$\text{d}\tilde{s}^2 = a^2(\tau)e^{2}\tilde{\zeta}_s(\tau,\tilde{x}) \text{ d}\tilde{x}^2 = a^2(\tau)e^{2\tilde{\zeta}_s(\tau,\tilde{x})+2\zeta_l} \text{ d}x^2$$

So spatial dilations generate large-scale curvature.

$$\tilde{\zeta}_s(x) = \tilde{\zeta}_s(\tilde{x}) = \tilde{\zeta}_s(x(1 + \zeta_l)) = (1 + \zeta_l(x \cdot \nabla))\tilde{\zeta}_s$$

In terms of long/short split, ansatz is $\zeta_s = (1 + \frac{6}{5}\zeta_l f_{NL})\zeta_s^G$

$$\Rightarrow f_{NL} = \frac{5}{6}x \cdot \nabla \sim -\frac{5}{6} \frac{\text{d} \ln \sqrt{\Delta^2(k)}}{\text{d} \ln k} = -\frac{5}{12}(n_s - 1)$$
Shapes of Non-Gaussianity

- This was ‘squeezed’ limit.

- \( \langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle \) characterizes non-gaussianity

- Function of 3*3 - 3 (momentum conservation) - 3 (rotational invariance) - 1 (scaling invariance) = 2 variables = leg ratios

- We just computed \( k_2/k_3 << k_1/k_2 \): i.e long modes interacting with short mode

- \( f_{\text{Equil.}}^{\text{NL}} \sim \mathcal{O}(1) \)
Non-Gaussianity in the EFT of Inflation
EFT Idea (Following Cheung et al. 2008 + Wayne’s Slides)

- Write down most general $\mathcal{L}$ consistent with symmetries of the problem

- Inflation: Want $\mathcal{L}$ with broken time diffs describing perturbations around flat FRW

  $\mathcal{L}(\mathcal{R}_{\mu\nu\sigma\rho}, g^{00}, K_{\mu\nu}, \nabla_{\mu}, t)$

- Lowest Order Operators should be most important
Unitary Gauge Action

\[ S = \int d^4 x \sqrt{-g} \left[ \frac{1}{2} M_{Pl}^2 R + \sum_{n=0}^{\infty} \frac{1}{n!} M_n^4(t)(g^{00} + 1)^n ight. \\
- \frac{1}{2} \bar{M}_1(t)^3 (g^{00} + 1) \delta K_\mu^\mu \\
- \frac{1}{2} \bar{M}_2(t)^2 (\delta K_\mu^\mu)^2 - \frac{1}{2} \bar{M}_3(t)^2 \delta K_\nu^\mu \delta K_\mu^\nu + \ldots \] 

- (From now on neglect the extrinsic curvature terms)
- \( M_0 \) and \( M_1 \) terms are linear in perturbations: Fixed by requiring the background to be FRW
- Not diff invariant: Time symmetry broken by design
- Theory has 3 d.o.f.: 2 graviton helicities + 1 scalar mode
The Stueckelberg Trick: Make the Scalar d.o.f into a Scalar Field

- Perform a time-diffeomorphism $t \rightarrow \tilde{t} = t + \xi(t, x^i)$ (i.e. a change in time-slicing).

- See how the action changes: This is the scalar d.o.f.

- Promote the parameter $\xi(t, x^i)$ to a scalar field $\pi(t, x^i)$ (with a slightly strange transformation rule $\pi(x) \rightarrow \pi(x) - \xi^0(x)$).

- We will have a diff invariant theory with the symmetries/background/perturbations we want.
How does the Action change?

- \( t \rightarrow t + \pi \); \( g^{00} \rightarrow \frac{\delta(t+\pi)}{\delta x^\mu} \frac{\delta(t+\pi)}{\delta x^\nu} g^{\mu\nu} \); \( M_n^4(t) \rightarrow M_n^4(t + \pi) \)

- In general, many time-space mixing terms in action like:
  \( \delta_i \pi g^{0i} \), \( \dot{\pi} g^{00} \), ...

