On Separate Universes:

Real & Fake

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(Heavily) Based on:

Arxiv: 1605.01412
1609.01701

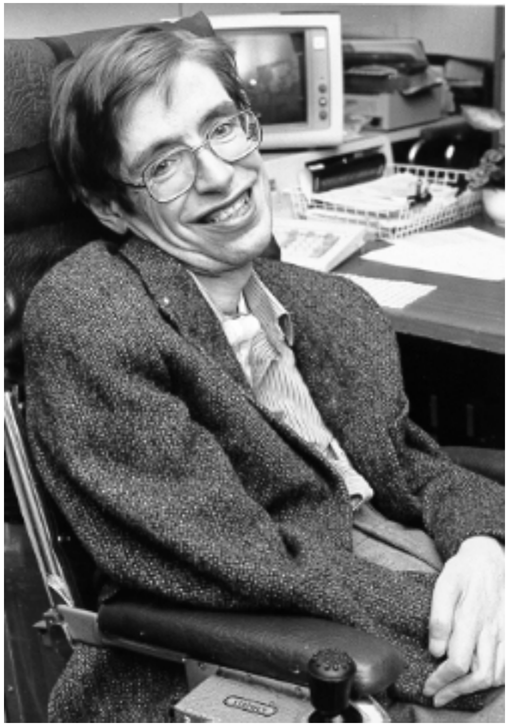
Also see:
Arxiv: 1504.00351
1409.6294
1405.3624

The separate universe assumption

"In GR a density perturbation behaves locally (on scales much smaller than the wavelength of the mode) as a separate universe with different background density and curvature"

(Dai L., Pajer E., Schmidt F., 2015)

Not a new idea...



Mon. Not. R. astr. Soc. (1974) **168,** 399–415.

BLACK HOLES IN THE EARLY UNIVERSE

B. J. Carr and S. W. Hawking

(Received 1974 February 25)

expansion is zero. If this positive curvature extended over a sufficiently large region, the spacelike hypersurface would close up on itself to form a disconnected compact 3-space of radius about $\mu^{-1/2}$. In this case the region would form a separate closed universe which was completely disconnected from our Universe. Such a situation

Why do we care?

Impact of a long-wavelength cosmological perturbation on small scale observables → change in the background cosmology

Inflationary context: (see Sam's talk)

1. Squeezed N-point functions
2. Evolution of isocurvature perturbations
3. …….. (see references)

Late universe context:
Impact of long wavelength perts on:

The power spectrum (position dependence)
Halo Bias
Model effects into nonlinear regime

Why do I care?

Effects of long wavelength matter pert. into the measured dark energy eq. of state? (still thinking on that – vague idea)

Real & Fake SU

Consider the multi-component case – not only gravitational forces – introduction of pressure and a **Jeans scale**

Question: We said (assumption) that we can always absorb δ → change in the expansion rate.

Evolution of the expansion rate described by Friedmann equations?

If δ larger than the total Jeans scale answer is **YES**

↓

Real separate universe

If δ shorter that total Jeans scale answer **NO** → Introduction of fictitious components → Effective F.E.

↓

Fake separate universe

Formalism

Local Expansion and Density

Matter perturbation: $\delta = \delta\rho_m/\overline{\rho}_m$

Absorb into the background of a SU:

$$\overline{\rho}_{m,W}(a) = \overline{\rho}(a)[1 + \delta(a)]$$

Introduce SU scale factor: $\rho_{mW} \propto a_W^{-3}$

Defines matter density parameters:

$$\frac{\Omega_{mW} h_W^2}{a_W^3} = \frac{\Omega_m h^2}{a^3}(1 + \delta)$$

"Initial condition":

$$\lim_{a\to 0} a_W(a) = a, \quad \lim_{a\to 0} \delta(a) = 0$$

This gives:

$$\Omega_{mW} h_W^2 = \Omega_m h^2$$

Scale factors differ:

$$a_W = \frac{a}{(1+\delta)^{1/3}} \cong a\left(1 - \frac{\delta}{3}\right)$$

And thus **expansion rates**:

$$\delta H^2 = H_W^2 - H^2 \cong -\frac{2}{3} H^2 \delta'$$

$$(' \equiv d/d\ln a)$$

Relativistic Formulation

Universal time → **Synchronous Gauge** $g_{00} = -1$

00 and trace *ii* **Einstein equations:** (see lecture notes #3)

$$-\frac{k^2 - 3K}{(aH)^2}\eta_T + \frac{1}{2}h'_L = \frac{4\pi G}{H^2}\sum_J \delta\rho_J$$

$$h''_L + \left(2 + \frac{H'}{H}\right)h'_L = -\frac{8\pi G}{H^2}\sum_J(\delta\rho_J + 3\delta p_J)$$

Also continuity & Navier-Stokes

$$\delta\rho'_J + 3(\delta\rho_J + \delta p_J) = -\frac{k\overline{\rho}_J}{aH}u_J - \frac{\overline{\rho}_J + \overline{p}_J}{2}h'_L$$

$$\overline{\rho}_J u'_J + (\overline{\rho}_J - 3\overline{p}_j)u_J = \frac{k}{aH}\left[\delta p_J - \frac{2}{3}\left(1 - \frac{3K}{k^2}\right)p_J u_J\right]$$

In order for the separate universe construction to hold **exactly**, the previous perturbation equations should be able to **reabsorded** into the (background) **Friedmann:**

$$H^2 + \frac{K}{a^2} = \sum_J \frac{8\pi G}{3}\overline{\rho}_J$$

Acceleration:

$$H^2 + \frac{1}{2}\frac{dH^2}{d\ln a} = -\frac{4\pi G}{3}\sum_J(\overline{\rho}_J + 3\overline{p}_J)$$

and **continuity** equation:

$$\overline{\rho}'_J + 3(\overline{\rho}_J + \overline{p}_J) = 0$$

Perturb them!

Matter continuity gives: $\delta' = -\frac{1}{2}h'_L = -3\frac{\delta H}{H}$

Conversion of derivatives: $\frac{d}{d\ln a_W} \cong \left(1 - \frac{\delta H}{H}\right)\frac{d}{d\ln a}$

Perturb acceleration:

$$\left(\frac{\delta H}{H}\right)' + \left(2 + \frac{H'}{H}\right)\frac{\delta H}{H} = -\frac{4\pi G}{3H^2}\sum_J (\delta\rho_J + 3\delta p_J)$$ ✔

Perturb Friedman: $$\delta H^2 + \frac{\delta K}{a^2} = \sum_J \frac{8\pi G}{3}\delta\rho_J$$ ✔

But: Curvature must be constant !!

Perturb conserv. equation: $\delta\rho'_J + 3(\delta\rho_J + \delta p_J) + 3\frac{\delta H}{H}(\bar\rho_J + \bar p_J) = 0$ ✔

If $u_J \cong 0$

To match equations (SU to be valid)
we need **negligible non-gravitational flows**
(from continuity eq.) and and from the Friedmann equation the **curvature has to be constant.**

In fact these are the same criteria
Analytical treatment shows that the critical scale is the **Jeans scale**

But what is the difference?

Matching the Universes

(or how to absorb the perturbations)

Background Universe:

$$\frac{H^2}{H_0^2} = \frac{\Omega_m}{a^3} + \Omega_Q F_Q(a) + \frac{\Omega_K}{a^2}$$

Separate Universe:

$$\frac{H_W^2}{H_{0W}^2} = \frac{\Omega_{mW}}{a_W^3} + \Omega_{QW} F_Q(a_W) + \frac{\Omega_{KW}}{a_W^2} + \Omega_{SW} F_S(a_W)$$

$$\frac{dF_i}{d\ln a} = -3(1+w_i)$$

More calculations....

$$\frac{\delta\Omega_m}{\Omega_m} = \frac{\delta\Omega_Q}{\Omega_Q} = -2\frac{\delta h}{h}$$

$$\Omega_{SW} + \Omega_{KW} = 1 - \Omega_{mW} - \Omega_{QW}$$

Real & Fake SU revisited

Do Ω_{SW} and Ω_{KW} truly represent an energy density and curvature (respectively) in the local universe?

In a real separate Universe the answer is YES!

Ω_{SW} It comes from an entropy perturbation

In a fake separate Universe the answer is NO!