- If we gauge-fix by choosing spatially flat gauge, can neglect all the spatial metric perturbations
  Justifies neglecting extrinsic curvature terms: their contributions will be... fourth order?

- See Wayne’s notes for a better explanation!
Goldstone Action in Spatially Flat Gauge to Cubic Order

\[ S_\pi = \int d^4x \sqrt{-g} \left[ (M^2_{\text{Pl}} \epsilon_H H^2 + 2M_2^4) \dot{\pi}^2 - M^2_{\text{Pl}} \epsilon_H H^2 \frac{(\delta_i \pi)^2}{a^2} ight. \\
\left. + 2M_2^4 (\dot{\pi}^3 - \dot{\pi} \frac{(\delta_i \pi)^2}{a^2}) - \frac{4}{3} M_3^4 \ddot{\pi}^3 \right] + \text{fourth order} \]

- Recall \( \epsilon_H = -\frac{\dot{H}}{H^2} \)
- Removed the background-fixed terms
Effective Sound Speed of $\pi$

- $c_s^{-2} = 1 + \frac{2M_2^4}{M_{Pl}^2 \epsilon_H H^2}$
  temporal/spacial coefficients at quadratic order

- Because $M_2$ is both in cubic and quadratic terms in Lagrangian expect non-gaussianity if $c_s << 1$

- $\frac{\dot{\pi}(\delta_i \pi)^2}{a^2 \dot{\pi}^2} \sim \frac{k \pi_{\text{rms}}}{c_s a}$

- ($\pi$ related to curvature fluctuations by $\zeta = -H \pi$ in single-field slow-roll)

$$f_{\text{NL}}^{\text{equil}} \sim \frac{1}{c_s^2}$$
Observational Constraints
From CMB

- Measure from 3-point functions of T and E
  \[ f_{\text{local}}^{NL} = 0.8 \pm 5.0; \quad f_{\text{equil}}^{NL} = 4 \pm 43; \quad f_{\text{ortho}}^{NL} = 26 \pm 21 \]

- But most interesting parameter range is \( f_{\text{NL}} \sim \mathcal{O}(1) \)

- How can we reach this? S4?

<table>
<thead>
<tr>
<th>Type</th>
<th>Planck actual (forecast)</th>
<th>CMB-S4</th>
<th>CMB-S4 + low-(\ell) Planck</th>
<th>Rel. improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local</td>
<td>(\sigma(f_{\text{NL}}) = 5 (4.5))</td>
<td>(\sigma(f_{\text{NL}}) = 2.6)</td>
<td>(\sigma(f_{\text{NL}}) = 1.8)</td>
<td>2.5</td>
</tr>
<tr>
<td>Equilateral</td>
<td>(\sigma(f_{\text{NL}}) = 43 (45.2))</td>
<td>(\sigma(f_{\text{NL}}) = 21.2)</td>
<td>(\sigma(f_{\text{NL}}) = 21.2)</td>
<td>2.1</td>
</tr>
<tr>
<td>Orthogonal</td>
<td>(\sigma(f_{\text{NL}}) = 21 (21.9))</td>
<td>(\sigma(f_{\text{NL}}) = 9.2)</td>
<td>(\sigma(f_{\text{NL}}) = 9.1)</td>
<td>2.4</td>
</tr>
</tbody>
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(S4 science book)
Multi-field models often give a scale dependent halo bias

\[ b(k) \propto \frac{f_{\text{squeeze}}^{\text{NL}}}{k^2} \]  

(Dalal et al. 2007)

From 800,000 photometric quasars (Leistedt 2014):

\[-49 < f_{\text{squeeze}}^{\text{NL}} < 31\]

LSST forecasts:

Systematics free \( \sigma(f_{\text{NL}}) \sim O(1) \).

Systematics-full \( \sigma(f_{\text{NL}}) \sim O(30) \).

Systematics-cleaned? \( \sigma(f_{\text{NL}}) \sim O(5) \).

Is it worth doing the analysis for DES - test bed for LSST?
Conclusion

- Non-gaussianity is a powerful probe of inflation
- EFT language makes it clear $f_{NL}$ related to effective sound speed of inflation
- On the verge of an interesting parameter space
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