Ω_{SW} compensates for non-conservation of curvature

Example: Scalar Field DE

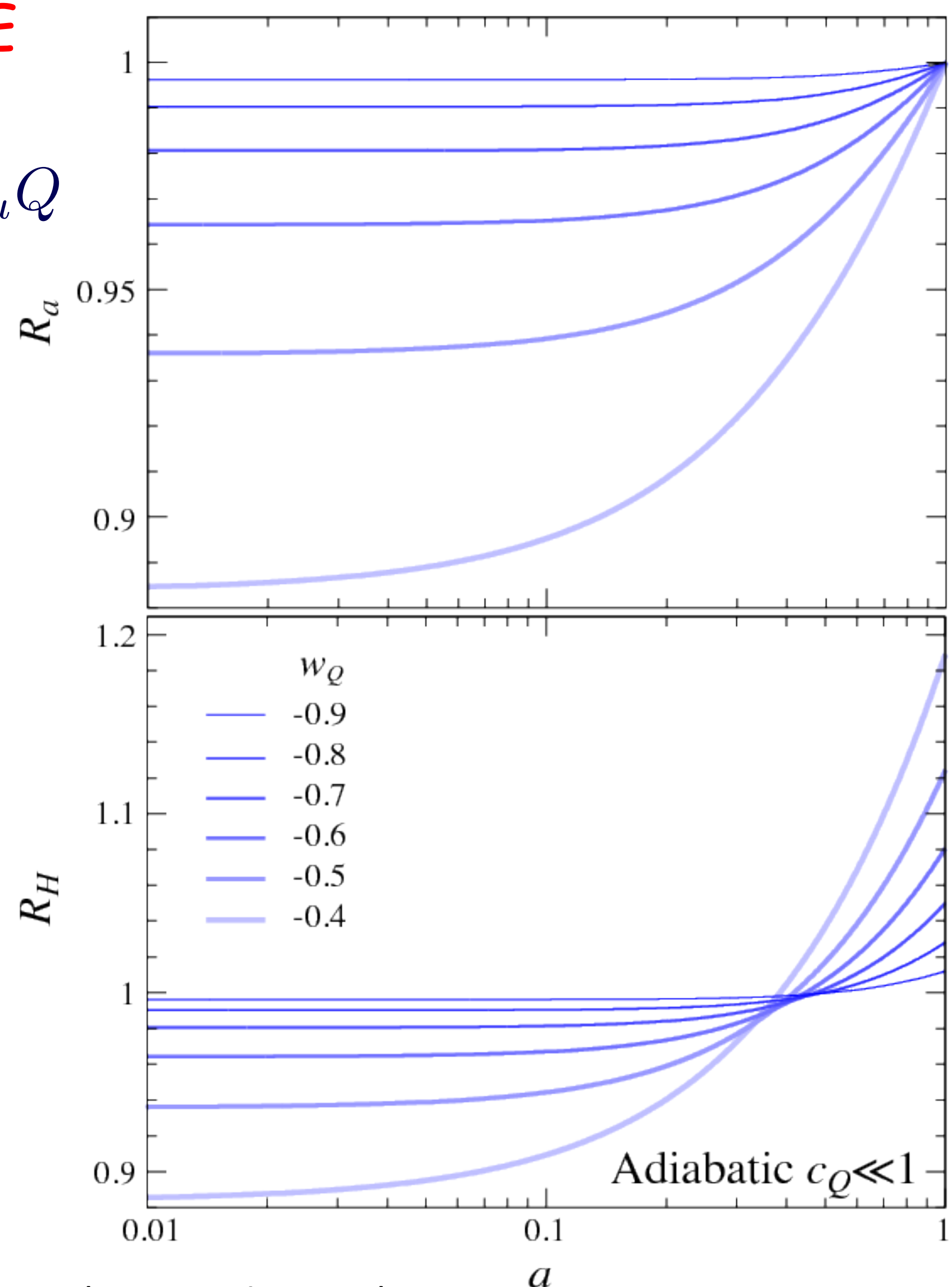
$$\mathcal{L} = P(X, Q), \quad X = -\frac{1}{2}\nabla^\mu Q \nabla_\mu Q$$

$$R_a \equiv \frac{\delta \ln a_+}{\delta \ln a_-}$$

$$R_H \equiv \frac{\delta \ln H_+}{\delta \ln H_-}$$

$$\delta \ln a = \ln a_W - \ln a$$

Etc…



(Hu et. al. 2016)

So why am I interested in that?

Is the measured eq of state of Dark energy the actual one?

A naive idea – from the simplistic sph. Collapse model

Let our observable / "local" universe overdence compared to the background universe.

Then $a_{loc} \neq a_{bu}$

Local sc. factor

Background/global scale factor

If DE homogeneous

True evolution of the DE

$$\rho_{de} \sim \frac{1}{a_{bu}^{3(1+w)}}$$

Apparent evolution of DE

$$\rho_{de} \sim \frac{1}{a_{loc}^{3(1+w')}}$$

This result shows that the transition to overcivility occurs between the values 2 and 3 given by Giftcourt (1956), respectively, Bookshelf (1956), a result which should be capable of direct experimental confirmation. The author hopes to deal with this problem next Saturday afternoon.

Thank You!!